UCLA Department of Electrical Engineering
EE101 – Engineering Electromagnetics
Winter 2009
Final, March 19 2009, (3 hours)

Name ______________________ Student number ______________________

This is a closed book exam – you are allowed 2 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit.

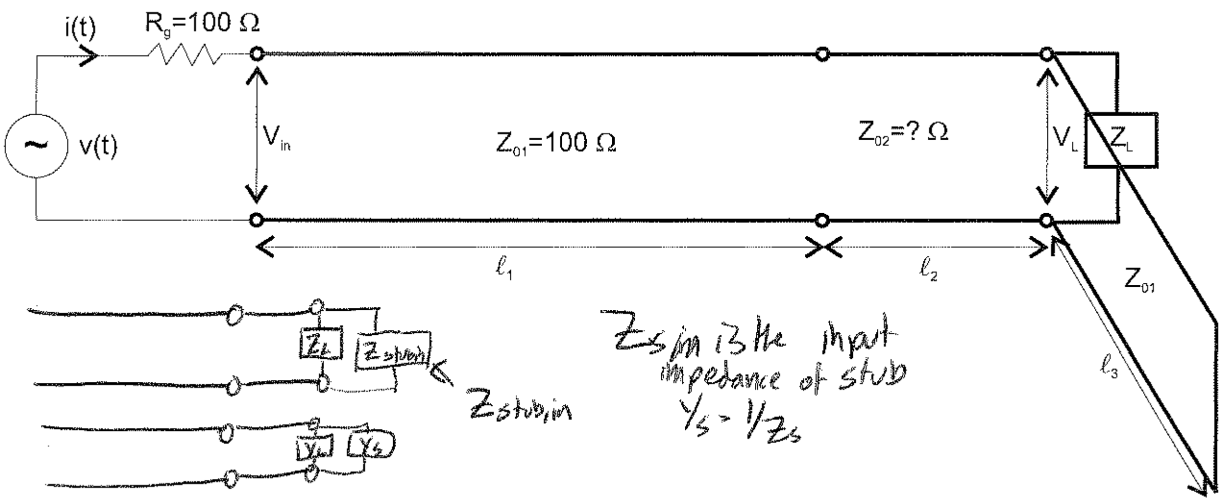
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Impedance matching	30	
Problem 2	Transmission lines and fields	25	
Problem 3	Plane wave reflection	20	
Problem 4	Vector Calculus	25	
Total		100	

1. Transmission line – Impedance Matching (30 points)

The goal of this problem is to design an impedance matching network for the load Z_L=50-j150 Ω, to a transmission line of characteristic impedance Z_{01}=100 Ω, so that all of the power on the transmission line is coupled into the load. A shorted stub with characteristic impedance Z_{01} is placed in parallel with the load. You may choose the parameters l_3 (length of the stub), l_2 (length of the section of extra line), and Z_{02} (characteristic impedance of section of extra line). You may use any method you wish to solve this problem.



$Z_{s,in}$ is the input impedance of stub $Y_s = 1/Z_s$

(a) (10 points) In terms of the wavelength λ, specify lengths l_2 and l_3 necessary to obtain an impedance match.

Strategy: use stub to balance out imaginary part of Load admittance Y_L. Then use λ/4 wave transformer to match real part of ~~load adm~~ Y_L to Z_{01}.

$\boxed{l_2 = \lambda/4}$

Now find l_3: $Z_L = 50 - j150\,\Omega$ $\quad z_L = \frac{Z_L}{Z_{01}} = 0.5 - j1.5$ ($\boxed{A}$ on Smith chart)

Use Smith chart to convert to admittance ($\boxed{B}$ on Smith Chart)

$y_L = 0.2 + j0.6 \quad Y_L = 0.002 + 0.006j\ S$

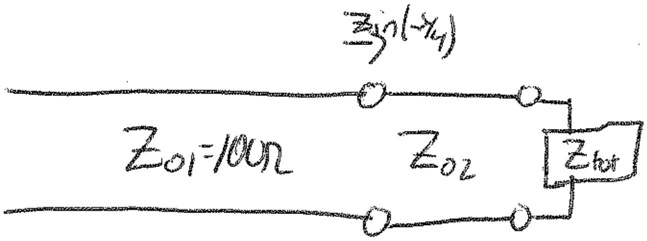
We should design stub to have input admittance ~~y~~ $y_{s,in} = -0.6j$

A stub terminated in a short requires rotation by $\boxed{l_3 = 0.165\lambda}$ toward generator to obtain $y_{sin} = -0.6j$ $\boxed{D}$

$y_{s,in} + y_L = 0.002\ S \quad Y_{s,in} + Y_L = 0.002\ S \qquad \left(\frac{1}{Z_{s,in}} + \frac{1}{Z_L}\right)^{-1} = 500\,\Omega =$

(b) (5 points) Specify characteristic impedance Z_{02} necessary to obtain impedance match. Give a number in Ohms.

Now we choose Z_{02} to satisfy the $\frac{\lambda}{4}$ matching condition.



$Z_{tot} = 500\Omega$

$$Z_{in}(-\lambda/4) = \frac{Z_{02}^2}{Z_{tot}} = Z_{01} \quad \text{Matching condition}$$

$$\boxed{\begin{aligned} Z_{02} &= \sqrt{Z_{01} Z_{tot}} \\ Z_{02} &= 223\,\Omega \end{aligned}}$$

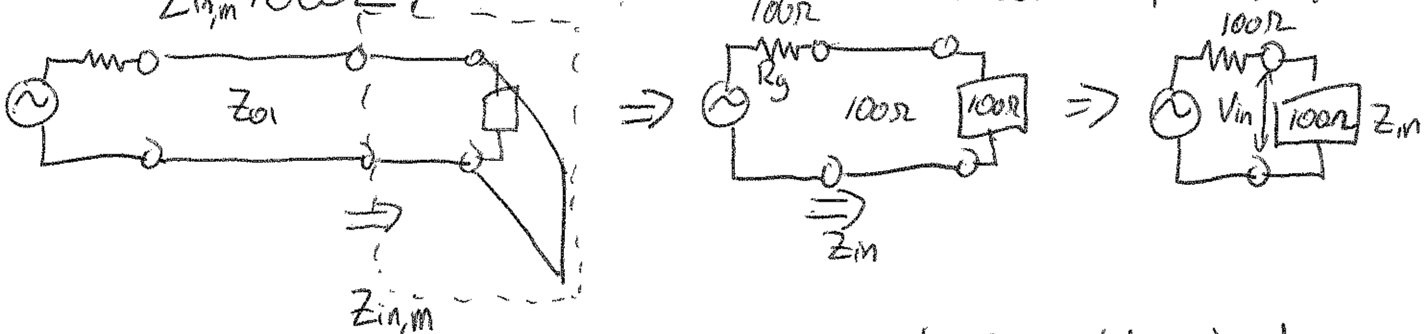
(c) (5 points) If the transmission line with impedance Z_{01} is a parallel-plate transmission line, with dimensions d=0.53 mm, w=1 mm, material parameters $\mu=\mu_0$, $\varepsilon=4\varepsilon_0$, and the frequency is f=100 MHz, what is the wavelength on the line (in meters)?

$$u_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{2} = 1.5\times10^8 \text{ m/s}$$

$$\lambda = \frac{u_p}{f} = 1.5\text{m}$$

(d) (5 points) At the matched condition, if $v(t)$ at the generator is $v(t) = \mathrm{Re}\{\tilde{V}_0 e^{j\omega t}\} = 50\cos(\omega t)$ Volts, how much average power is dissipated in the load?

At the input of the matching section, we see an input impedance of $Z_{in,m} = 100\Omega$ which means $Z_{in} = 100\Omega$ at input of line.

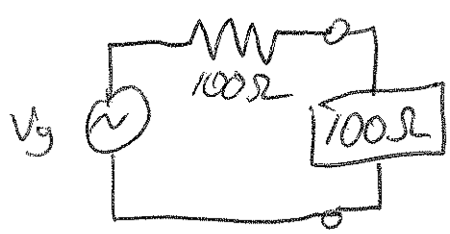


All power dissipated in input impedance must be dissipated in load, since the load contains only resistive element.

$$P_{av} = \frac{1}{2}\mathrm{Re}\{V_{in} I_{in}^*\} = \frac{|V_{in}|^2}{2Z_{in}} = \frac{|25\,V|^2}{2 \times 100\Omega} = \boxed{3.13\,W}$$

$V_{in} = V_g/2$ since circuit is voltage divider

(e) (5 points) At the matched condition, what fraction of the total power dissipated in the circuit is dissipated in the load?



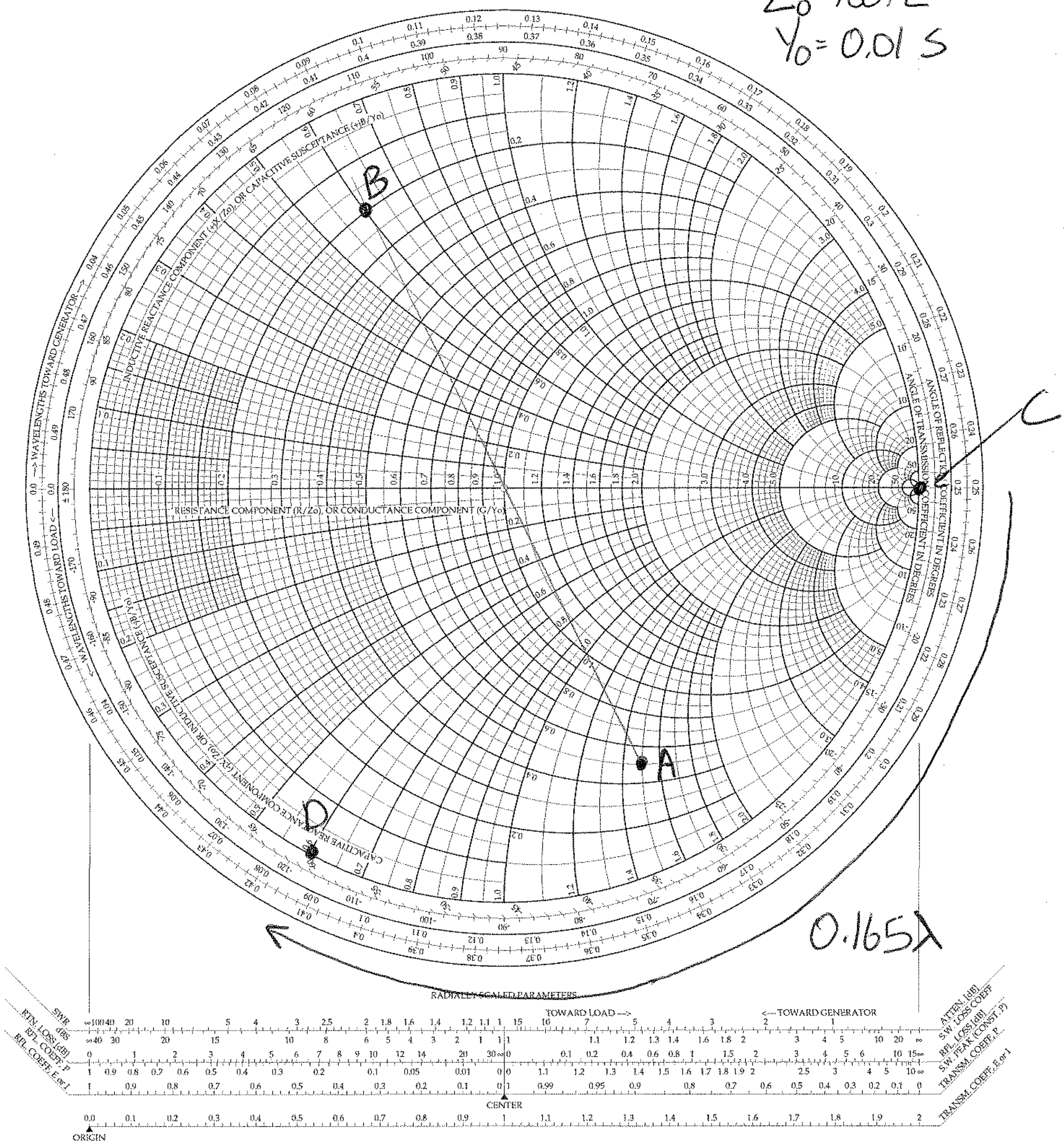
total power dissipated $= \frac{I V_g}{2}$

I

$$\boxed{\frac{\{I^* V_{in}\}}{\{I^* V_g\}} = \frac{1}{2}}$$

Smith Chart for 1a

$Z_0 = 100\Omega$
$Y_0 = 0.01\ S$

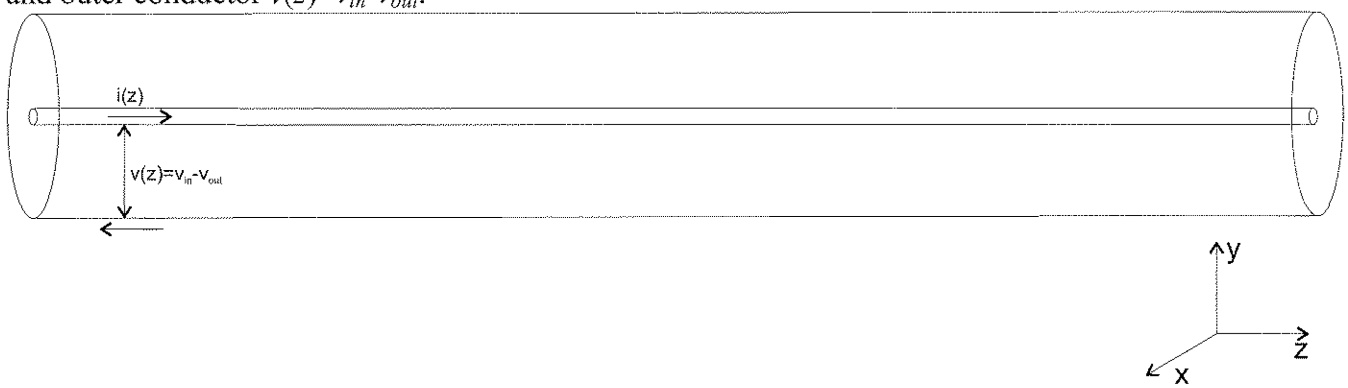


A: $Z_L = 0.5 - j1.5$
B: $Y_L = 0.2 + j0.6$
C: $Y = \infty$ for short
D: $Y_{stub,in} = -0.6j$

$Y_{total} = Y_L + Y_{stub\,in} = 0.2$
$\underline{Y}_{tot} = 0.002\ S$
$Z_{tot} = 500\Omega$

2. Fields on a transmission line (25 points)

Consider a coaxial cable transmission line with characteristic impedance Z_0 that has an "open" termination ($Z_L=\infty$). The voltage difference $v(z,t)$ is given by the potential difference between the inner and outer conductor $v(z)=v_{in}-v_{out}$.



The phasors of the voltage and current solutions on a TL are given as follows:

$\tilde{V}(z)=V_0^+e^{-j\beta z}+V_0^-e^{j\beta z}$ $\tilde{I}(z)=\frac{V_0^+}{Z_0}e^{-j\beta z}-\frac{V_0^-}{Z_0}e^{j\beta z}$

(a) (5 points) What are the phasors $\tilde{V}(z)$ and $\tilde{I}(z)$ as a function of z, only in terms of β, V_0^+, and Z_0? Sketch $\tilde{V}(z)$ and $\tilde{I}(z)$.

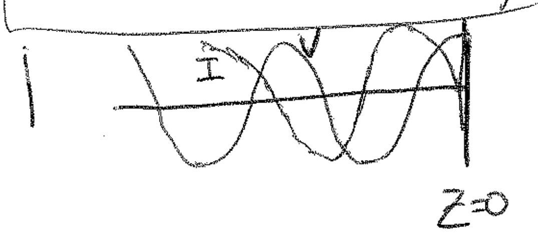
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$

$V(z)= V_0^+(e^{-j\beta z}+e^{j\beta z})$

$\boxed{\tilde{V}(z)=2V_0^+\cos\beta z}$

$I(z)=\frac{V_0^+}{Z_0}(e^{-j\beta z}-e^{j\beta z})$

$\boxed{\tilde{I}(z)=-2j\frac{V_0^+}{Z_0}\sin\beta z}$



(b) (5 points) Write an expression for the voltage $v(z,t)$ and $i(z,t)$ – the voltage and current as a function of time.

$V(z,t)=Re\{\tilde{V}(z)e^{j\omega t}\}=Re\{2V_0^+\cos\beta z\, e^{j\omega t}\}$

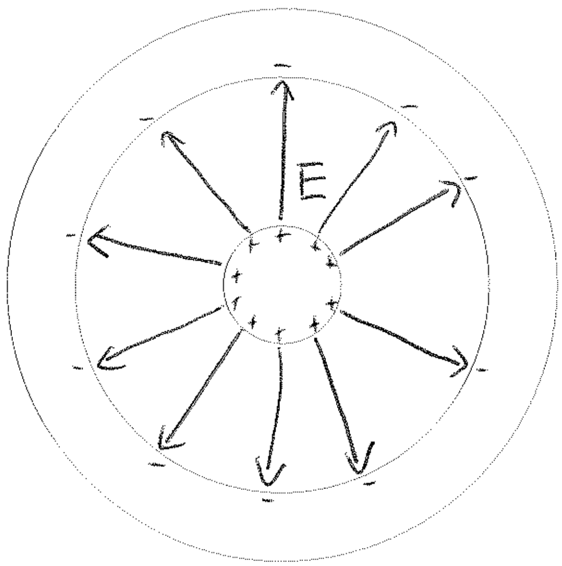
$\boxed{v(z,t)=2V_0^+\cos(\beta z)\cos(\omega t)}$

$i(z,t)=Re\{\frac{-2V_0^+}{Z_0}\sin\beta z\; j(\cos\omega t+j\sin\omega t)\}$

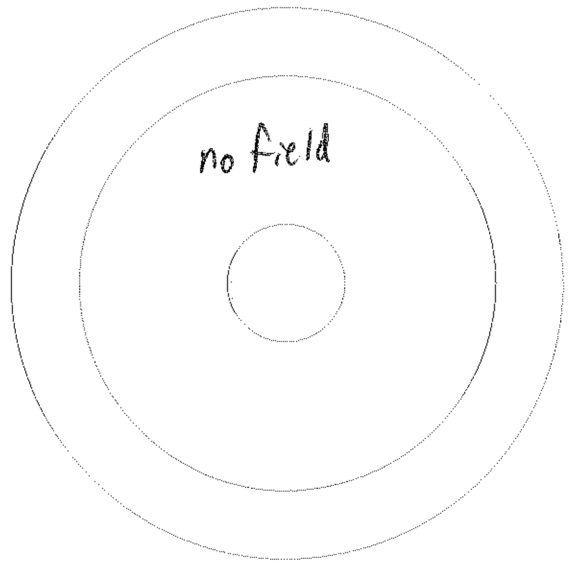
$\boxed{i(z,t)=\frac{2V_0^+}{Z_0}\sin(\beta z)\sin(\omega t)}$

(c) (5 points) Sketch the transverse electric field $\mathbf{E}(x,y)$ at the position $z=0$ at times $t=0$, $t=\pi/2\omega$, $t=\pi/\omega$, $t=3\pi/2\omega$. Accurately sketch the sign and location of any free charge.

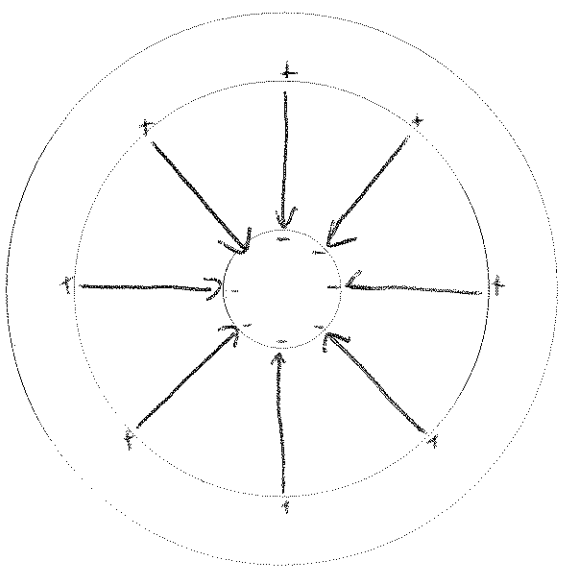
Charge on inner/outer surfaces within δ_s



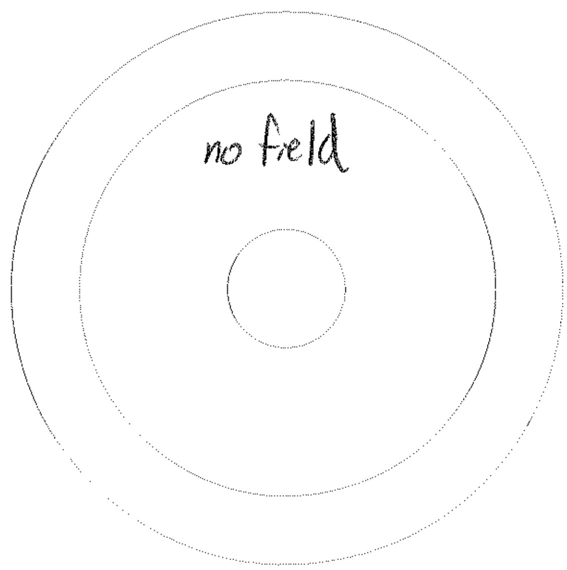
$t=0$



$\omega t=\pi/2$



$\omega t=\pi$

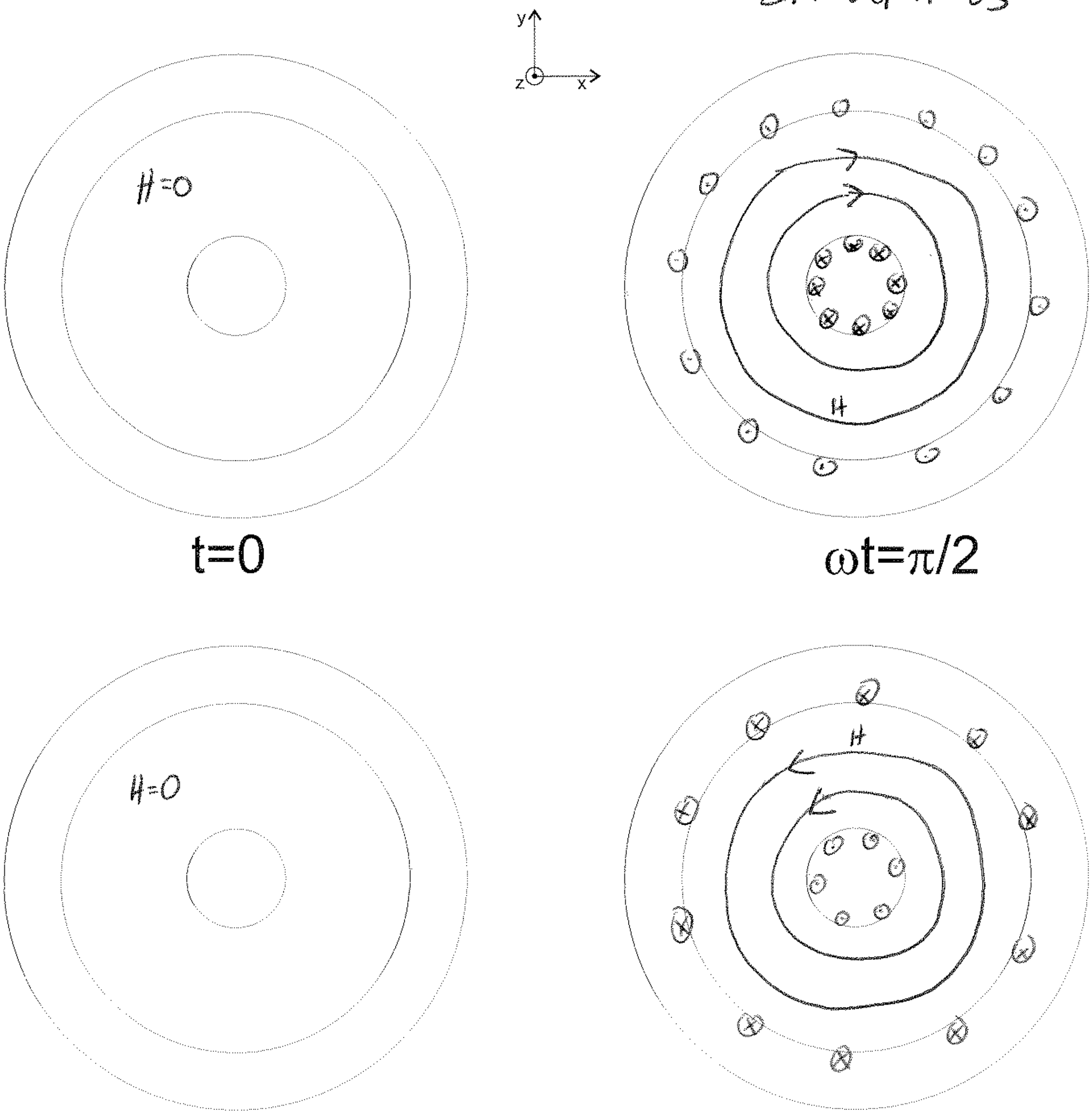


$\omega t=3\pi/2$

$$E(r,\phi)=\frac{V(z)}{\ln(b/a)}\frac{1}{r}\hat{r}$$

(d) (5 points) Sketch the transverse magnetic field $\mathbf{H}(x,y)$ at the position $z=-\lambda/4$ at times $t=0$, $t=\pi/2\omega$, $t=\pi/\omega$, $t=3\pi/2\omega$. *Accurately* sketch the direction and location of any free current that is flowing. (The z-direction points out of the page.)

Current flows within approx skin depth δ_s



ωt=π

ωt=3π/2

$$\vec{H}(r,\phi,z) = \frac{i(z)}{2\pi r}\hat{\phi}$$

(e) (5 points) Assume the inner and outer conductors of the coaxial cable are made of Copper with conductivity $\sigma = 6\times10^7$ S/m. As a result, there will be some absorption of waves (loss) that propagate due to the finite resistance of the conductors. As the frequency of the waves is increased, will this loss increase, or decrease? Why? You must justify your answer.

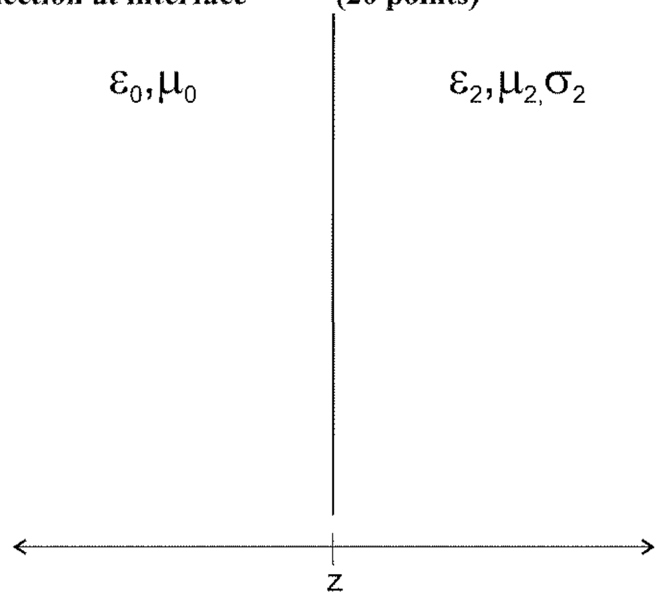
As frequency increases, skin depth decreases.

$$\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

As a result, surface resistance increases: $R_s = \frac{1}{\sigma \delta_s}$

This causes R' to increase + causes increase in loss.

3. Plane wave reflection at interface (20 points)



Consider the interface between two media with the parameters listed above. An infrared plane wave with a wavelength of 1 μm is normally incident upon the interface from the left. The electric fields are given in region 1 and 2 by:

$$\tilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}}\left[E_0^i e^{-jk_1 z} + E_0^r e^{jk_1 z}\right]$$
$$\tilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}}\left[E_0^t e^{-j\beta_2 z} e^{-\alpha_2 z}\right].$$

Medium 1 is vacuum, and medium 2 has $\varepsilon_2 = 4\varepsilon_0$, $\mu_2 = \mu_0$, and finite conductivity $\sigma_2 = 1\ \Omega^{-1}\ \mathrm{m}^{-1}$? .

(a) (5 points) What is the field reflection coefficient $\Gamma = E_0^r / E_0^i$, and the field transmission coefficient $\tau = E_0^t / E_0^i$? Make sure your answer is in terms of the material parameters stated above. You should also give a number for your answer. You may make any reasonable approximations if you justify them.

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\quad \tau = 1 + \Gamma$

$\lambda = 1\,\mathrm{um} \Rightarrow f = 300\,\mathrm{THz} \rightarrow \omega = 1.9\times 10^{15}\,\mathrm{s}^{-1}$

$\varepsilon_2' = 4\varepsilon_0 = 3.54\times 10^{-11}\ \mathrm{F/m}$ $\quad \varepsilon_2'' = \frac{\sigma}{\omega} = 5.26\times 10^{-16}\ \mathrm{F/m}$

$\varepsilon_2'' \ll \varepsilon_1' \rightarrow$ Material is poor conductor

thus $\eta_2 \approx \sqrt{\frac{\mu}{\varepsilon_2}} = \frac{\eta_0}{2}$. Reflection is not affected by conductivity

$\boxed{\Gamma = -\frac{1}{3}}$ $\quad \boxed{\tau = \frac{2}{3}}$

(b) (5 points) Write an expression for the magnetic field phasor $\tilde{\mathbf{H}}_t(z)$ in both regions in terms of E_0^i, and the various material parameters. Make sure you include vector.

$$H_1 = \hat{y}\,\frac{E_0^i}{\eta_1}\left(e^{-jk_1 z} - \Gamma e^{jk_1 z}\right) \qquad H_2 = \hat{y}\,\frac{E_0^i}{\eta_2}\tau\left(e^{-jk_2 z}\,e^{-\alpha_2 z}\right)$$

$$k_1 = \omega\sqrt{\varepsilon_0 \mu_0} \qquad k_2 = \omega\sqrt{\varepsilon_2 \mu_0}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \eta_2 \approx \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\alpha_2 \approx \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon_2}}$$

(c) (5 points) What fraction of the incident intensity (power per unit area) is transmitted past the interface?

Transmitted power $= 1 - |\Gamma|^2 = 1 - \frac{1}{9} = \frac{8}{9}$

(d) (5 points) At what wavelength does material 2 change from being a good conductor to a poor conductor (low-loss material)? How will the qualitative behavior of the reflection and transmission change?

Change occurs when $\varepsilon' \approx \varepsilon''$

$$4\varepsilon_0 = \frac{\sigma}{\omega}$$

$$\omega = \frac{\sigma}{4\varepsilon_0} = 2.82\times10^{10}\ s^{-1}$$

$$f = 4.5\ GHz$$

$$\lambda_0 = 6.7\ cm \quad \text{(in free space)}$$

(e) (5 points) What is the free current density $\mathbf{J}_f(x,y,z)$ in ~~material 1 and~~ material 2?

~~$J_f = 0$ in region 1~~

~~$J_f = \sigma_2 E_2$ in region 2.~~

~~$J_{f,2} = \sigma_2 \hat{x}\, \tau E_0^i e^{-jk_2 z}$~~

4. Vector calculus (20 points)

(a) (7 points) Starting from Maxwell's equations, and using any vector identities necessary, derive the current continuity law $\oint_S \mathbf{J}_f \bullet d\mathbf{S} = -\int_V \frac{\partial \rho_f}{\partial t} dV$. Be sure to go step-by-step with descriptions to demonstrate that you understand the proof.

Start with Ampere's Law

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t}$$

Apply Div operator

$$\underbrace{\nabla \cdot (\nabla \times H)}_{\Downarrow \text{ by vector identity } \, 0} = \nabla \cdot J + \varepsilon \frac{\partial}{\partial t}(\nabla \cdot E)$$

$\Downarrow$ apply Gauss's Law

$$\nabla \cdot J = -\frac{\partial \rho_f}{\partial t}$$

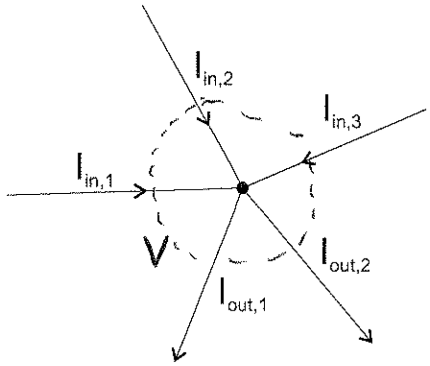
To obtain integral formulation, apply divergence theorem over a volume V.

$$\int_V \nabla \cdot J \, dV = -\int_V \frac{\partial \rho_f}{\partial t} dV$$

$\Downarrow$ div theorem

$$\oint_S J \cdot dS = -\int_V \frac{\partial \rho_f}{\partial t} dV$$

(b) (5 points) Explain qualitatively how the current continuity law that you derived above is related to Kirchoff's current law (the sum of the currents flowing into a circuit node is equal to the sum of the currents flowing out: $\sum_n I_{in,n} = \sum_m I_{out,m}$).



$\oint J \cdot dS \Rightarrow \sum_n I_n$

In magneto quasi static limit, currents are constant, and $\frac{\partial E}{\partial t} \to 0$ so $\nabla \times H \approx J$.

The theorem above in part (a) becomes $\oint \vec{J} \cdot d\vec{S} = 0$, which states that the net flux of current in + out of a volume is zero - equivalent to KCL.

(c) (8 points)

Here is a "proof" that there is no such thing as magnetism. Magnetic Gauss's law states that: $\nabla \bullet \mathbf{B} = 0$,
When we apply the divergence theorem, we find:

$$\int_V (\nabla \bullet \mathbf{B})\, dV = \int_S \mathbf{B} \bullet d\mathbf{S} = 0 \qquad (1)$$

Because **B** has zero divergence, we are able to define **B** as the curl of the vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$ If we combine the last two equations, we obtain:

$$\int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S} = 0 \qquad (2)$$

Next we apply Stokes's theorem to the above result to obtain:

$$\int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S} = \oint_C \mathbf{A} \bullet d\mathbf{l} = 0$$

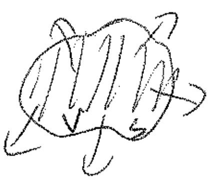
Thus we have shown that the circulation of **A** is path independent. It follows that we can write $\mathbf{A} = \nabla \psi$ where ψ is some scalar function. Since the curl of a gradient is zero, we arrive at the remarkable conclusion that:

$$\mathbf{B} = \nabla \times (\nabla \psi) = 0$$

That is, the magnetic field is zero everywhere!

Obviously I made a mistake somewhere in this proof. Explain where I went wrong.
(Hint: pay careful attention to the definitions of the various laws and theorems – it may be helpful to make sketches).

The problem is in Step 1 + 2, where the divergence theorem is applied. Div theorem is applied over a closed surface S, thus (2) should read



$$\oint_S (\nabla \times \vec{A}) \cdot d\vec{S} = 0$$

The closed surface of the divergence theorem is in compatible with the open surface of the Stokes theorem.