UCLA Department of Electrical Engineering
EE101 – Engineering Electromagnetics
Winter 2012
Final Exam, March 20 2012, (3 hours)

Name ________________________________ Student number____________________________

This is a closed book exam – you are allowed 2 page of notes (each page front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit.

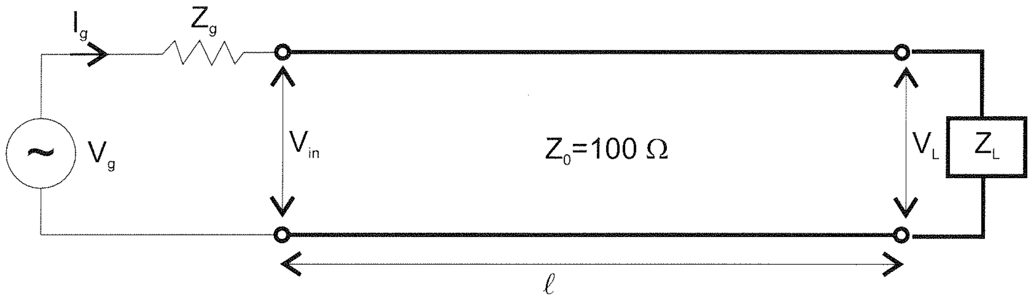
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Smith Chart	10	
Problem 2	Impedance Matching	30	
Problem 3	Vector calculus and phasors	30	
Problem 4	Parallel plate transmission line	18	
Problem 5	Plane wave	12	
Total		100	

1. Smith chart basics (10 points)

Consider the generic transmission line problem as shown below. Assume that the transmission line is a coaxial transmission line of filled with a material $\varepsilon=9\varepsilon_0$, $\mu=\mu_0$.



(a) (5 points) For each of the following loads, mark their position on the Smith chart below (using the letter as a label), and write below the reflection coefficient (magnitude and phase angle).

A: Z_L= 25 Ω. $\Gamma =$ $0.61 \angle 180^\circ = -0.61$

B: Z_L = 40-j200 Ω. $\Gamma =$ $0.85 \angle -52^\circ$

(b) (5 points) What is the input impedance of the transmission line $Z_{in}(-l)$ for each of the loads if l=1.25 m and f=10 MHz? Label each point on the Smith Chart using A', B'

A: Z_L= 25 Ω A' Z_{in} = $52 + j88\ \Omega$

$\cancel{Y_L} =$ $Z_{in} = 0.52 + j.88$

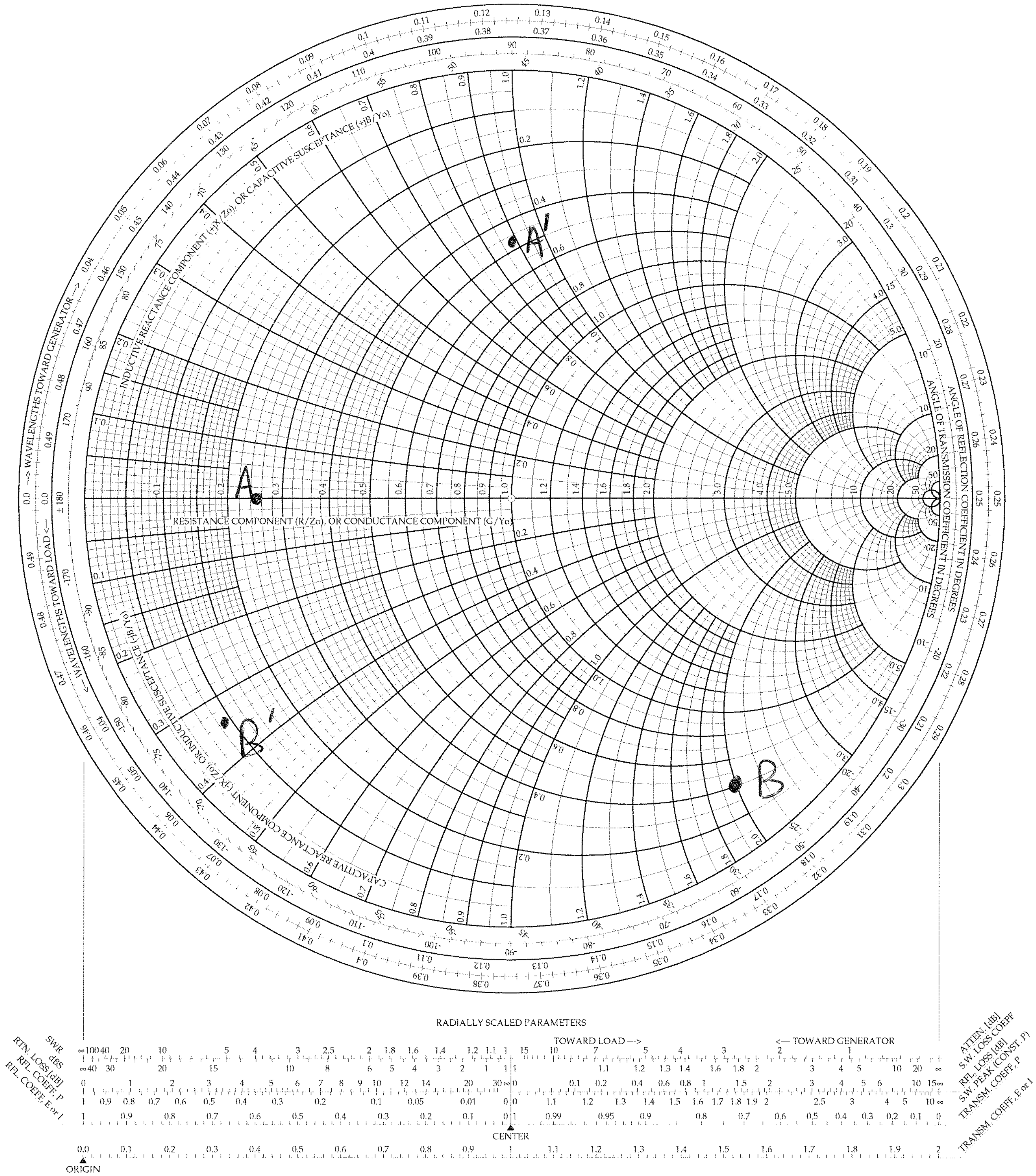
B: Z_L = 40-j200 Ω. B' Z_{in} = $90 - j35\ \Omega$

Phase velocity $u_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{3} = 10^8$ m/s

$\lambda = \frac{u_p}{f} = 10$m $\qquad l = \lambda/8 = 0.125\lambda$

($\pi/2$ rotation on circle)

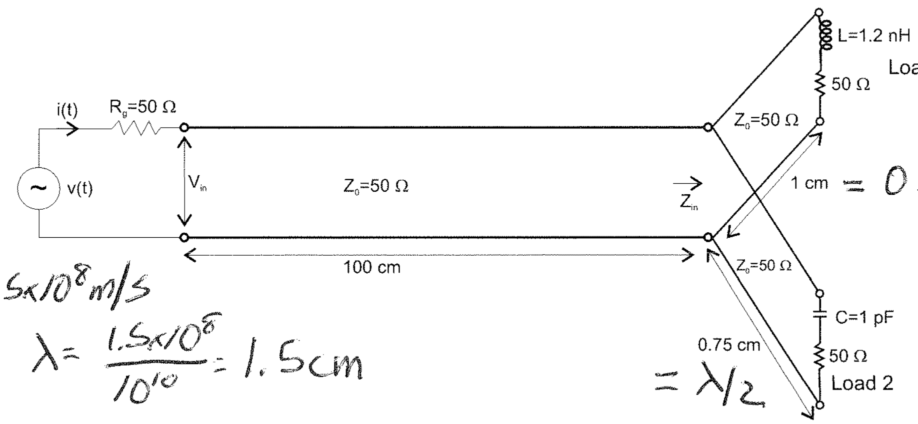
Smith chart for problem 1



2. **Transmission line – Impedance Matching (30 points)**

This problem involves a transmission line which is lossless coaxial cable filled with a dielectric material $\varepsilon=4\varepsilon_0$, $\mu=\mu_0$. The frequency of operation is f=10 GHz.

For these problems, you may use any methods you wish, including the Smith chart (not required).



$= 0.66\lambda$

$= \lambda/2$

$u_p = \frac{c}{2} = 1.5\times10^8 \text{ m/s}$

$f = 10^{10}$ Hz $\quad \lambda = \frac{1.5\times10^8}{10^{10}} = 1.5$ cm

(a) (points) At the frequency given above (f=10 GHz), what is the load impedance 1 (Z_{L1}) and load impedance 2 (Z_{L2})?

$f = 10$ GHz $\quad \omega = 2\pi f \approx 6.28\times10^{10} s^{-1}$

For C=1pF $\quad \omega C = 0.0628 \quad \frac{1}{\omega C} = 15.9$

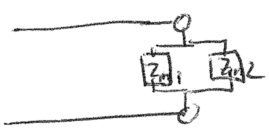
$\boxed{Z_{L2} = 50 - j16\ \Omega}$

For L=1.2nH $\quad \omega L = 75\Omega$

$\boxed{Z_{L1} = 50 + j75\ \Omega}$

(b) (points) What is the input impedance Z_{in} looking into the junction of the two lines?

$Z_{in}^{-1} = Z_{in,1}^{-1} + Z_{in,2}^{-1}$



$Y_{in} = Y_{in1} + Y_{in2}$

See Smith Chart $\quad z_{L2} = 1 - 0.32j \quad z_{L1} = 1 + j1.5$

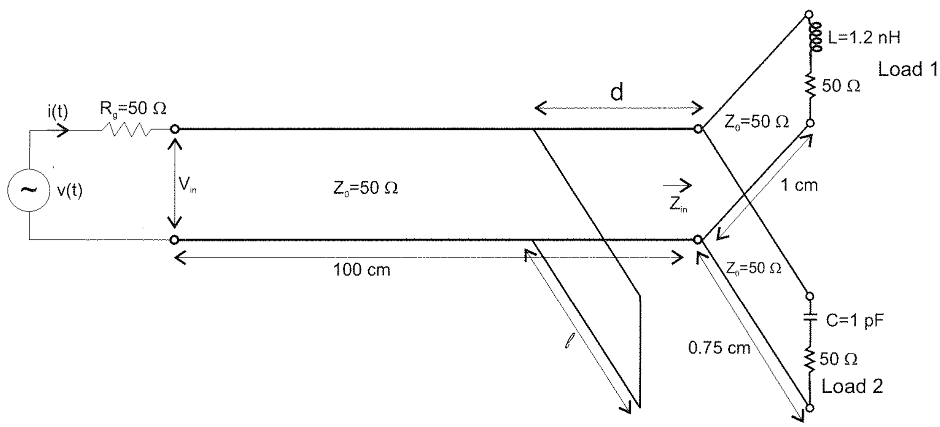
$y_{L2} = 0.9 + j0.3 \quad y_{L1} = 0.3 - j0.46$

since $l_2 = \frac{\lambda}{2}$ $\quad y_{in2} = 0.9 + j0.3 \quad y_{in1} = 0.33 + j0.57$

$y_{in} = y_{in1} + y_{in2} = 1.23 + j0.87$

$z_{in} = 0.54 - j0.38$

$\boxed{Z_{in} = 27 - j19\ \Omega}$



(c) (points) Consider the using a shorted stub as shown to prevent any reflections back on the feedline and into the generator. What values of *d* and *l* should be chosen to achieve matching to the line? Give these values in term of both wavelengths~~, and in meters.~~

Start with $y_{in} = 1.23 + j0.87$

1st choose d such that $y_{in}(-d) = 1$

Solution 1

$\boxed{d = 0.167\lambda}$

$y_{in}(-d) = 1 - j0.8$

choose l such that $y_{in}(-l) = +j0.8$

$\boxed{l = 0.357\lambda}$

Solution 2

$\boxed{d = 0.478\lambda}$

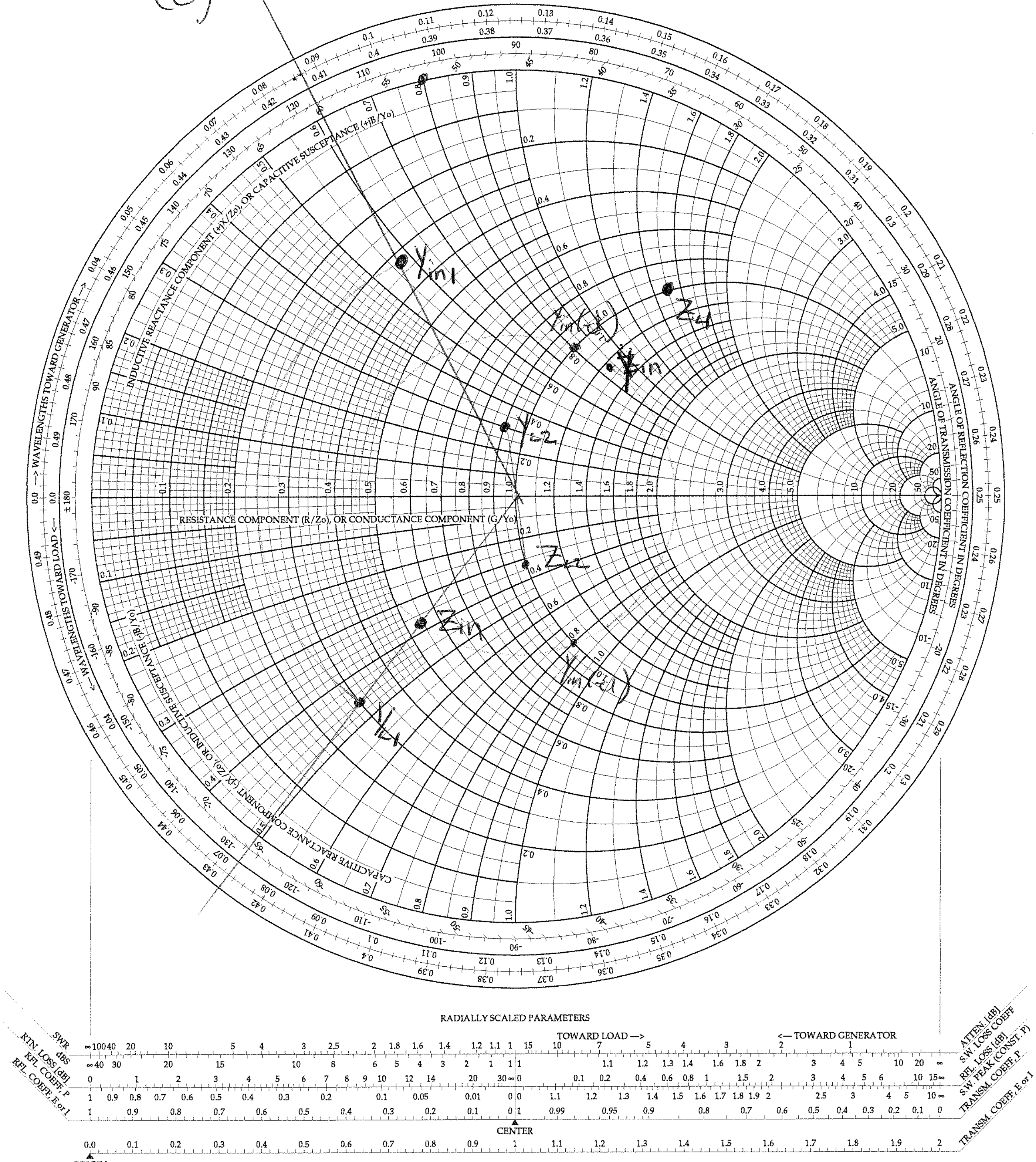
$y_{in}(-d) = 1 + j0.8$

$\boxed{l = 0.143\lambda}$

The Complete Smith Chart

Black Magic Design

Part (b)
(c)



3. Vector Calculus and phasors (30 points)

(a) Write the following phasor quantities in the time domain. (Do not include the expression Re{} in your answer). Assume E_0, H_0, and A are real quantities.

i. $\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}} E_0 e^{jkz}$ $\quad$ $\mathbf{E}(z,t) = ?$ $\quad E_0 \cos(\omega t - kz)\,\hat{x}$

$\mathrm{Re}\{E_0 \, e^{j(\omega t - kz)}\}$

ii. $\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}} j H_0 e^{jkz}$ $\quad$ $\mathbf{H}(z,t)=?$ $\quad -H_0 \sin(\omega t - kz)\,\hat{y}$

$H_0 \,\mathrm{Re}\{ j\cos(kz - \omega t) \mp \sin(kz - \omega t)\}$

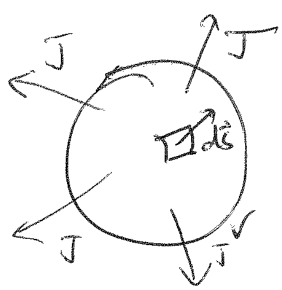
iii. $\tilde{F} = 3A(1+j)$ $\quad$ $F(t)=?$ $\quad 3A\sqrt{2}\cos(\omega t + \phi)$

where $\phi = 45°$ or $\pi/4$ rad

$\mathrm{Re}\left\{ 3A\sqrt{2}\,\frac{(1+j)}{\sqrt{2}}\, e^{j\omega t}\right\}$

$= \mathrm{Re}\{3A\sqrt{2}\, e^{j\pi/4} e^{j\omega t}\} =$

(b) Consider the current continuity equation: $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$. Apply this equation to a volume V with a surface defined by differential elements $d\mathbf{S}$, and rewrite this equation in integral form. Give a physical explanation.



Apply to a volume V with closed surface $d\vec{S}$ using divergence theorem

$$\int \nabla \cdot \mathbf{J}\, dV = \int -\frac{\partial \rho}{\partial t}\, dV$$

$$\oint_S \mathbf{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_V \rho \, dV$$

$$\boxed{I_{out,total} = -\frac{\partial}{\partial t} Q_{enclosed}}$$

The total net current that flows out of a closed surface is equal to the time rate of change of the enclosed charge.

(c) Consider the case of fields, charges, and currents that vary harmonically with angular frequency ω. Rewrite Maxwell's equations in phasor form using the phasors: $\tilde{\rho}, \tilde{\mathbf{J}}, \tilde{\mathbf{E}}, \tilde{\mathbf{H}}, \tilde{\mathbf{B}}, \tilde{\mathbf{D}}$.

(given in Equations)

$$\nabla \cdot \tilde{D} = \tilde{\rho} \qquad \nabla \cdot \tilde{B} = 0$$

$$\nabla \times \tilde{E} = -j\omega \tilde{B} \qquad \nabla \times \tilde{H} = \tilde{J} + j\omega \tilde{D}$$

(d) Prove that for harmonically varying fields, charges, and currents that: $\tilde{\rho} = 0$. in a medium that obeys Ohm's Law

Start with the current continuity equation in phasor form.

$$\nabla \cdot \tilde{J} = -j\omega \tilde{\rho}$$

Substitute Ohm's Law $\quad \tilde{J} = \sigma \tilde{E}$

$$\sigma \nabla \cdot \tilde{E} = -j\omega \tilde{\rho}$$

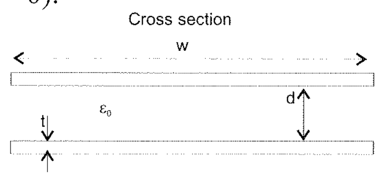
Substitute Gauss's Law for a homogeneous medium $\quad \nabla \cdot \tilde{E} = \tilde{\rho}/\varepsilon$

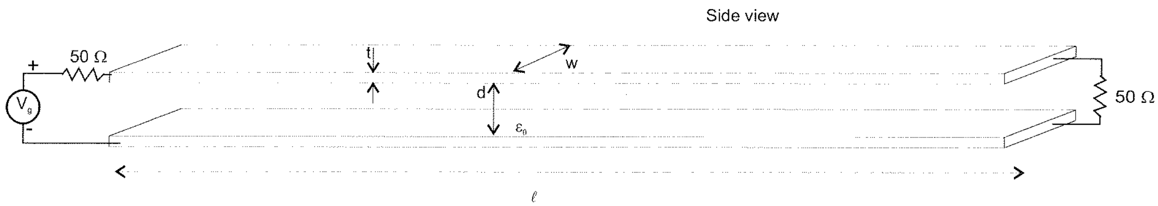
$$\boxed{\frac{\sigma}{\varepsilon}\tilde{\rho} = j\omega \tilde{\rho}}$$

The only way this equation can be true is if $\tilde{\rho} = 0$

4. Parallel plate transmission line (18 points)

Consider a parallel plate transmission line with dimensions width w=1mm, height d=0.1 mm, and metal thickness t=0.01 mm. It is made of copper with σ_c= 5.9x10^7 S/m and $\mu_c = \mu_0$. The plates are separated by air (i.e. $\epsilon = \epsilon_0, \mu = \mu_0$, and $\sigma = 0$).





(a) (6 points) What is the resistance per unit length R' for this transmission line at f=10 kHz? Give a number.

Calculate skin depth at 10 kHz: $\delta_s = 0.66$ mm which is much larger than metal thickness $t = 0.01$mm. Thus we assume current flows uniformly through metal cross section.

$$\boxed{R' = \frac{2}{\sigma_c w t} = 3.4\ \Omega/m}$$

(Factor of two from two conductors)

(b) (6 points) What is the resistance per unit length R' for this transmission line at f=10 GHz? Give a number.

At 10 GHz $\delta_s = 0.66\ \mu m \ll t$. Thus current flows only at surfaces (inner) of conductors.

$$\boxed{R' = \frac{2}{\sigma_c w \delta_s} = 51.4\ \Omega/m}$$

(c) (6 points) At which frequency (10 kHz or 10 GHz) is it a better approximation to consider this a "lossless" transmission line? Explain why.

A lossless TL has Z_o real
and a propagation constant β = real
$\alpha = 0$.

We can say a line is a good approximation of lossless if $Re\{Z_o\} >> Im\{Z_o\}$
and $\beta >> \alpha$.

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{R' + j\omega L'}{j\omega C'}}$$

since $G' = 0$ for this case

Inductance per unit length

$$L' = \frac{\mu_0 d}{W} = 0.1\mu_0$$

at f =	10 kHz	10 GHz
$\omega L'$ =	0.0079Ω	$7900\ \Omega$
ωC	$5.6\times10^{-6}\Omega$	$5.56\ \Omega$

At 10 kHz $R' >> \omega L'$ so $Z_0 \approx \sqrt{\frac{R'}{j\omega C'}} \approx \frac{1-j}{\sqrt{2}}\sqrt{\frac{R'}{\omega C'}}$

At 10 GHz $R' << \omega L'$ so $Z_0 \approx \sqrt{\frac{L'}{C'}}$ approximately.

So at 10 GHz we can better make the approximation of a lossless line.

5. **Plane waves** **(12 points)**

(a) (6 points) Consider a linearly polarized plane wave in free space with an electric field phasor given by: $\tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}} + \hat{\mathbf{y}})\frac{E_0^+}{\sqrt{2}}e^{-jkz}$. Write the corresponding H-field phasor.

$\vec{E}, \vec{H}, \vec{k}$ form a right handed triple for a plane wave.

e^{-jkz} indicates a wave propagating in $+z$ direction $\vec{k} = k\hat{z}$

$$\boxed{\tilde{\vec{H}}(z) = (\hat{y} - \hat{x})\frac{E_0^+}{\sqrt{2}\,\eta_0}e^{-jkz}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega$$

(b) (6 points) Consider a circularly polarized plane wave in free space with an electric field phasor given by: $\tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}} - j\hat{\mathbf{y}})\frac{E_0^+}{\sqrt{2}}e^{+jkz}$. Write the corresponding H-field phasor.

e^{+jkz} - propagation in $-z$ direction

$$\boxed{\tilde{\vec{H}}(z) = (-\hat{y} - j\hat{x})\frac{E_0^+}{\sqrt{2}\,\eta_0}e^{jkz}}$$

Maxwell's Equations:

$$\nabla \cdot \mathbf{D} = \rho_f$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

Auxillary Fields: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$, $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$

In linear media: $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$, $\mathbf{M} = \chi_m \mathbf{H}$, $\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$

Ohm's Law: $\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$

Electrostatic Potential: $\mathbf{E} = -\nabla V$ Vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

Electrodynamic Potential: $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$

Gradient Theorem: $\int_a^b (\nabla f) \cdot d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\int_V (\nabla \cdot \mathbf{A})\, dV = \oint_S \mathbf{A} \cdot d\mathbf{S}$

Stokes's Theorem: $\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$

Electric energy density: $W_e = \frac{1}{2}\mathbf{E} \cdot \mathbf{D}$ or $W_e = \frac{1}{2}\varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m = \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$ or $W_m = \frac{1}{2}\mu H^2$ (in linear media)

Power dissipation density (Joule/Ohmic) = $\mathbf{E} \cdot \mathbf{J}$ or σE^2 (in Ohm's law media)

Poynting Theorem: $\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) - \mathbf{E} \cdot \mathbf{J}$

Poynting Vector: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Time averaged Poynting vector: $\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\}$

Capacitance: $C = \frac{Q}{V}$

Inductance: $L = \frac{\Lambda}{I} = N\frac{\Phi}{I}$

Vector identities

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$
$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$
$$\nabla \times (\nabla f) = 0$$
$$\nabla \cdot \nabla f = \nabla^2 f$$
$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$
$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$