EE101 Midterm; 11/08/2007, Thursday // **Student Name:** **Student ID#**

(**OPEN BOOKS AND NOTES**)

(1-a) (**5 pts**) Evaluate $\int_0^{\pi}\int_0^{2\pi}\left[\left|a\hat{x}+a\hat{y}+a\hat{z}\right|+\left|a\hat{x}+a\hat{y}+a\hat{z}\right|^2+\left|a\hat{x}+a\hat{y}+a\hat{z}\right|^4+\left|a\hat{x}+a\hat{y}+a\hat{z}\right|^6\right]\,d\phi\,d\theta = ?$

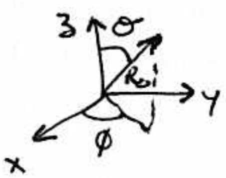
(R, θ, ϕ) and (x,y,z) define the Spherical and Cartesian coordinate systems, respectively.
'a' is just a constant where $a \neq 0$.

$$= \int_0^{\pi}\int_0^{2\pi}\left[(a\sqrt{3}) + (a\sqrt{3})^2 + (a\sqrt{3})^4 + (a\sqrt{3})^6\right] d\phi\, d\theta$$

$$\boxed{= (\pi)(2\pi)\left(\sqrt{3}a + 3a^2 + 9a^4 + 27a^6\right)}$$

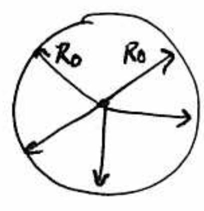
(1-b) (**4 pts**) Evaluate $\int_0^{\pi}\int_0^{2\pi} a\left(x\hat{x}+y\hat{y}+z\hat{z}\right)\,d\phi\,d\theta = ?$ given the fact that $x^2+y^2+z^2=R_o^2$.

(R, θ, ϕ) and (x,y,z) define the Spherical and Cartesian coordinate systems, respectively
'a' and R_o are just non-zero constants.



$$x\hat{x} + y\hat{y} + z\hat{z} = R_o\hat{R}$$

$$\int_0^{\pi}\int_0^{2\pi} a R_o \hat{R}\, d\phi\, d\theta = \boxed{0}$$



(2) (**10 pts**) We define a vector: $\bar{T} = \hat{x}(9-x^3)+\hat{y}(z^3-x^3)+\hat{z}(x^3-y^3)$, where $(\hat{x},\hat{y},\hat{z})$ define the unit vectors in Cartesian coordinate system. Can $\bar{T}$ be a magnetic flux density vector (i.e., $\bar{B}$) that satisfies the Maxwell's 4th Equation ($\nabla\cdot\bar{B}=0$)? Under what conditions can $\bar{T}$ satisfy $\nabla\cdot\bar{B}=0$?

$$\nabla\cdot\bar{T} = \frac{\partial}{\partial x}(9-x^3) + \frac{\partial}{\partial y}(z^3-x^3) + \frac{\partial}{\partial z}(x^3-y^3)$$

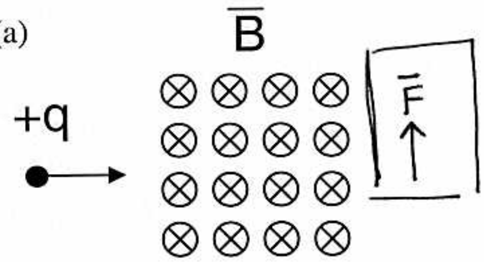
$$= -3x^2 + 0 + 0$$

$$\boxed{\nabla\cdot\bar{T} = -3x^2 \neq 0}$$ So $\bar{T}$ cannot be $\bar{B}$.

* The x-component of $\bar{T}$ must be independent of x.

(3) (**20 pts**) Determine the **initial** direction of the **deflection (i.e., force acting on the particle)** of the charged particles as they enter the magnetic fields as shown below. The charged particle is entering the magnetic field region from the left (as shown below) with a constant speed. Draw your answers on the figures shown below. Also EXPLAIN your result briefly!

(a)



$\overline{F} = q\,\overline{u} \times \overline{B}$

$\overline{F} = q u B \sin\theta$

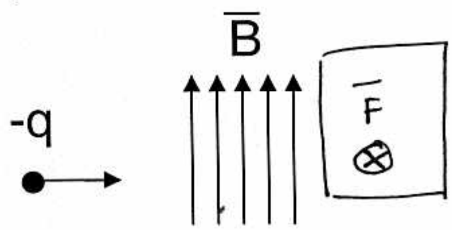
Right hand rule

F = force (deflection)

u = velocity

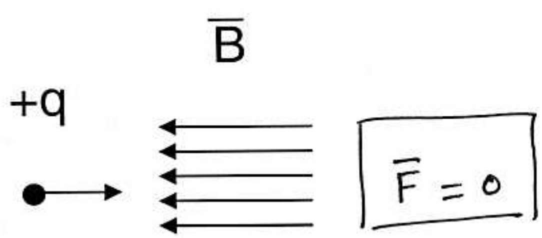
$\overline{B}$ = magnetic flux density

(b)



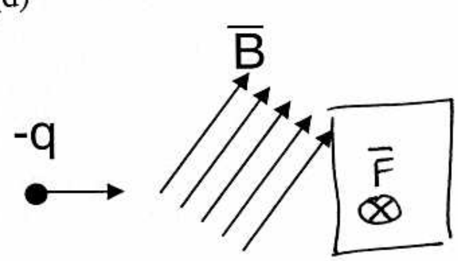
Since charge is negative, force direction reverses.

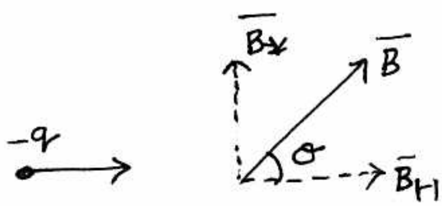
(c)



no force acting as angle between $\overline{u}$ and $\overline{B}$ = 180°

(d)

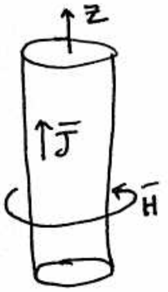




$\overline{B}$ can be decomposed into $\overline{B_H}$ and $\overline{B_V}$

$\overline{u} \parallel B_H$ so no force and $\overline{u} \perp^r \overline{B_V}$ so force into paper (due to negative charge)

(4) (**15 pts**) A uniform current density of $\bar{J} = J_o\hat{z}$ flows thorough a long cylindrical wire of radius a. The current density is uniform for $a \ge r \ge 0$. Find magnetic flux density (i.e., $\bar{B}$) everywhere in space (i.e., $0 < r < \infty$).



Using Ampere's Law,

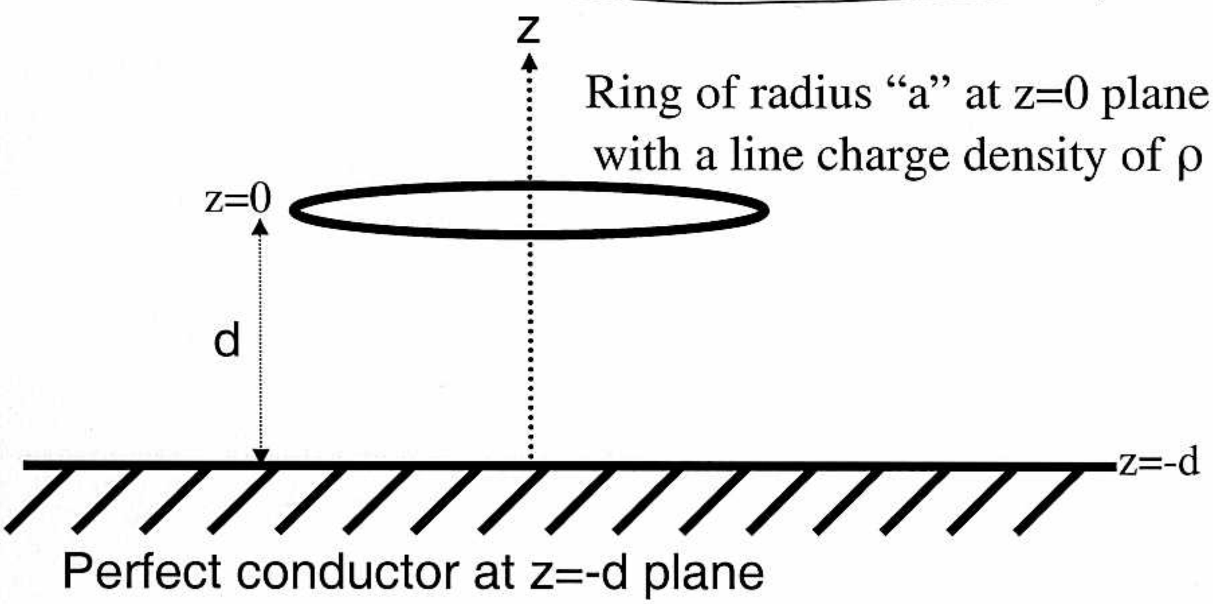
$0 < r < a$ $\quad \oint \bar{H}\cdot d\bar{l} = I \qquad \oint \bar{H}\cdot d\bar{l} = \oint \bar{J}\cdot d\bar{s}$

$\bar{H}(2\pi r) = J_0(\pi r^2) \qquad \bar{H} = \hat{\phi}\,\dfrac{J_0 r}{2}$

$r > a$ $\quad \bar{H}(2\pi r) = J_0(\pi a^2) \qquad \bar{H} = \hat{\phi}\,\dfrac{J_0 a^2}{2r}$

$$\boxed{\bar{B} = \begin{cases} \dfrac{\mu_0 J_0 r}{2}\,\hat{\phi} & 0 < r < a \\ \dfrac{\mu_0 J_0 a^2}{2r}\,\hat{\phi} & r > a \end{cases}}$$

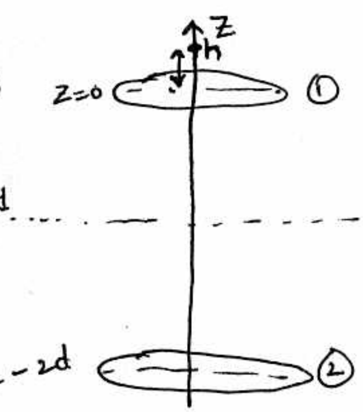
(5) (**20 pts**)



(a) Find the electric field ($\bar{E}$) on the z-axis for **z = h > 0** ?
(b) Find the electric field ($\bar{E}$) on the z-axis for **z = -k where k>d**?
(You can use any derived formula in your text book or class notes regarding a charged ring.)

pg. 159. (Eq 4.23) $\bar{E}$ due to ring of charge $\quad \bar{E} = \dfrac{\rho_l a\cdot z}{2\varepsilon_0(a^2+z^2)^{3/2}}\,\hat{z}$

a)



$\bar{E}(h) = \bar{E}_1(h) + \bar{E}_2(h)$

$$\boxed{\bar{E}(h) = \frac{\rho_l a h}{2\varepsilon_0(a^2+h^2)^{3/2}}\,\hat{z} - \frac{\rho_l a(h+2d)}{2\varepsilon_0(a^2+(h+2d))^{3/2}}\,\hat{z}}$$

Image theory

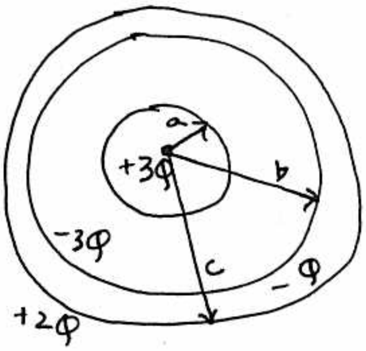
b) $\boxed{\bar{E} = 0}$, inside a conductor

(6) (**20 pts**) A solid ***insulating*** sphere of radius 'a' has a net charge of 3Q, uniformly distributed throughout its volume. Surrounding this sphere is a spherical ***conducting*** shell with an inner radius of 'b' and an outer radius of 'c'. This shell has a net charge of –Q. (Note that a < b < c)

(a) Find the magnitude of $\bar{E}$ in r<a, a<r<b, b<r<c and r>c.

(b) Determine the charge per unit area on the inner surface of the shell.

(c) Determine the charge per unit area on the outer surface of the shell.



$$\boxed{\bar{E} = \begin{cases} \dfrac{3Qr}{4\pi\varepsilon_0 a^3}\hat{R} & r<a \\ \dfrac{3Q}{4\pi\varepsilon_0 r^2}\hat{R} & a<r<b \\ 0 & b<r<c \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{R} & r>c \end{cases}}$$

a) $\oint \bar{E}\cdot d\bar{s} = \dfrac{Q_{en}}{\varepsilon_0}$

$r<a$ $\quad \bar{E}(4\pi r^2) = \dfrac{\rho_v}{\varepsilon_0}\left(\dfrac{4}{3}\pi r^3\right)$ $\quad \rho_v = \dfrac{Q_{enc.}}{\text{volume}} = \dfrac{3Q}{\frac{4}{3}\pi a^3} = \dfrac{9Q}{4\pi a^3}$

$$\bar{E} = \left(\frac{9Q}{4\pi a^3}\right)\frac{1}{\varepsilon_0}\left(\frac{4}{3}\pi r^3\right)\frac{1}{4\pi r^2} = \frac{3Qr}{4\pi\varepsilon_0 a^3}\hat{R}$$

$a<r<b$ $\quad \bar{E}(4\pi r^2) = \dfrac{3Q}{\varepsilon_0}$ $\quad \bar{E} = \dfrac{3Q}{4\pi\varepsilon_0 r^2}\hat{R}$

$b<r<c$ $\quad \bar{E} = 0$, inside conductor

$r>c$ $\quad \bar{E}(4\pi r^2) = \dfrac{(3Q-Q)}{\varepsilon_0}$ $\quad \bar{E} = \dfrac{Q}{2\pi\varepsilon_0 r^2}\hat{R}$

b) Total charge on inner shell $= -3Q$ $\quad \boxed{\text{Charge per unit area} = \dfrac{-3Q}{4\pi b^2}}$

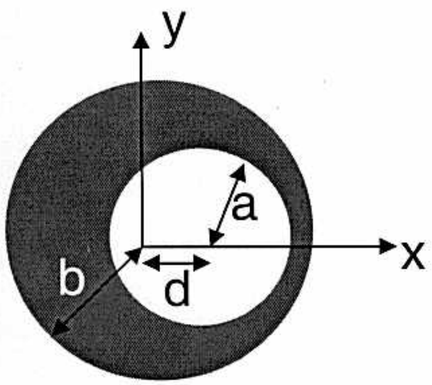
c) Total charge on outer shell $= 2Q$ $\quad \boxed{\text{Charge per unit area} = \dfrac{Q}{2\pi c^2}}$

(7) (**6 pts**) The cross section shown below is a cylindrical geometry that extends to infinity along +z and –z directions. The outer radius of the circular cross section is ‘b’. In the same cross section there exists a **cylindrical cavity** that has free space given with a radius of ‘a’. As shown in the figure the center of this **cylindrical void cavity** is offset from the center of the cylinder by ‘d’.

This structure carries a uniform current density of J_o along the z direction. **Assume μ_o for both the cavity and the conductor region.**

Find $\bar{B}$ within the cavity!

Hint: You can hypothetically create a similar cavity by considering a current density of $-J_o$ that flows within a cylinder of radius ‘a’. Then, use the superposition of two uniform cylindrical conductors with radii ‘b’ and ‘a’ that carry opposite charge densities of J_o and $-J_o$, respectively.

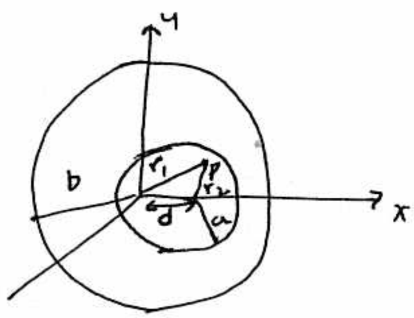


$$\oint \bar{H}\cdot d\bar{l} = I = \int_s \bar{J}\cdot ds$$

$$\bar{H} = \frac{J_o r}{2}\hat{\phi} \qquad \bar{B} = \mu_o \bar{H} = \frac{\mu_o J_o}{2} r\,\hat{\phi}$$

If there is no cavity, for J_o

$$\bar{B}_1 = \frac{\mu_o J_o r_1}{2}\hat{\phi}_1$$

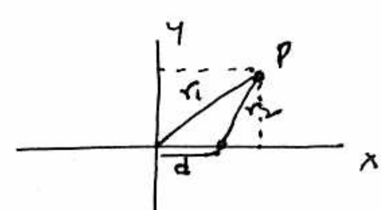


$$\hat{\phi}_1 = -\hat{x}\sin\phi_1 + \hat{y}\cos\phi_1$$

$$\hat{\phi}_1 = \frac{-y_1}{\sqrt{x_1^2+y_1^2}}\hat{x} + \frac{x_1}{\sqrt{x_1^2+y_1^2}}\hat{y} \qquad r_1^2 = x_1^2+y_1^2$$

$$\bar{B}_1 = -\frac{\mu_o J_o y_1}{2}\hat{x} + \frac{\mu_o J_o x_1}{2}\hat{y}$$

For $-J_o$ in the cavity region



$$\bar{B}_2 = \frac{\mu_o J_o y_2}{2}\hat{x} - \frac{\mu_o J_o x_2}{2}\hat{y}$$

$x_1 = x_2 + d$

$y_1 = y_2$

By superposition

$$\bar{B} = \bar{B}_1 + \bar{B}_2 = -\frac{\mu_o J_o y_1}{2}\hat{x} + \frac{\mu_o J_o x_1}{2}\hat{y} + \frac{\mu_o J_o y_1}{2}\hat{x} - \frac{\mu_o J_o (x_1-d)}{2}\hat{y}$$

$$\boxed{\bar{B} = \frac{\mu_o J_o d}{2}\hat{y}}$$ Constant magnetic flux density