Name: Solutions

February 15, 2006

UCLA EE101
MIDTERM

This is a closed book, closed note examination. If you need to know an integral or the value of a physical constant which is not given, please feel free to ask. The formula sheets are attached at the back so just tear them off.

Please work neatly and clearly, and please clearly indicate your final answer. <u>What we cannot read, we cannot grade.</u> Correct results derived from incorrect or incomplete work will receive little if any credit. The problems are not in any particular order of difficulty. **Good luck!!!!**

Problem	Score
1	
2	
3	
4	
Total	

1. For this problem let $Z_1 = \sqrt{3} + j$ and $Z_2 = 1 + j\sqrt{3}$ and at given point in space the vectors **A** and **B** are given in spherical coordinates by

$$\mathbf{A} = \hat{R}4 + \hat{\theta}2 - \hat{\phi}$$
$$\mathbf{B} = -\hat{R}2 + \hat{\theta}4 + \hat{\phi}0$$

Find:

(5) a) The ratio $\frac{Z_1}{Z_2}$.
(5) b) $\mathbf{A} \times \mathbf{B}$.
(5) c) The unit vector along **A**.
(5) d) The projection of **B** along **A**.
(5) e) The vector component of **B** perpendicular to **A**.

a) $$\frac{\sqrt{3}+j}{1+j\sqrt{3}} = \frac{\sqrt{3}+j}{1+j\sqrt{3}} \cdot \frac{1-j\sqrt{3}}{1-j\sqrt{3}} = \frac{\sqrt{3}-3j+j+\sqrt{3}}{4} = \frac{\sqrt{3}}{2} - \frac{1}{2}j = e^{-j\pi/6}$$

$$= \frac{2e^{j\pi/6}}{2e^{j\pi/3}} \qquad = e^{-j\frac{\pi}{6}}$$

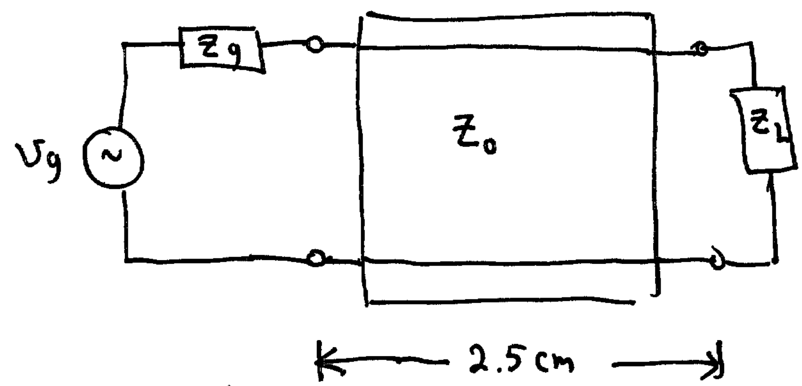
b) $$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\varphi} \\ 4 & 2 & -1 \\ -2 & 4 & 0 \end{vmatrix} = \hat{R}(+4) + \hat{\theta}2 + \hat{\varphi}20$$

c) $$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{R}4 + \hat{\theta}2 - \hat{\varphi}}{\sqrt{16+4+1}} = \frac{1}{\sqrt{21}}\left(\hat{R}4 + \hat{\theta}2 - \hat{\varphi}\right)$$

d) $$(\vec{A}\cdot\vec{B})\hat{A} = (-8+8+0)\hat{A} = 0\hat{A}$$

e) $$\vec{B} - (\vec{A}\cdot\vec{B})\hat{A} = \vec{B} - 0 = \vec{B}$$

2. A voltage generator with $v_g = 5sin(2\pi \times 10^9)V$ and internal impedance $Z_g = 50\Omega$ is connected to a 100 Ω lossless air-spaced transmission line with $v_\phi = c/3$. The line length is 2.5 cm and it is terminated in a load with impedance $Z_L = 300\Omega$. Find the following:



(5) a) Γ at the load.
(5) b) Z_{in} at the input to the transmission line.
(5) c) the input voltage $\tilde{V}$ and input current $\tilde{I}$.
(10) d) the current $i(z,t)$ along the line.

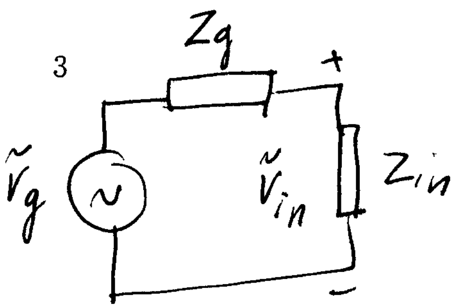
a) $\Gamma = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{300-100}{300+100} = 0.5$

b) $v_\phi = 10^8 = \lambda f \quad f = \dfrac{\omega}{2\pi} = 10^9 \rightarrow \lambda = 0.1^{m} = 10^{cm}$

$l = \lambda/4$ quarter-wavelength T.L.

$Z_{in} = \dfrac{Z_o^2}{Z_L} = \dfrac{100^2}{300} = 33.3\ \Omega$

c) $\tilde{V}_{in} = \tilde{V}_g \dfrac{Z_{in}}{Z_{in}+Z_g}$



$$\tilde{V}_g = 5\angle -90^\circ$$

$$\tilde{V}_{in} = 5\angle -90^\circ \; \frac{33.3}{33.3+50} = 2\angle -90^\circ$$

$$\tilde{I}_{in} = \frac{\tilde{V}_{in}}{Z_{in}} = \frac{2\angle -90^\circ}{33.33} = 0.06\angle -90^\circ$$

d)

$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{+j\beta z} = I_o^+ \left(e^{-j\beta z} - \Gamma e^{j\beta z} \right)$$

$$I(t)\Big|_{z=-2.5\text{ cm}} = \tilde{I}_{in} = I_o^+ \left(e^{+j\pi/2} - \underset{\underset{0.5}{\uparrow}}{\Gamma} e^{-j\pi/2} \right) = 0.06\angle -90^\circ$$

$$\beta z = \frac{2\pi}{\lambda}\cdot\left(-\frac{\lambda}{4}\right) = -\pi/2$$

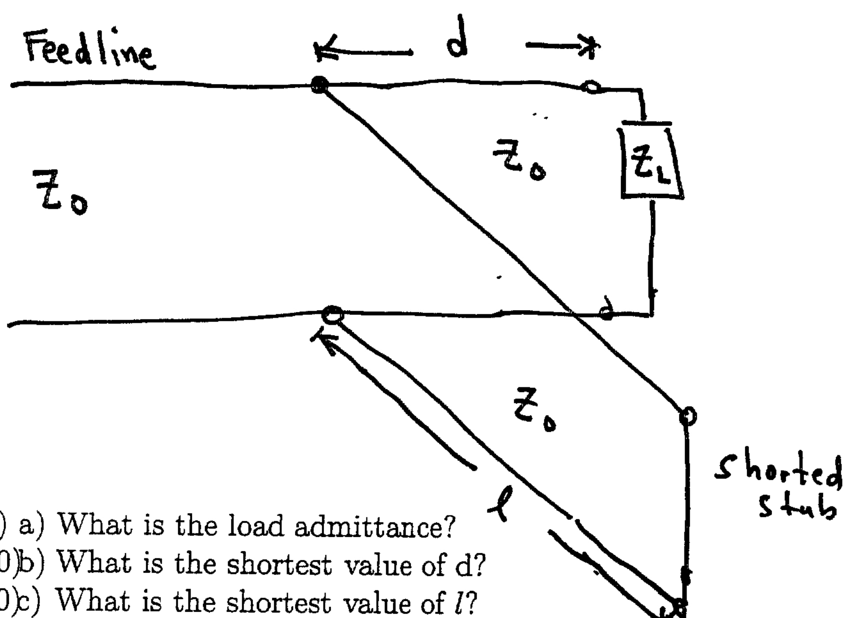
$$I_o^+ = \frac{0.06\angle -90^\circ}{j + 0.5j} = 0.04\angle -\pi = -0.04$$

$$\tilde{I}(z) = -0.04\left(e^{-j\beta z} - 0.5 e^{j\beta z}\right)$$

$$i(z,t) = -0.04 \cos(2\pi \times 10^9 t - \beta z) + 0.04 \times 0.5 \cos(2\pi \times 10^9 t + \beta z)$$

where $\beta = \frac{2\pi}{\lambda}$ & $\lambda = 0.1^{m}$

3. A 100Ω transmission line is connected to an antenna with load impedance $Z_L = 50 - j100\Omega$. Find the position (the length d) and length (the length l) of the short circuited stub required to match the line. For this problem use attached Smith Chart. There is an extra in case you need it. Do your best job to draw circles if you don't have a protractor. You will be graded for showing points on the chart, drawing appropriate circles and rotating around the chart in the proper direction. Carefully label all points on the chart. If you read numbers off the chart properly but your answer is incorrect because you have not drawn a perfect circle you will receive full credit. (Hint: The impedance and the admittance of the shorted stub is purely imaginary, i.e., it is reactive).



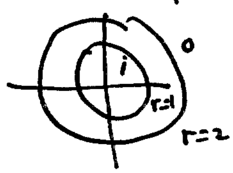
(5) a) What is the load admittance?
(10) b) What is the shortest value of d?
(10) c) What is the shortest value of l?

see example 2-12
normalized admittances are identical

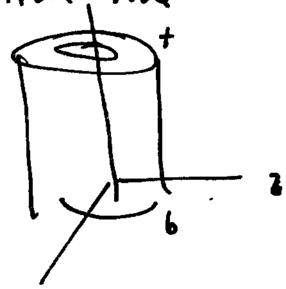
4. A vector field $\mathbf{D} = \hat{r}r^3 + \hat{\phi}0 + \hat{z}0$ exists in the region between two concentric cylindrical surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Evaluate the following: evaluate the following:

(10) a) $\oint \mathbf{dS} \cdot \mathbf{D}$.
(5) b) The divergence of $\mathbf{D}$.
(10) c) $\int dv \nabla \cdot \mathbf{D}$. (Explicitly carry out this integral)

a) From top



from side



four surfaces

$$\oint d\vec{S} \cdot \vec{D} = \int_{\text{inside}} + \int_{\text{outside}} + \int_{\text{top}} + \int_{\text{bottom}}$$

$$= \int_{r=1} dz\, r\, d\varphi\, (-\hat{r}) \cdot \vec{D} + \int_{r=2} dz\, r\, d\varphi\, (+\hat{r}) \cdot \vec{D} + \overbrace{0 + 0}^{\hat{z} \cdot \vec{D} = 0}$$

$$= -\int_0^5 dz \int_0^{2\pi} d\varphi\, 1 + \int_0^5 dz \int_0^{2\pi} d\varphi\, 2^4$$

$$= -2\pi 5 + 2\pi \times 5 \times 16$$

$$= 150\pi$$

b) In cylindrical coordinates $\nabla \cdot \vec{D} = \frac{1}{r}\frac{\partial}{\partial r} r D_r + \frac{1}{r}\frac{\partial}{\partial \varphi} A_\varphi + \frac{1}{z}\frac{\partial A_z}{\partial z}$

$$= 4r^2$$

c) $$\int_0^5 dz \int_1^2 r\, dr \int_0^{2\pi} d\varphi\, 4r^2 = 5 \times 2\pi \int_1^2 dr\, 4r^3$$

$$= 5 \times 2\pi\, r^4 \Big|_1^2 = 150\pi \checkmark$$