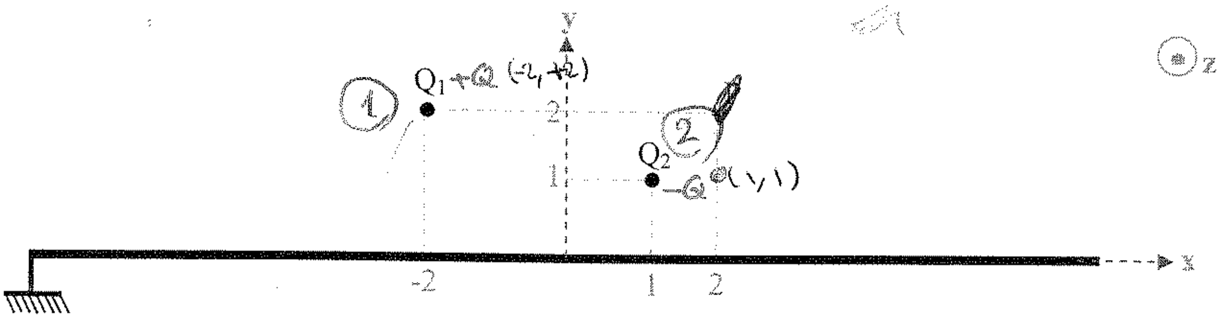
EE101 Midterm; 10/27/2010, Wednesday // **Student Name:** **Student ID#:**
(**OPEN BOOKS AND NOTES**)

1) 12 PTS TOTAL

Two point charges, $Q_1 = +Q$ and $Q_2 = -Q$ are located at (-2, 2, 0) and (1, 1, 0), respectively, above a <u>grounded</u> (V=0) conducting metallic plate coincident with xz plane.



+Q (+1, -1)
4

a) Find the total force **F** acting on Q_1.
3
b) Find **V** at P(2,1,0).
-Q (-2, -2)

a) F at $Q_1 = Q_1 \times E \big|_{at\, Q_1} = +Q\, E_{total}$

$$\vec{E}_{total} = \vec{E}_3\Big|_{at(1)} + E_4\Big|_{at(1)} + E_2\Big|_{at(1)}$$

$$E_3\Big|_{at(1)} = \frac{-Q}{4\pi\epsilon_0}(4\hat{y})/4^3 = \frac{Q}{4\pi\epsilon_0} \times \frac{1}{16}(-\hat{y})$$

$$E_4\Big|_{at(1)} = \frac{+Q}{4\pi\epsilon_0}\left(\frac{-3\hat{x} + 3\hat{y}}{((-3)^2 + (+3)^2)^{3/2}}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{-\hat{x} + \hat{y}}{6\sqrt{18}}\right)$$

$$E_2\Big|_{at(1)} = \frac{-Q}{4\pi\epsilon_0}\left(\frac{-3\hat{x} + \hat{y}}{((-3)^2 + 1^2)^{3/2}}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{+3\hat{x} - \hat{y}}{10\sqrt{10}}\right)$$

$$\Rightarrow E_{total} = \frac{Q}{4\pi\epsilon_0}\left(\hat{x}\left(\frac{-1}{6\sqrt{18}} + \frac{3}{10\sqrt{10}}\right) + \hat{y}\left(\frac{-1}{16} + \frac{1}{6\sqrt{18}} - \frac{1}{10\sqrt{10}}\right)\right)$$

$$\Rightarrow F = \vec{E} \times Q = \frac{+Q^2}{[illegible]}\left(\hat{x}\left(\frac{-1}{[illegible]} + \frac{3}{[illegible]}\right) + \hat{y}\left(\frac{-1}{16} + \frac{1}{18\sqrt{2}} - \frac{1}{10\sqrt{10}}\right)\right)$$

$$V(2,1,0) = V_1 + V_2 + V_3 + V_4$$

$$V_1 = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{4^2+(-1)^2}} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{17}}\right)$$

$$V_2 = \frac{-Q}{4\pi\epsilon_0} \times \frac{1}{\sqrt{1^2+0^2}} = \frac{Q}{4\pi\epsilon_0}(-1)$$

$$V_3 = \frac{-Q}{4\pi\epsilon_0} \frac{1}{\sqrt{4^2+3^2}} = \frac{Q}{4\pi\epsilon_0}\left(\frac{-1}{5}\right)$$

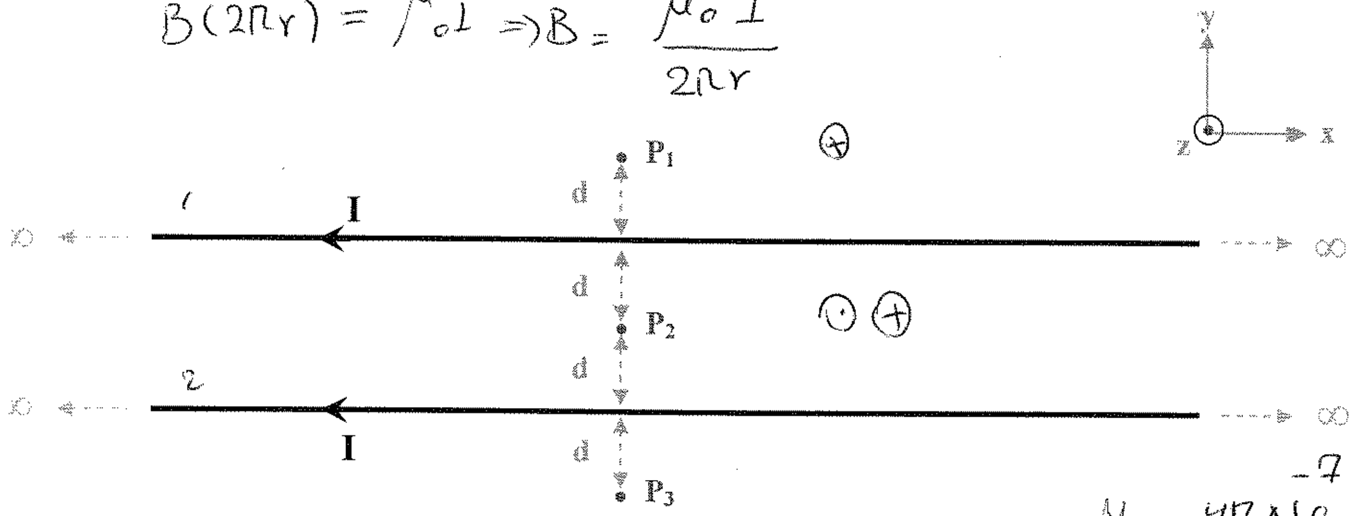
$$V_4 = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{1^2+2^2}} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow V\Big|_{at(P)} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{17}} - 1 + \frac{1}{\sqrt{5}} - \frac{1}{5}\right)$$

2) 12 PTS TOTAL

Determine the magnetic flux density at the points P_1, P_2, and P_3.

$$B(2\pi r) = \mu_o I \Rightarrow B = \frac{\mu_o I}{2\pi r}$$



$\mu_o = 4\pi \times 10^{-7}$

$\mu_o / 2\pi = 2 \times 10^{-7}$

$$P_1 : \vec{B}_1 = \frac{\mu_o I}{2\pi d}\hat{z} \qquad B_2 = \frac{\mu_o I}{2\pi(3d)}\hat{z} \Rightarrow B\Big|_{P_1} = \frac{\mu_o I}{2\pi}\left(\frac{1}{d} + \frac{1}{3d}\right)\hat{z} = \frac{8 d I \times 10^{-7}}{3d^2}\hat{z}$$

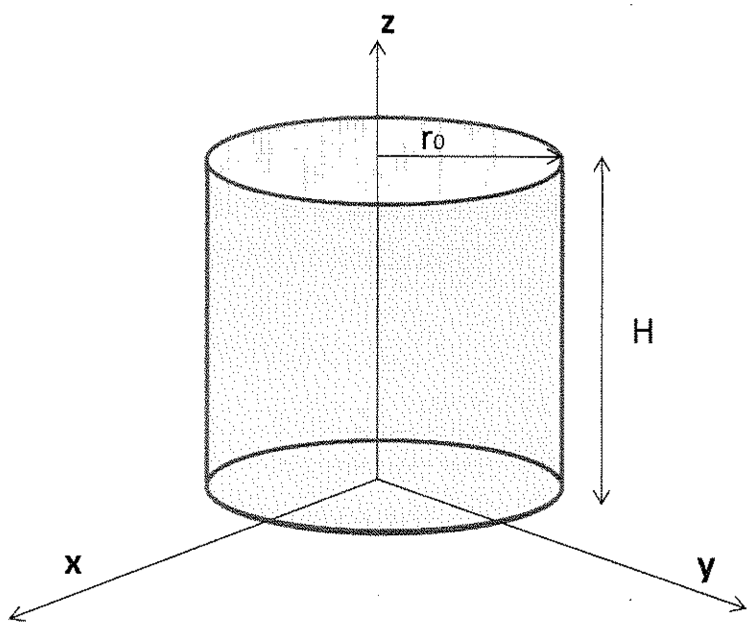
$\frac{4d}{3d^2}$

$$P_2 : \vec{B}_1 = -\vec{B}_2 \Rightarrow \text{total } \vec{B} = 0$$

P_3 : $\vec{B}_3 = -\vec{B}_1$ (the same calculati as P_1 just finally it is going to be $\frac{8 d I \times 10^{-7}}{3d^2}(-\hat{z})$)

3) 7 PTS TOTAL

Given a vector function $\vec{A} = Cos(\varphi)\hat{r}$. Calculate $\oiint \vec{A}.\overrightarrow{ds}$ over the whole surface of a cylinder that is shown below (the center of the bottom circular surface is placed at (0, 0, 0)).



$$\oiint A.ds = \underset{\text{side walls}}{\iint} A.ds + \underset{\text{on top}}{\iint} A.ds + \underset{\text{on bottom}}{\iint} A.ds$$

on top: $=0 \quad (\hat{r}.\hat{z})=0$

on bottom: $=0 \quad (\hat{r}.\hat{z})=0$

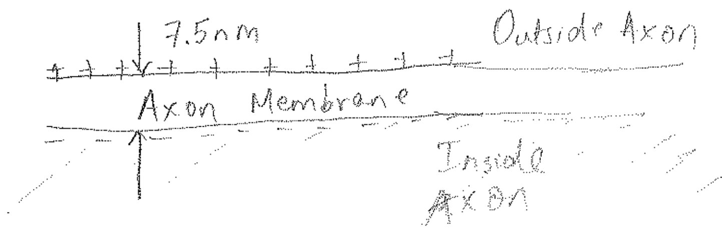
$$\Rightarrow \oiint A.ds = \underset{\text{on side wall}}{\iint} A.ds = \int_0^{2\pi}\int_{z=0}^{H} \cos\varphi \, r_0 \, d\varphi \, dz = \int_0^H dz \underbrace{\int_0^{2\pi} \cos\varphi \, d\varphi}$$

r_0: constant

$$= H \times 0 = 0$$

Sample Midterm Question

1) Cell Membranes (the walled enclosure around a cell) are typically about 7.5nm thick. They are partially permeable to allow charged material to pass in and out, as needed. Equal but opposite charge densities build up on the inside and outside faces of such a membrane, and these charges prevent additional charges from passing through the cell wall. We can model a cell membrane as a capacitor, with the membrane itself containing proteins embedded in an organic material to give the membrane a dielectric constant of about 10.
   a. What is the capacitance per square centimeter of such a cell wall?
   b. In its normal resting state, a cell has a potential difference of 85mV across it membrane. What is the electric field inside this membrane?



Solution:

IDENTIFY: $C = KC_0 = K\epsilon_0 \frac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0\text{ cm}^2 = 1.0\times10^{-4}\text{ m}^2$.

EXECUTE: (a) $C = (10)\frac{(8.85\times10^{-12}\text{ F/m})(1.0\times10^{-4}\text{ m}^2)}{7.5\times10^{-9}\text{ m}} = 1.18\ \mu F$ per cm^2.

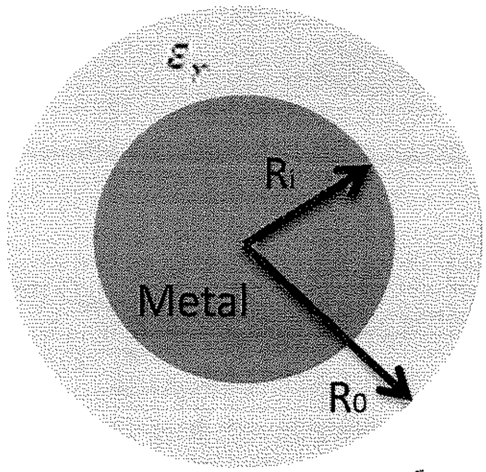
(b) $E = \frac{V}{\cancel{K}d} = \frac{85\text{ mV}}{7.5\times10^{-9}\text{ m}} = 1.13\times10^7\text{ V/m}$.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

The following question (question 5) is a <u>bonus</u> question.

5) 7 PTS

Consider a conducting sphere with diameter R_i which is covered by a spherical dielectric shell with dielectric constant ε_r, in the way that the outer diameter of the sphere is R_o (as you see below). If the charge Q is placed on conducting sphere surface, calculate electric field and potential everywhere in the space ($0 < R < \infty$).



$\vec{D} = \epsilon \vec{E}$

$\Rightarrow$ $\boxed{\begin{array}{ll} \bar{D} = \epsilon_0 \bar{E} & R > R_o \\ \bar{D} = \epsilon_r \epsilon_0 \bar{E} & R_i < R < R_o \end{array}}$

$\iint \bar{D} \cdot ds = Q \qquad \iint E \cdot ds = Q/\epsilon$

$R > R_o \qquad \bar{D}(4\pi R^2) = Q \Rightarrow \bar{D} = \frac{Q}{4\pi R^2}\hat{R} \rightarrow \vec{E} = \frac{Q}{4\pi \epsilon_0 R^2}\hat{R}$

$R_i \le R \le R_o \qquad D(4\pi R^2) = Q \Rightarrow \vec{D} = \frac{Q}{4\pi R^2}\hat{R} \rightarrow \vec{E} = \frac{Q}{4\pi \epsilon_r \epsilon_0 R^2}\hat{R}$

$0 < R \le R_i$ $\longrightarrow$ Metallic shell (sphere) $\rightarrow$

$\vec{E} = 0$

$\vec{D} = 0$

for $R > R_0 \longrightarrow \vec{E} = \dfrac{Q}{4\pi\epsilon_0 R^2}\hat{R} \Rightarrow$

$$V = -\int_{+\infty}^{R} \frac{Q}{4\pi\epsilon_0 R^2} dR\, \hat{R} = \frac{Q}{4\pi\epsilon_0 R} \qquad (R > R_0)$$

$R_i <$ for $\leq R_0$ $\quad \vec{E} = \dfrac{Q}{4\pi\epsilon_r\epsilon_0 R^2}\hat{R} \Rightarrow$

$$V = \int_{\infty}^{R_0} E \cdot dR + \int_{R_0}^{R} E \cdot dR = \frac{Q}{4\pi\epsilon_0 R} + \int_{R_0}^{R} \frac{Q}{4\pi\epsilon_r\epsilon_0 R^2} dR \Rightarrow$$

$$V = \frac{Q}{4\pi\epsilon_0 R} + \frac{Q}{4\pi\epsilon_r\epsilon_0}\left(\frac{1}{R} - \frac{1}{R_0}\right) \qquad (R_i > R \geq R_0)$$

for $0 < R \leq R_i \longrightarrow V = V(R = R_i)$

the previous part

$$= \frac{Q}{4\pi\epsilon_0 R_i} + \frac{Q}{4\pi\epsilon_r\epsilon_0}\left(\frac{1}{R_i} - \frac{1}{R_0}\right)$$

Reason: The metallic surface is the equi-potential surface.