(5) $Z_0 = \sqrt{\frac{L'}{C'}} = 50 \longrightarrow L' = 1.67 \times 10^{-7}\ (H)$

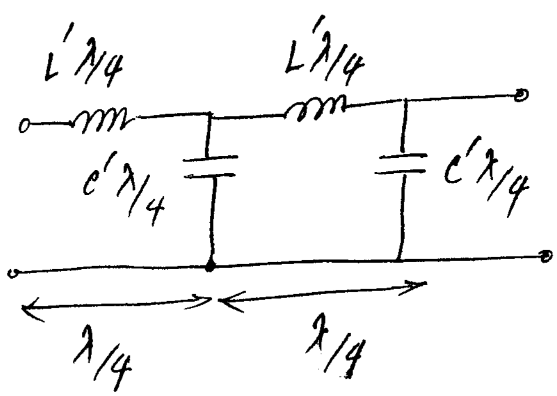
$L'C' = \mu\varepsilon \qquad C' = 6.67 \times 10^{-11}\ (F)$

b) $u_p = c \quad (air)$

(5) $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^8} = 1\ (m) \qquad \beta = \frac{2\pi}{\lambda} = 2\pi\ (1/m) = 6.287$

c)

(5)



d) $\varepsilon_r = 4 \qquad L' = 2\,(L'_{in\ part\ a}) = 3.34 \times 10^{-7}\ (H)$

(10) $\qquad c' = 2\,(c'_{in\ part\ a}) = 1.33 \times 10^{-10}\ (F)$

b) $u_p = \frac{c}{\sqrt{\varepsilon_r}} = c/2 = 1.5 \times 10^8\ (m/s)$

$\lambda = \frac{u_p}{f} = \frac{1.5 \times 10^8}{300 \times 10^6} = 0.5\ (m) \qquad \beta = \frac{2\pi}{\lambda} = 4\pi = 12.57\ (m)$

2)

a) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 - 120j - 100}{60 - 120j + 100} = 0.2 - 0.6j = 0.6325\, e^{-j1.25}$

4

b) $S = \frac{1 + 0.6325}{1 - 0.6325} = 4.44$

4

c) V_{max} @ $\frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} \longrightarrow l_{max} = \frac{-1.25\lambda}{4\pi} + \frac{1 \times \lambda}{2} = 2$ cm

4

l_{min} for voltage $= l_{max}$ for max current

$l_{min} = 2 - \lambda/4 = 0.75$ cm

4

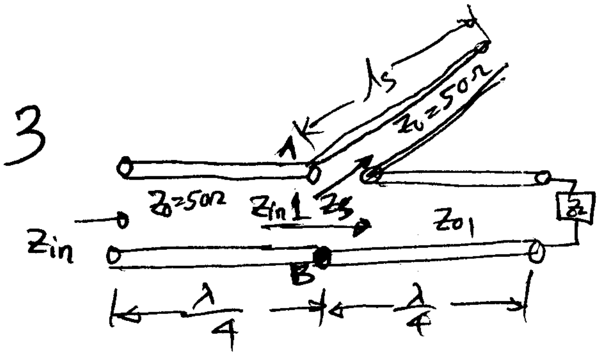
d) No, $S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$ and since $|\Gamma|$ remains the same (only its phase changes) $\rightarrow S$ remains constant.

e) $|V_0^+| = 1 \longrightarrow |V_{max}| = |V_0^+|(1 + |\Gamma|) = 1.6325$ (Volts)

4

$|V_{min}| = |V_0^+|(1 - |\Gamma|) = 0.3675$ (V)

#3



(a) For Quarter wavelength transformer

5 points

$$Z_{in}1 = \frac{Z_{01}^2}{Z_L} = \frac{(80)^2}{100} = 64 \ (\Omega)$$

For open circuit of length $l_s = \frac{\lambda}{4}$

$$Z_s = Z_{in}^{oc} = -jZ_0 \cot\beta l = -jZ_0\cot(\pi/2) = 0 \ (\Omega)$$

∴ Total Input Impedance at AB Z_{in}' is

$$Z_{in}' = Z_{in}1 + Z_s = 64 + 0 = 64 \ (\Omega)$$

$$\therefore Z_{in} = \frac{Z_0^2}{Z_{in}'} \quad \text{(Quarter wavelength transformer)}$$

$$= \frac{(50)^2}{64} \simeq 39 \ (\Omega)$$

(b) To impedance match

5 points

$Z_{in}' = Z_0 = 50$

$\therefore Z_{in}' = Z_{in}1 + Z_s = 50 \qquad \therefore Z_{in}1 = 50$

$Z_{in}1 = \frac{Z_{01}^2}{Z_L} \quad \therefore Z_{01} = \sqrt{Z_{in}1 \times Z_L} = \sqrt{50 \times 100} \simeq 70.7 (\Omega)$

(c) Now $Z_L = (100 - j100)\ \Omega$.

10 points

To impedance match

$Z_{in}' = Z_{in}1 + Z_s = 50$

$$Z_{in}1 = \frac{(Z_{01})^2}{Z_L} = \frac{(Z_{01})^2}{100 - j100} = \frac{(Z_{01})^2(100+j100)}{(100)^2+(100)^2} = \frac{(Z_{01})^2(1+j)}{200}$$

and

$$Z_s = -j Z_0 \cot \beta l_s$$

$$\therefore \; Z_{in}' = Z_{in}1 + Z_s = 50$$

$$\Rightarrow \frac{(Z_{01})^2(1+j)}{200} + (-j Z_0 \cot\beta l_s) = 50$$

$$\frac{(Z_{01})^2}{200} + \frac{j(Z_{01})^2}{200} - j Z_0 \cot\beta l_s = 50$$

to satisfy this equation

$$\frac{(Z_{01})^2}{200} = 50 \quad \text{and} \quad \frac{j(Z_{01})^2}{200} - j Z_0 \cot\beta l_s = j0$$

$$\therefore Z_{01} = \sqrt{50 \times 200} = 100 \,(\Omega)$$

and

$$\frac{(Z_{01})^2}{200} = Z_0 \cot \beta l_s$$

$$\Rightarrow \frac{(100)^2}{200} = 50 \cot \beta l_s$$

$$\therefore \cot\beta l_s = 1 \qquad \therefore \cot\beta l_s = \cot\frac{\pi}{4}$$

$$\therefore \frac{2\pi}{\lambda} l_s = \pi/4 \,, \quad l_s = \frac{\lambda}{8}$$

$$\therefore l_s = n\frac{\lambda}{2} + \frac{\lambda}{8} \quad (n = 0, 1, 2, \ldots)$$

$$\boxed{Z_{01} = 100\,\Omega \,, \quad l_s = n\frac{\lambda}{2} + \frac{\lambda}{8}}$$

#4

1(a) $Z_L = (50 - j25)\,\Omega$ ($\bar{z}$ represents normalized value on z)

4 points

$\therefore$ Normalized Impedance $\bar{z}_L = 1 - j0.5$

Plotting this value in Smith chart, we get

$$\Gamma \approx 0.24 \angle -76^\circ$$

4 points

1(b) SWR circle intersects two points in real axis Γ_r.

Point in the right hand side gives $r_L > 1$. ($s = r_L \geq 1$)

$\therefore$ $s \approx 1.63$

4 points

(c) Center and $\bar{z}_L$ point connected line intersects the WTG scale at 0.356λ.

$\bar{z}_{in}$ at ~~...~~ 0.3λ from the load ~~is g~~ will be at

$0.356\lambda + 0.3\lambda - 0.5\lambda = 0.156\lambda$ point.

$\therefore$ $\bar{z}_{in} \approx 1.08 + j0.5$

$\therefore$ $Z_{in} = (1.08 + j0.5) \times 50 = 54 + j25\,(\Omega)$.

4 points

(d) Rotating the impedance by $\frac{\lambda}{4}$, we get the admittance.

$\therefore$ $\bar{y}_{in}$ will be at $0.156\lambda + 0.25\lambda = 0.406\lambda$

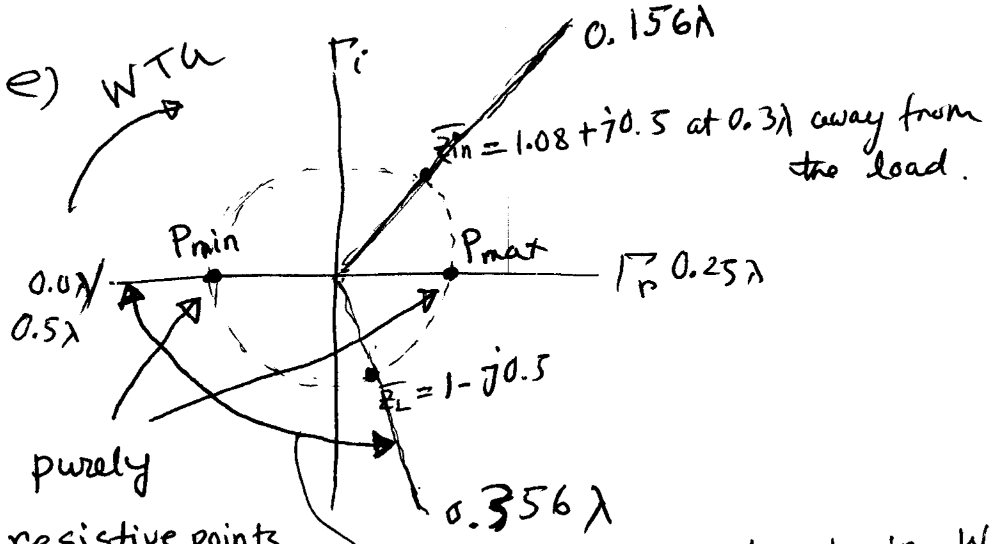
From smith chart $\bar{y}_{in} \approx 0.77 - j0.35$

$$\therefore\ Y_{in} = \bar{y}_{in} Y_0 = \bar{y}_{in}/Z_0 = \frac{0.77 - j0.35}{50}$$

$$= 0.0154 - j0.007\ S$$

$$\left[\ y = \frac{Y}{Y_0} = \frac{G}{Y_0} + j\frac{B}{Y_0} = g + jb\ ,\ \therefore\ Y = Y_0 g + jY_0 b = \frac{g}{Z_0} + j\frac{b}{Z_0}\ \right]$$

5 points (e) WTG



Shortest length in WTG scale for purely resistive input impedance.

∴ Shortest length $0.5\lambda - 0.356\lambda = 0.144\lambda$.

4 points (f) Voltage maximum at Pmax point

∴ Position is $0.144\lambda + 0.25\lambda = 0.394\lambda$

(you can calculate as $0.25\lambda - 0.356\lambda + 0.5\lambda$
$= 0.394\lambda$)

5

5 points (a) According to Stoke's theorem for vector quantity $\mathbb{A}$.

$$\int_s (\nabla \times \mathbb{A}) \cdot ds = \oint_c \mathbb{A} \cdot dl \quad [\ \mathbb{A} \text{ represents vector of } A\ ]$$

$$\therefore \oint_c \mathbb{E} \cdot dl = \int_s (\nabla \times \mathbb{E}) \cdot ds$$

Faraday's law gives $\oint_c \mathbb{E} \cdot dl = -\int_s \frac{\partial \mathbb{B}}{\partial t} \cdot ds$

$$\therefore \int_s (\nabla \times \mathbb{E}) \cdot ds = -\int_s \frac{\partial \mathbb{B}}{\partial t} \cdot ds$$

Taking derivative on both side (S arbitrary)

$$\boxed{\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}}$$

(b) From divergence theoram, for vector quantity $\mathbb{A}$

5 points

$$\int_V \nabla \cdot \mathbb{A}\, dv = \oint_S \mathbb{A} \cdot ds$$

$$\therefore \int_S \mathbb{D} \cdot ds = \int_V \nabla \circ \mathbb{D}\, dv$$

From Gauss's law

$$\int_S \mathbb{D} \cdot ds = \int_V \rho_v\, dv$$

$$\therefore \int_V \nabla \cdot \mathbb{D}\, dv = \int_V \rho_v\, dv$$

(volume arbitrary)

Take derivative in both side

$$\boxed{\nabla \circ \mathbb{D} = \rho_v}$$