UCLA Department of Electrical Engineering
EE101 – Engineering Electromagnetics
Winter 2011
Final Exam, March 14 2011, (3 hours)

Name ______________________ Student number ______________________

This is a closed book exam – you are allowed 2 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

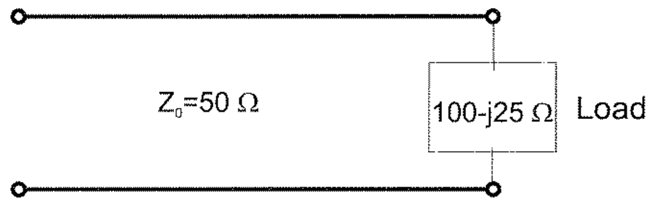
Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Impedance matching	40	
Problem 2	Solenoid revisited and phasors	30	
Problem 3	Plane waves	30	
Total		100	

1. **Transmission line – Impedance Matching** **(40 points)**

Consider a transmission line terminated with a load of 100-j25 Ω. The goal of this problem is to design three different impedance matching networks that prevents any reflections from the network and maximizes the power delivered to the load. Assume the transmission line is lossless coaxial cable filled with a dielectric material $\varepsilon=9\varepsilon_0$, $\mu=\mu_0$, and the frequency of operation is f=3 GHz.

For these problems, you may use any methods you wish, including the Smith chart (except (b)).



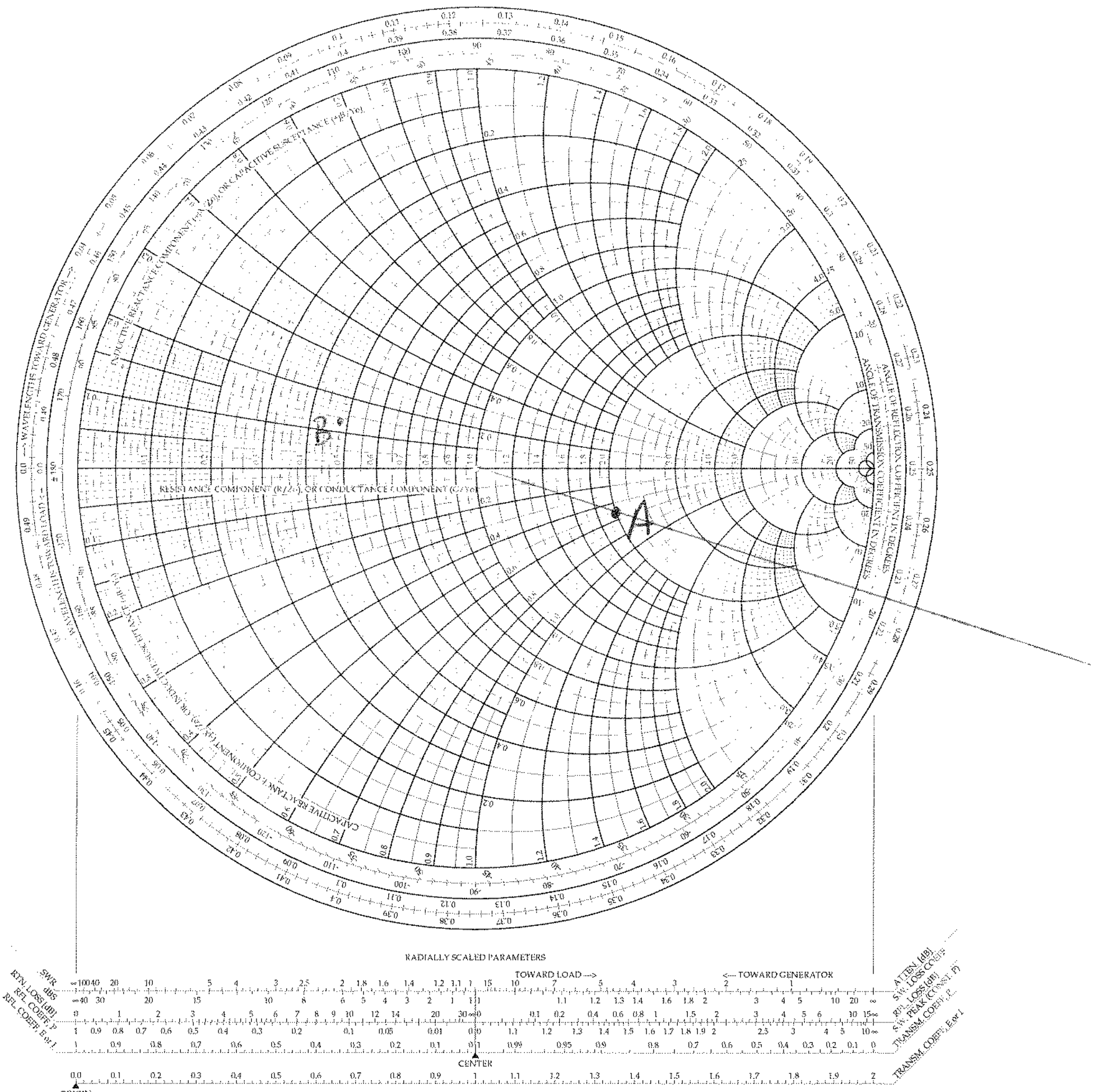
(a) (5 points) What is the wavelength for this transmission line at f=3 GHz?

$$\text{Phase velocity} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{3} = 10^8 \text{ m/s}$$

$$\lambda = \frac{u_p}{f} = \frac{10^8}{3\times10^9} = \frac{1}{3}\times10^{-1} = 0.033 \text{ m}$$

(b) (5 points) On the Smith chart below indicate the location of the normalized load impedance (mark with the letter A) and the normalized load admittance (mark with letter B). Write the value of the reflection coefficient Γ from the load (magnitude and phase) and load admittance Y.

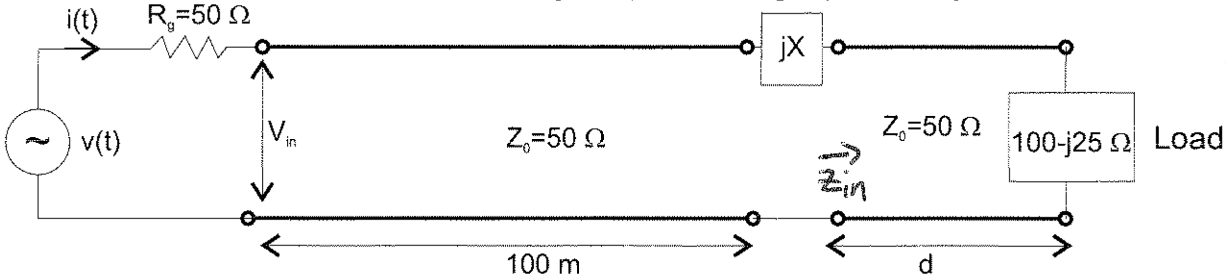
$Y_L = 0.0094 + j0.0024\ \Omega^{-1}$ $\Gamma = 0.37\angle -18^\circ$



$Z_L = 100 - j25$ $z_L = 2 - j0.5$

$Y_L = 0.47 + 0.12j$

(c) (10 points) In this case, we consider placing a series reactance X into the transmission line a distance d from the load. Find the value X and the length d (in wavelengths) necessary to match the load.

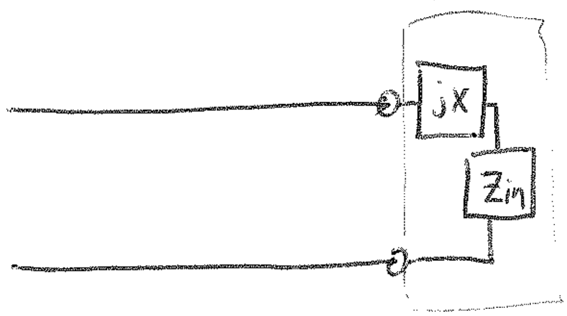


$Z_{in}(-d) = R_{in} + jX_{in}$

$z_{in} = r_{in} + jx_{in}$

Choose d so that $R_{in} = 50\Omega$ and then choose $X = -X_{in}$ so that

$Z_{tot} = Z_{in} + jX = 50\Omega$



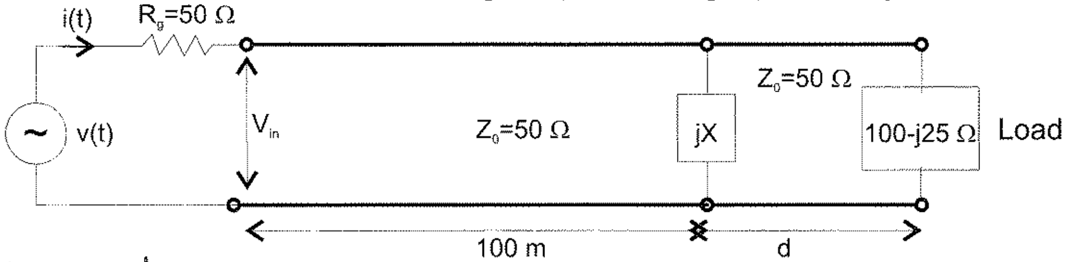
Solution 1: Choose $\boxed{d = 0.071\lambda}$ at this point $r_{in} = 1$ $x_{in} = -0.8$ (B)

Choose $x = 0.8$ $\Rightarrow$ $\boxed{X = 40\Omega}$

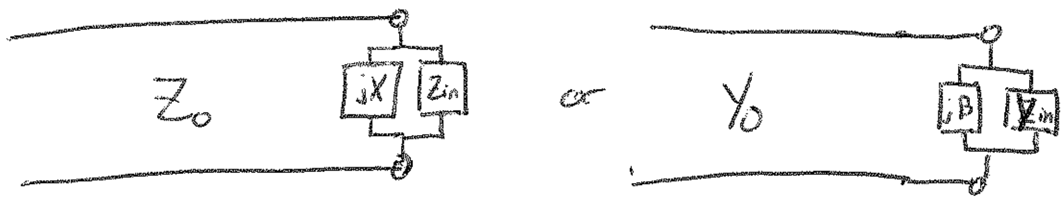
Solution 2: Choose $\boxed{d = 0.379\lambda}$ at this point $r_{in} = 1$ $x_{in} = 0.8$ (C)

choose $x = -0.8$ $\Rightarrow$ $\boxed{X = -40\Omega}$

(d) (10 points) Now our strategy is similar, except we will place the reactance X in parallel across the line. Find the value X and the length d (in wavelengths) necessary to match the load.



Since the matching element is a shunt, it makes more sense to work with admittance. $y_L = 0.47 + j0.12$



Choose d to give $y_{in} = g_{in} + jb_{in}$ so that $g_{in} = 1$ $\left(G_{in} = \frac{1}{50\Omega}\right)$

then choose $B = -\frac{1}{X} \Rightarrow b = -\frac{1}{x}$ so that $b = -b_{in}$.

Solution 1: Choose $\boxed{d = 0.13\lambda}$ at this point $g_{in} = 1$ $b_{in} = 0.8$ (B)

Choose $b = -0.8 \rightarrow x = \frac{-1}{b} = 1.25$

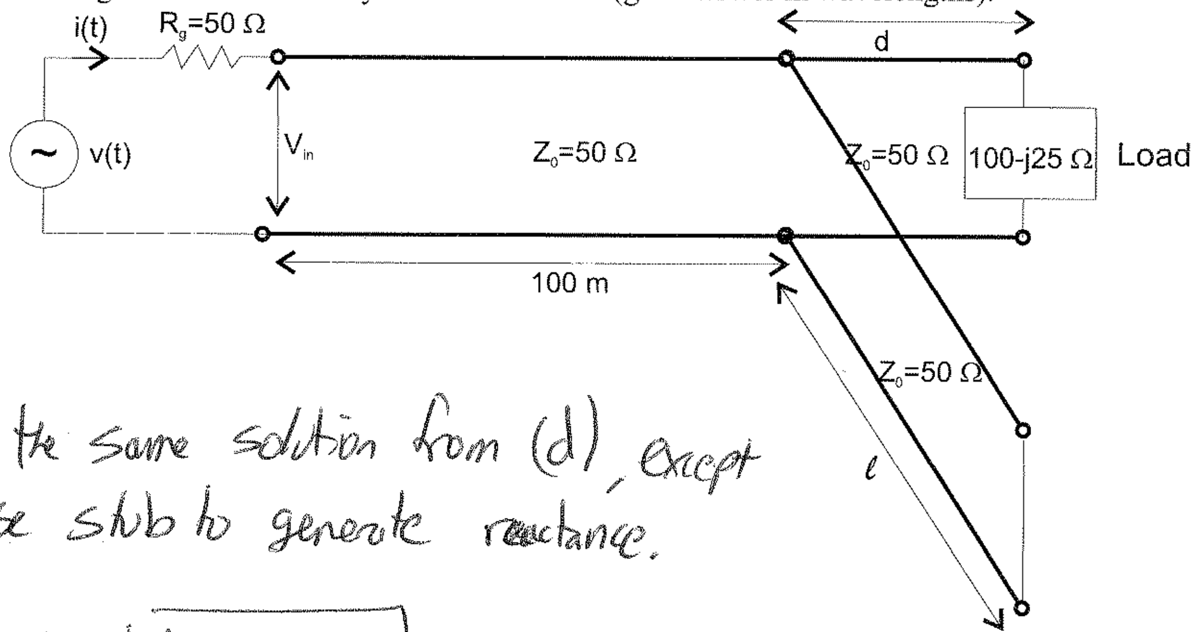
$$\boxed{X = 62.5\,\Omega}$$

Solution 2: Choose $\boxed{d = 0.322\lambda}$ $g_{in} = 1$ $b_{in} = -0.8$ (C)

Choose $b = 0.8 \rightarrow x = -\frac{1}{b} = -1.25$

$$\boxed{X = -62.5\,\Omega}$$

(e) (10 points) Now instead of a lumped element reactance, we use a shorted stub of length l. Find the lengths l and d necessary to match the load (give answer in wavelengths).



Use the same solution from (d), except use stub to generate reactance.

Solution 1: $\boxed{d = 0.13\lambda}$

Need stub to give $b_s = -0.8$

$\boxed{l = 0.142\lambda}$

D

Solution 2: $\boxed{d = 0.322\lambda}$

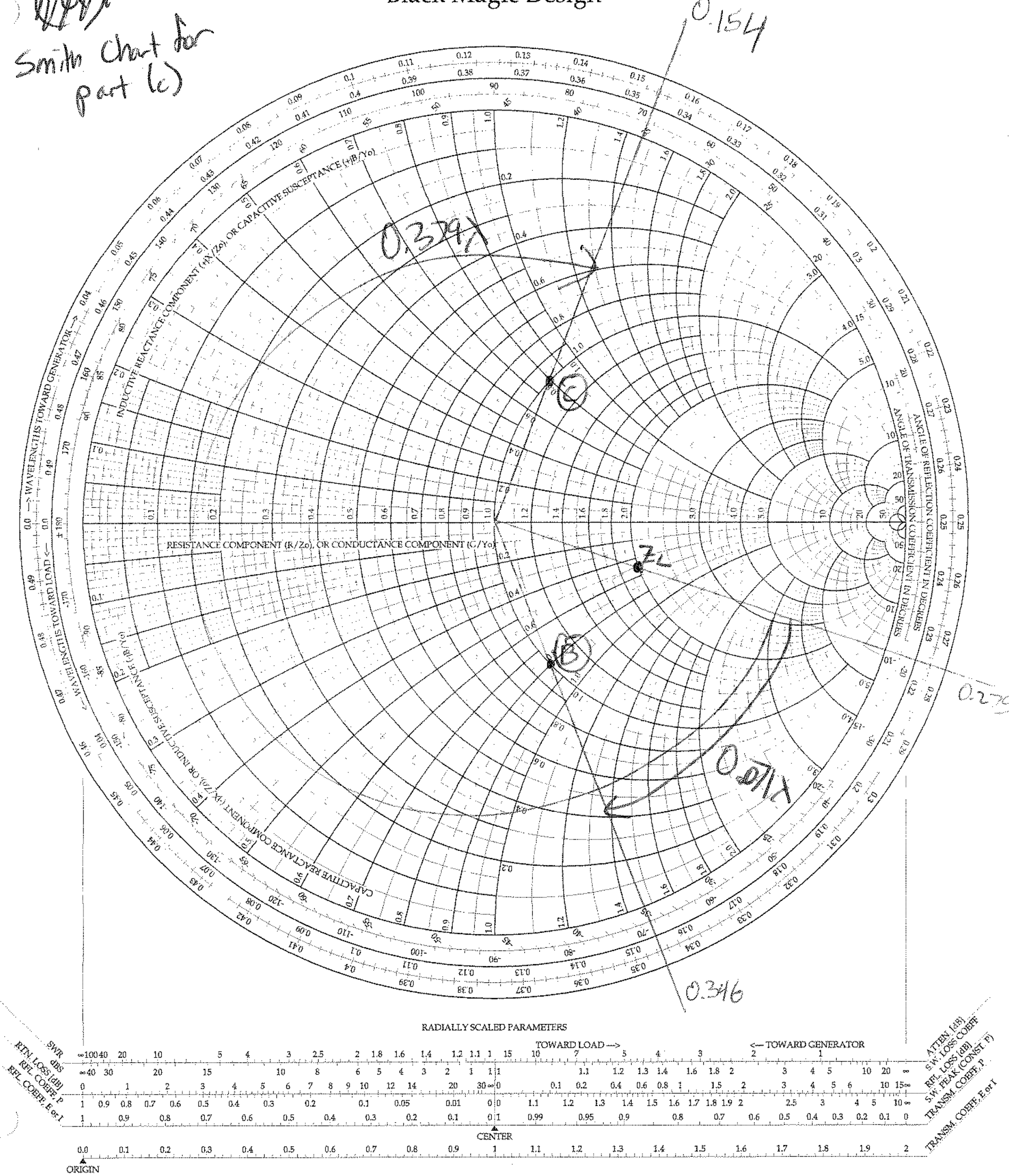
$\boxed{l = 0.357\lambda}$

E

The Complete Smith Chart

Black Magic Design

Smith Chart for part (c)

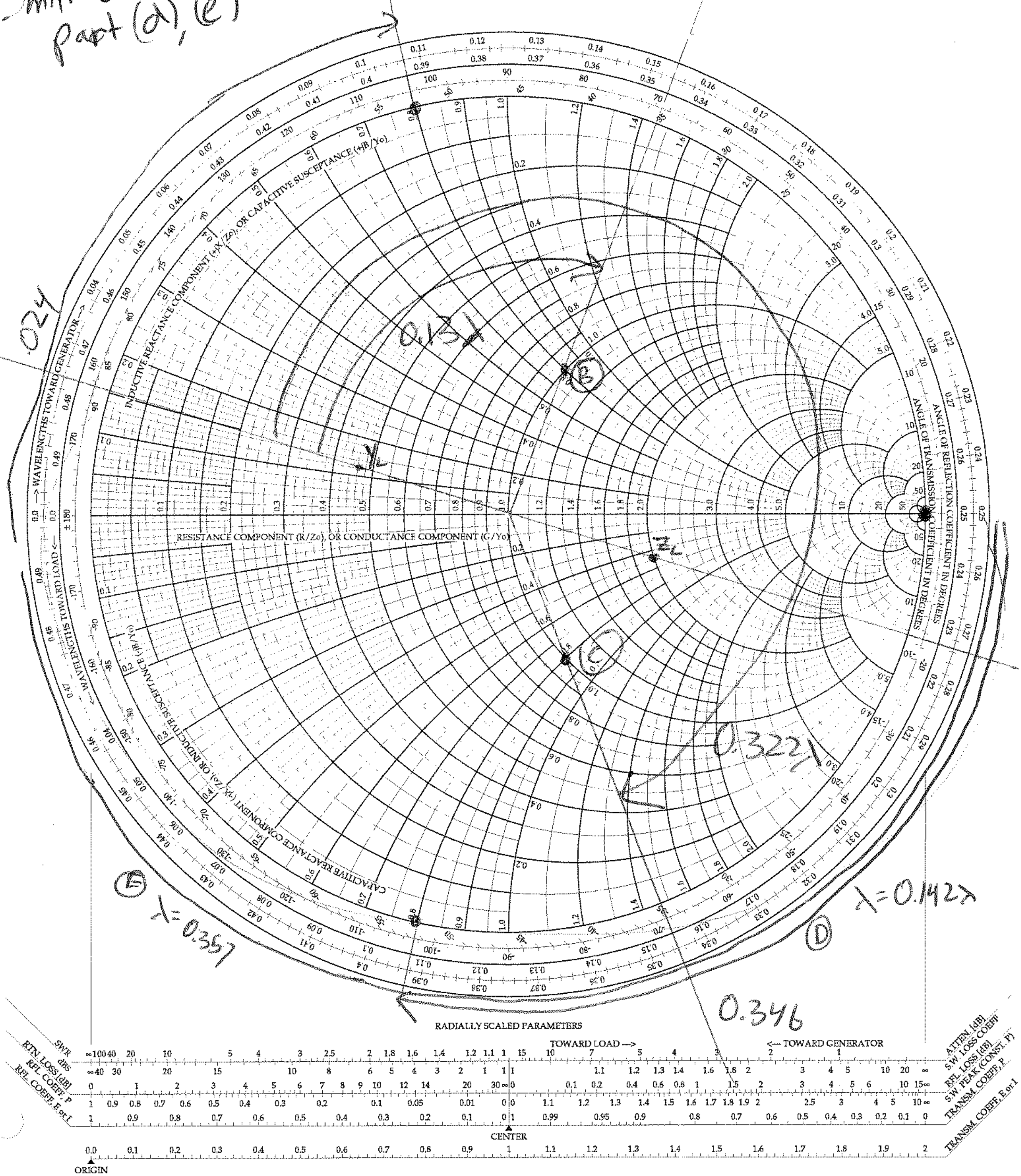


(b)

Smith Chart for part (d), (e)

The Complete Smith Chart

Black Magic Design

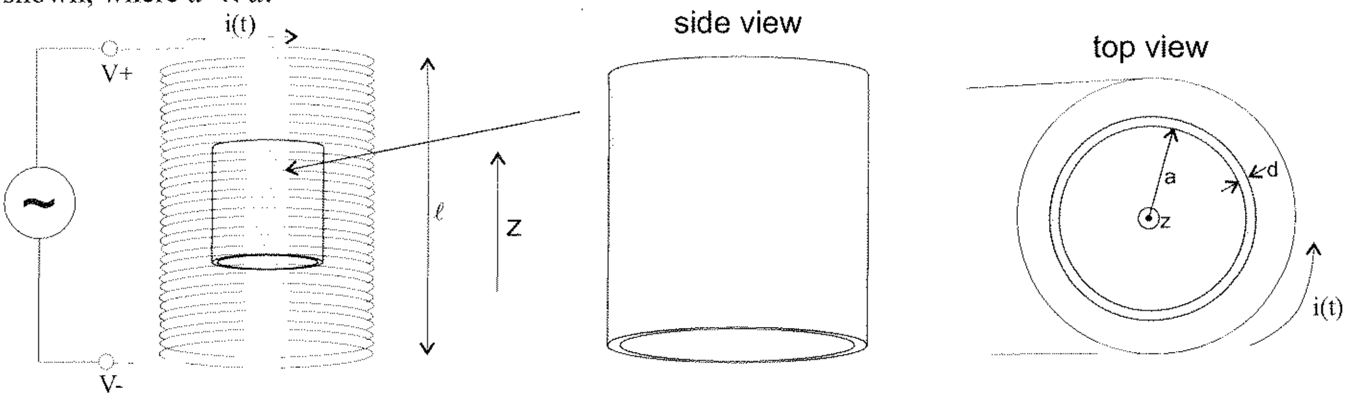


$$\begin{array}{r} 0.392 \\ -.250 \\ \hline .142 \end{array}$$

.107

2. Solenoid (30 points)

Consider a cylindrical metal shell with conductivity σ placed inside a long solenoid of length l with N turns driven to produce a magnetic field. The current in the solenoid is $i(t) = I_0 \cos(\omega t) = \mathrm{Re}\{\tilde{I}_0 e^{j\omega t}\}$ (I_0 is real). The direction of positive current is as indicated in the figure. The shell has dimensions as shown, where $d \ll a$.



It may be helpful to refer to the relationship we derived in class for a metal shell in a z-oriented H-field:

$$\frac{\partial}{\partial t}H_{in} + \frac{H_{in}}{\tau_m} = \frac{H_0}{\tau_m},$$

where $\tau_m = d\sigma a\mu_0/2$ is the magnetic diffusion time, and where $H_1(t)$ is the field generated by currents in the metal shell, $H_0(t)$ is the externally applied H field in the z-direction resulting from the solenoid, and $H_{in}(t)=H_1(t)+H_0(t)$ is the total axial H-field inside the metal shell (all H-fields are defined to be oriented in the z-direction).

a. (10 points) Since the driving current $i(t)$ is sinusoidal, we will solve this problem in the harmonic case using phasors. In other words, we assume that all time varying quantities such as $H_0(t)$, $H_{in}(t)$, $H_1(t)$ etc. have the same sinusoidal dependence and can also be written as phasors. Rewrite the equation above, and convert it to phasor notation (i.e. using $\tilde{H}_{in}, \tilde{H}_1, \tilde{H}_0$ etc.).

Write phasor eq $\frac{\partial}{\partial t} \to j\omega$

$$\boxed{j\omega \tilde{H}_{in} + \frac{\tilde{H}_{in}}{\tau_m} = \frac{\tilde{H}_0}{\tau_m}}$$

$$\tilde{H}_{in}\left(j\omega + \frac{1}{\tau_m}\right) = \frac{\tilde{H}_0}{\tau_m}$$

$$\frac{\tilde{H}_{in}}{\tilde{H}_0} = \frac{1}{1+j\omega\tau_m}$$

b. (10 points) What is the current density phasor $\tilde{J}_\phi$ flowing in the metal shell in terms of $\tilde{I}_0$? Be careful about the sign (i.e. direction of current flow). Your answer should be as a function of dimensional and material parameters, ω, etc. (H should not appear in your final answer).

The current flowing in the metal shell is related to H_1

$\vec{H}_1 = \hat{z} J_\phi d$ (J is volumetric current density)

Solve for H_1: $H_{in} = H_1 + H_0$

$$H_{in}\left(j\omega + \frac{1}{\tau_m}\right) = \frac{H_0}{\tau_m}$$

$$(H_1 + H_0)\left(j\omega + \frac{1}{\tau_m}\right) = \frac{H_0}{\tau_m}$$

$$H_1 = H_0 \frac{-j\omega}{j\omega + \frac{1}{\tau_m}}$$

$$H_\phi = H_0\left(\frac{-j\omega\tau_m}{1+j\omega\tau_m}\right)$$

$$\boxed{\tilde{H}_1 = \frac{\tilde{I}N}{\ell}\left(\frac{-j\omega\tau_m}{1+j\omega\tau_m}\right)}$$

Convert to J_ϕ

$$\boxed{\tilde{J}_\phi = -\frac{\tilde{I}_0 N}{d\ell}\left(\frac{j\omega\tau_m}{1+j\omega\tau_m}\right)}$$

c. (10 points) Consider the low frequency limit ($\omega\tau_m \ll 1$). In this limit, write an expression for the time dependent current density $J_\phi(t)$ in terms of I_0 and the various parameters.

$$\omega \tau_m \ll 1$$

$$\tilde{J}_\phi = -j\frac{\tilde{I}_0 N}{d\ell}\omega\tau_m$$

$$J_\phi(t) = \mathrm{Re}\{\tilde{J}_\phi e^{j\omega t}\}$$

$$= -\frac{N}{d\ell}\omega\tau_m\left(jI_0\cos\omega t - I_0\sin\omega t\right)$$

$$\boxed{J_\phi(t) = \frac{N\omega\tau_m I_0}{d\ell}\sin(\omega t)}$$

or

$$J_\phi(t) = \frac{N\omega\sigma a\mu_0 I_0}{2\ell}\sin(\omega t)$$

3. **Plane waves - Reflection from Interface** **(30 points)**

Consider a plane wave propagating in the z-direction that is normally incident upon a boundary between air and a Gallium Arsenide semiconductor wafer. Medium 1 is air, and we can approximate $\varepsilon_1 = \varepsilon_0$, and $\mu_1 = \mu_0$. Medium 2 has $\varepsilon_2 = 13\varepsilon_0$, and $\mu_2 = \mu_0$.

The electric fields are given in region 1 and 2 by:
$$\tilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}} E_0^i \left[e^{-jk_1 z} + \Gamma e^{jk_1 z} \right]$$
$$\tilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}} E_0^i \left[\tau e^{-jk_2 z} \right].$$

(a) (7 points) Write expressions for the H-field phasors in regions 1 and 2 in terms of the quantities specified above.

$$\tilde{H}_1 = \hat{y} \frac{E_0^i}{\eta_1} \left(e^{-jk_1 z} - \Gamma e^{jk_1 z} \right)$$

$$\tilde{H}_2 = \hat{y} \frac{E_0^i}{\eta_2} \left(e^{-jk_2 z} \right)$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0$$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}} = \frac{\eta_0}{\sqrt{13}}$$

(b) (8 points) What fraction of the incident power is reflected from and what fraction is transmitted through the interface?

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.56$$

$$\boxed{\begin{aligned} R &= |\Gamma|^2 = 0.319 \\ T &= 1 - R = 0.681 \end{aligned}}$$

(c) (8 points) Now consider that medium 2 is made to be conductive (for example by n-type doping) so that it has a conductivity of $\sigma = 4\text{x}10^4$ S. At the frequency of f=100 GHz, is this material a good conductor or a poor conductor?

$$\varepsilon_2' = \frac{8.85\times10^{-12}}{13}\ F/m \qquad \varepsilon_2'' = \frac{\sigma}{\omega} = \frac{4\times10^4}{2\pi\times100\times10^9} = 6.37\times10^{-8}\ F/m$$

$\varepsilon_2'' \gg \varepsilon_2'$ Material is good conductor

(d) (7 points) Considering this new conductivity in (c), what fraction of the incident power is reflected from the interface?

(FYI – the sensitivity of reflection and phase shift of reflected high-frequency radiation can be used to probe carrier density and doping in semiconductor wafers – a very useful technique for non-contact wafer inspection.)

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}} \approx \sqrt{\frac{\mu_0}{-j\sigma/\omega}} = \frac{(1+j)}{\sqrt{2}}\sqrt{\frac{\mu_0\omega}{\sigma}} = (1+j)3.14 = \pi(1+j)$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_2'+j\eta_2''-\eta_1'}{\eta_2'+j\eta_2''+\eta_1'} = \frac{(\eta_2'-\eta_1')+j\eta_2''}{(\eta_2'+\eta_1')+j\eta_2''}$$

$$|\Gamma|^2 = \frac{(\eta_2'-\eta_1')^2+\eta_2''^2}{(\eta_2'+\eta_1')^2+\eta_2''^2} = \frac{(\pi-377)^2+\pi^2}{(\pi+377)^2+\pi^2} = \frac{139779}{144517}$$

$$R = \boxed{|\Gamma|^2 = 0.967}$$