MIDTERM SOLUTIONS

EE101 Midterm; **Student Name:** **Student ID#**
(OPEN BOOKS AND NOTES)

(1) (**7 pts**) Evaluate $\int_0^{\pi}\int_0^{2\pi}(|a\hat{x}\cdot\hat{x}+a\hat{y}\cdot\hat{y}+a\hat{z}\cdot\hat{z}|^2)\,d\varphi\,d\theta+\int_0^{\pi}\int_0^{2\pi}(|a\hat{x}+a\hat{y}+a\hat{z}|+a\hat{R})\,d\varphi\,d\theta$=?
(R,θ,φ) and (x,y,z) define the Spherical and Cartesian coordinate systems, respectively. 'a' is just a constant where $a\neq 0$.

2 $$= \int_0^{\pi}\int_0^{2\pi}\left(|a+a+a|^2\right)d\phi\,d\theta + \int_0^{\pi}\int_0^{2\pi}\left(a^2+a^2+a^2\right)^{1/2}d\phi\,d\theta$$

$$+\int_0^{\pi}\int_0^{2\pi}(a\,\hat{R})\,d\phi\,d\theta =$$

2 $$= \int_0^{\pi}\int_0^{2\pi}|3a|^2\,d\phi\,d\theta + \int_0^{\pi}\int_0^{2\pi}\sqrt{3}\,a\,d\phi\,d\theta + \int_0^{\pi}\int_0^{2\pi}a.\hat{R}\,d\phi\,d\theta =$$

2 $$= (9a^2)(2\pi)(\pi) + (\sqrt{3}\,a)(2\pi)(\pi)$$

$$+\int_0^{\pi}\int_0^{2\pi}a(\sin\theta\cos\phi\,\hat{x}+\sin\theta\sin\phi\,\hat{y}+\cos\theta\,\hat{z})\,d\phi\,d\theta \rightarrow 0$$

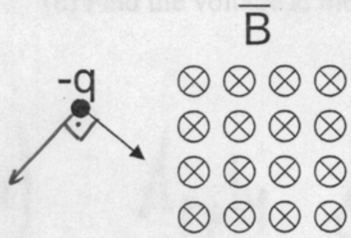
this integral does not need to be evaluated, there is a spherical (3D) symmetry.
(Appropriate explanation takes the same point)

1 $$= 2\pi^2\left(9a^2+\sqrt{3}\,a\right)$$

(2) (**<u>20 pts</u>**) Determine the **initial** direction of the **<u>deflection</u>** of the charged particles **<u>(i.e., the force acting on the particles)</u>** as they enter the magnetic fields as shown below. The charged particle is entering the magnetic field region from the left (as shown below) with a constant speed. The arrow on the charged particle indicates the initial velocity vector of the particle. Draw your answers on the figures shown below. Also EXPLAIN your result briefly!

(a)

5



negative sign of charge changes the direction of deflection.

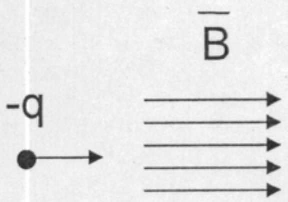
$$\vec{F} = \underline{q\,(\vec{V} \times \vec{B})} + q\vec{E}$$

→ Lorentz Law should be followed.

→ And right hand rule should be used for cross product.

(b)

5



No deflection, cross product of parallel vectors is zero.

(c)

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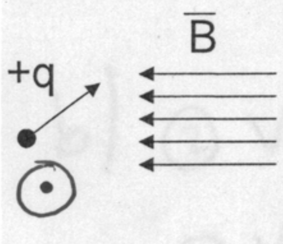

Deflection is out of the paper.

(d)

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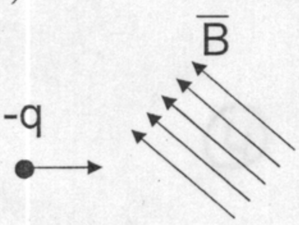

Deflection is into the paper due to the negative charge.

(3) (**21 pts**) A perfectly conducting metallic sphere of radius R is placed in vacuum and is loaded with a total charge of Q. You can assume Q is positive.

(a) Find the electric field (both magnitude and direction) everywhere in space, including within the conductor body.

(b) Find the voltage at the center (i.e., the core) of this metallic sphere.

(c) Find the voltage at the surface of this metallic sphere

a) Apply gauss' Law :

radius of gauss surface $r > R$

outside

4 $\oint \vec{E} \cdot d\vec{s} = \dfrac{Q}{\varepsilon_0} \longrightarrow E \cdot 4\pi r^2 = \dfrac{Q}{\varepsilon_0}$ [V/m]

$$E_{out} = \frac{Q}{4\pi\varepsilon_0 r^2} \longrightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad [V/m]$$

inside

4 $\vec{E}_{in} = 0$ in PEC

b) 2 $V = -\int \vec{E} \cdot d\vec{l}$ $\quad (\vec{E} = E \cdot \hat{r},\ d\vec{l} = dr \cdot \hat{r})$

4 $V = -\int_{\infty}^{0} E \cdot dr = -\cancel{\int_{R}^{0} E_{in}\, dr} - \int_{\infty}^{R} E_{out}\, dr$

$0\ (E_{in} = 0)$

4 $= -\int_{\infty}^{R} \dfrac{Q}{4\pi\varepsilon_0 r^2}\, dr = -\dfrac{Q}{4\pi\varepsilon_0}\left[-\dfrac{1}{r}\right]_{\infty}^{R} = \dfrac{Q}{4\pi\varepsilon_0 R}$ [V]

3 c) $V_{center} = V_{surface} = -\int_{\infty}^{R} E_{out}\, dr = \dfrac{Q}{4\pi\varepsilon_0 R}$ [V]

(4) (**<u>24 pts</u>**) You are given a uniform sphere of charge with a radius b and a volume charge density of ρ. This spherical charge density is located in vacuum.
Calculate the amount of electrostatic energy stored in the following regions:

(a) inside the sphere.
(b) outside the sphere.

(c) What would be the ***<u>additional energy</u>*** needed to double the ***radius*** of the charged sphere to $2b$, while still keeping the uniform charge density ρ the same.

(a) $R < b$ (inside)

(3) → Find $\vec{E}$ by Gauss Law: $\int_S \vec{E}\cdot d\vec{s} = \int_V \rho/\varepsilon \, dV$ $\quad (\vec{E} = E.\hat{R})$

$$E.\,4\pi R^2 = \frac{\rho}{\varepsilon}\,\frac{4\pi R^3}{3} \rightarrow E = \frac{\rho R}{3\varepsilon}$$

(5) → $W_e = \frac{1}{2}\int_V \varepsilon E^2 dV = \frac{1}{2}\frac{\rho^2}{3^2\varepsilon}\int_0^b\int_0^\pi\int_0^{2\pi} R^2\left(R^2 \sin\theta \, d\theta \, dR \, d\phi\right) =$

$$= \frac{1}{2}\frac{\rho^2}{3^2\varepsilon}\left[\frac{R^5}{5}\right]_0^b \left[\phi\right]_0^{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{2\pi}{45\varepsilon}\rho^2 b^5$$

(b) $R > b$ (outside)

(3) $E = \dfrac{\rho b^3}{3\varepsilon R^2}$ (Gauss Law applied as we did in part(a))

(5) $W_e = \frac{1}{2}\varepsilon E^2 dV = \frac{1}{2}\frac{\rho^2 b^6}{3^2\varepsilon}\int_b^\infty\int_0^\pi\int_0^{2\pi} \frac{1}{R^4} R^5 \sin\theta \, dR \, d\theta \, d\phi =$

$$= \text{- - - - - - -} = \frac{16\pi}{9}\rho^2 b^5$$

(4) (c) $R < b \rightarrow W_{add} = \frac{32\pi}{45}\rho^2 (2b)^5 - \frac{32\pi}{45}\rho^2 b^5 = \frac{32\pi}{45}\rho^2 . 31 . b^5$

(4) $R > b \rightarrow W_{add} = \frac{32\pi}{9}\rho^2 (2b)^5 - \frac{32\pi}{9}\rho^2 b^5 = \frac{32\pi}{9}\rho^2 . 31 . b^5$

"b" replaced by "2b"