

# Solution to Quiz 1

## Part 1

$$\begin{aligned}
 r &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
 &= \sqrt{R'G' \left(1 + j\omega \frac{L'}{R'}\right) \left(1 + j\omega \frac{C'}{G'}\right)} \quad \text{but } \frac{L'}{R'} = \frac{C'}{G'} \\
 &= \sqrt{R'G' \left(1 + j\omega \frac{C'}{G'}\right)^2} \\
 &= \left(1 + j\omega \frac{C'}{G'}\right) \sqrt{R'G'} = \alpha + j\beta
 \end{aligned}$$

$$\alpha = \sqrt{R'G'} \quad , \quad \beta = \omega \sqrt{L'C'}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$$

$\alpha$  and  $u_p$  are both independent of  $\omega$ , line is distortionless.

## Part 2

$$\frac{L'}{R'} = \frac{C'}{G'} \Rightarrow R'C' = G'L' \quad \text{or } G' = \frac{R'C'}{L'}$$

$$\begin{aligned}
 Z_0 &= \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}} = \sqrt{\frac{R'}{G'} \frac{(1 + j\omega \frac{L'}{R'})}{(1 + j\omega \frac{C'}{G'})}} = \sqrt{\frac{R'}{G'}} = \sqrt{\frac{L'}{C'}} \\
 \alpha &= \sqrt{R'G'} = R' \sqrt{\frac{L'}{C'}} = \frac{R'}{Z_0} \Rightarrow R' = \alpha Z_0 = 20 \times 10^{-3} \cdot 60 = 1.2 \frac{\Omega}{m}
 \end{aligned}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} \quad \text{and} \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$$

$$L' = \frac{Z_0}{u_p} = \frac{60}{0.6(3 \times 10^8)} = 333 \frac{nH}{m}$$

$$G' = \frac{\alpha^2}{R'} = \frac{400 \times 10^{-6}}{1.2} = 333 \frac{\mu S}{m}$$

$$u_p \cdot Z_0 = \frac{1}{C'}$$

$$C' = \frac{1}{u_p Z_0} = \frac{1}{0.6(3 \times 10^8) \cdot 60} = 92.59 \frac{pF}{m}$$