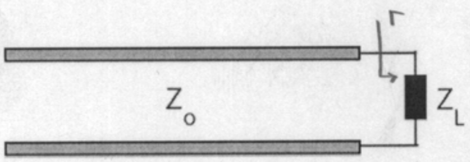
EE101 SAMPLE FINAL **Student Name:** **Student ID#**

(**OPEN BOOKS AND NOTES**)

(1) Assume a **LOSSLESS** transmission line as shown below:



(a) (1 pts) What is the reflection coefficient (at the load) Γ = ?

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \quad //$$

(b) (2 pts) Does Γ depend on the frequency of the generator source ? Explain.

Z_o is frequency independent

Z_L in general is frequency dependent

$\therefore$ Γ in general is frequency dependent //

(c) (6 pts) For the same lossless transmission line shown above **find the reflection coefficient** using the attached Smith chart **at each** of the following cases:

(a) $Z_L = 3\ Z_o$

(b) $Z_L = 2\,j\ Z_o$

(d) $Z_L = 0$

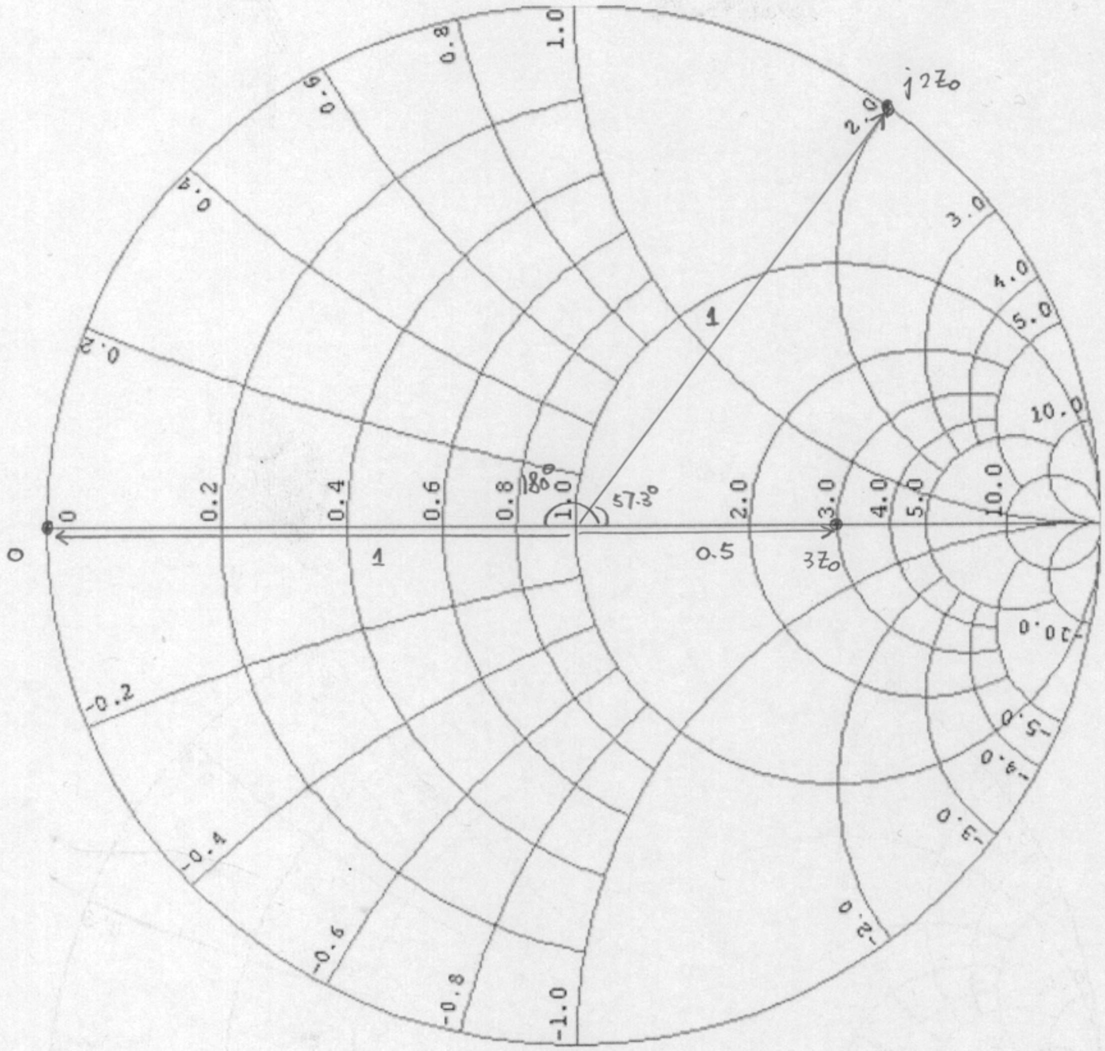
--Show your drawings on the Smith Chart that is attached to the end of the exam sheet.

--Verify the reflection coefficients that you found at the Smith chart using your answer to Question (a) above.

(a) $$\Gamma = \frac{3Z_o - Z_o}{3Z_o + Z_o} = 0.5 \quad //$$

(b) $$\Gamma = \frac{j2Z_o - Z_o}{j2Z_o + Z_o} = 0.6 + j0.8 = 1e^{j57.3^\circ} \quad //$$

(c) $$\Gamma = \frac{0 - Z_o}{0 + Z_o} = -1 = 1e^{j180^\circ} \quad //$$



$j2Z_0$
1
57.3°
180°
1
0.5
$3Z_0$
0

(2) (4 pts) Evaluate $\int_0^{\pi}\int_0^{2\pi}\left[\left|a\hat{x}+a\hat{y}+a\hat{z}\right|+\hat{R}+\left|\hat{R}\right|^3+\left|\hat{R}\right|^5\right]\,\mathrm{d}\phi\,\mathrm{d}\theta = ?$

(R, θ, ϕ) and (x,y,z) define the Spherical and Cartesian coordinate systems, respectively.
'a' is just a **constant** where a≠0.

$$\int_0^{\pi}\int_0^{2\pi} (\sqrt{3}a + \hat{R} + 2)\, d\phi\, d\theta$$

$$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\therefore \quad \sqrt{3}\,a\cdot(2\pi^2) + 0 + 2\cdot(2\pi^2)$$

$$2\pi^2(2+\sqrt{3}a) \quad //$$

(3) (8 pts) Using the **time varying** Maxwell's equations derive the **wave equation** in free space for the **H** field.

$$\nabla \times \underset{\sim}{E} = -\mu_0 \frac{\partial \underset{\sim}{H}}{\partial t}$$ assume source-free, $\underset{\sim}{J} = 0$

$$\nabla \times \underset{\sim}{H} = \epsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t} + \underset{\sim}{J}$$

$$\nabla \cdot \underset{\sim}{E} = 0$$

$$\nabla \cdot \underset{\sim}{H} = 0$$

using the Ampère's law,

$$\nabla \times \nabla \times \underset{\sim}{H} = \epsilon_0 \frac{\partial \nabla \times \underset{\sim}{E}}{\partial t} + \nabla \times \underset{\sim}{J}$$

$$\nabla(\nabla \cdot \underset{\sim}{H}) - \nabla^2 \underset{\sim}{H} = -\mu_0\epsilon_0 \frac{\partial^2 \underset{\sim}{H}}{\partial t^2} + \nabla \times \underset{\sim}{J} \leftarrow \underset{\sim}{J} = 0$$

$$\nabla^2 \underset{\sim}{H} - \mu_0\epsilon_0 \frac{\partial^2 \underset{\sim}{H}}{\partial t^2} = 0 \quad //$$

(4) (a) (4 pts) For the electric field phasor expression $\overline{E} = 2j(\hat{y} - j\hat{z})\exp(j\beta x)$, write down the **instantaneous time-varying form** of the electric field. You can assume an angular frequency of ω for the electric field. **Assume also that β is a real quantity for questions (a-d).**

$$\underline{E}(x,t) = Re\{(\hat{y}\,2e^{j\frac{\pi}{2}} + \hat{z}\,2)\,e^{j\beta x}\,e^{j\omega t}\}$$
$$= Re\{\hat{y}\,2e^{j\frac{\pi}{2}}e^{j\beta x}e^{j\omega t} + \hat{z}\,2e^{j\beta x}e^{j\omega t}\}$$
$$= \hat{y}\,2\cos(\omega t + \beta x + \tfrac{\pi}{2}) + \hat{z}\,2\cos(\omega t + \beta x) \quad //$$

(b) (3 pts) What is the propagation direction of the constant phase fronts of the electric field given in (a).

$-\hat{x}$ //

(c) (4 pts) What is the speed of propagation of the constant phase fronts of the electric field given in (a).

$$\beta = \frac{\omega}{c}$$
$$\Rightarrow c = \frac{\omega}{\beta} \quad //$$

(d) (4 pts) Does this time-varying electric field satisfy the Wave Equation in free space for the E field? Prove your answer!

$$\nabla^2 \underline{E} - \mu_o\epsilon_o \frac{\partial^2 \underline{E}}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial x^2} = -2\beta^2\cos(\omega t + \beta x + \tfrac{\pi}{2}), \qquad \frac{\partial^2 \underline{E}}{\partial t^2} = -\hat{y}\,2\omega^2\cos(\omega t + \beta x + \tfrac{\pi}{2}) - \hat{z}\,2\omega^2\cos(\omega t + \beta x)$$

$$\frac{\partial^2 E_z}{\partial x^2} = -2\beta^2\cos(\omega t + \beta x)$$

$$\therefore\ \hat{y}\frac{\partial^2 E_y}{\partial x^2} + \hat{z}\frac{\partial^2 E_z}{\partial x^2} = -\hat{y}\,2\beta^2\cos(\omega t + \beta x + \tfrac{\pi}{2}) - \hat{z}\,2\beta^2\cos(\omega t + \beta x)$$
$$= -\hat{y}\,2\left(\frac{\omega}{c}\right)^2\cos(\omega t + \beta x + \tfrac{\pi}{2}) - \hat{z}\,2\left(\frac{\omega}{c}\right)^2\cos(\omega t + \beta x)$$
$$= \frac{1}{c^2}\frac{\partial^2 \underline{E}}{\partial t^2} \quad // \qquad c = \frac{1}{\sqrt{\mu_o\epsilon_o}}$$

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(e) (8 pts) How would you modify your answers to the above questions (b), (c) and (d) if β was a complex quantity instead of being purely real? Assume for this question that $\beta = -a + jb$, where both a and b are **positive real** numbers.

$$\bar{E} = 2j(\hat{y} - j\hat{z})e^{j(-a+jb)x} = 2j(\hat{y} - j\hat{z})e^{-jax}e^{-bx}$$

(b) $+\hat{x}$ //

(c) $a = \frac{\omega}{c} \Rightarrow c = \frac{\omega}{a}$ //

(d) should still satisfy the time-harmonic Wave Eqn. //

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0$$

$\uparrow$ complex number $(-a+jb)^2$

(f) (6 pts) Assuming the same parameters as in part (e) for this electric field, what is the propagation distance that the intensity of this electric field drops to $\left(\frac{1}{e}\right)^2$? Also what is the propagation distance that the amplitude of this electric field drops to $\left(\frac{1}{e}\right)^2$? Electric field intensity is defined as the square of the amplitude.

$$\bar{E} = 2j(\hat{y} - j\hat{z})e^{-jax}e^{-bx} = \bar{E}_0(x)e^{-bx}$$

the magnitude of $\bar{E}_0$ is 2

at $x = 0$, $|\bar{E}| = |\bar{E}_0|e^{-b0} = |\bar{E}_0| = 2$

at $x = x_0$, $|\bar{E}| = |\bar{E}_0|e^{-bx_0} = 2e^{-bx_0} = \frac{1}{e^2}$

$\therefore -bx_0 + \ln 2 = -2$

$x_0 = \frac{1}{b}(2 + \ln 2)$ //

at $x = 0$, $|\bar{E}|^2 = 4$

at $x = x_1$, $|\bar{E}|^2 = 4e^{-2bx_1} = \frac{1}{e^2}$

$\therefore -2bx_1 + \ln 4 = -2$

$x_1 = \frac{1}{2b}(2 + 2\ln 2)$ //

(g) (6 pts) Assuming the same parameters as in part (e) for this electric field, what is the propagation distance that the constant phase fronts of this electric field accumulates $\pi/4$ phase?

$$ax = \frac{\pi}{4}$$

$$x = \frac{\pi}{4a} \quad //$$