EE 110
CIRCUIT ANALYSIS II
A. N. Willson, Jr.

NAME: WILLSON

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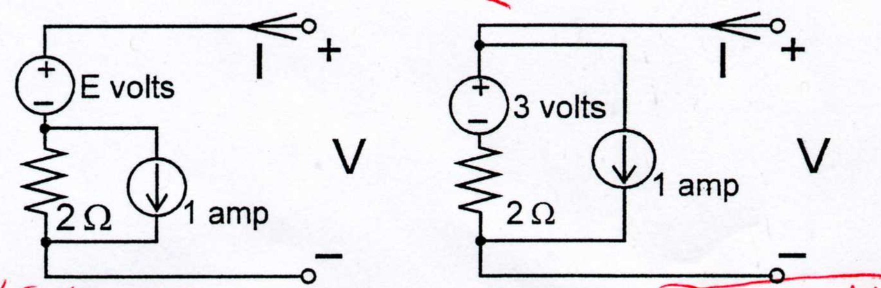
# MID-TERM EXAMINATION

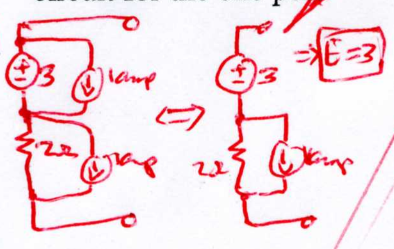
Do all work in this examination packet. There are four questions. Each counts 25 points. Good luck!

1A. (5 points) Find the value of E for which the two one-ports shown to the right have the same V vs. I relations.

1B. (5 points) Find the Thevenin's equivalent circuit for the one port.




1A

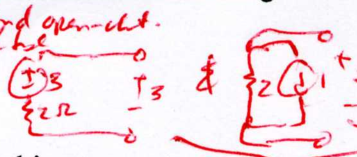



$\Rightarrow \boxed{E=3}$

1B

Set sources to zero:

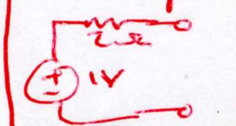

Find open-ckt. voltage



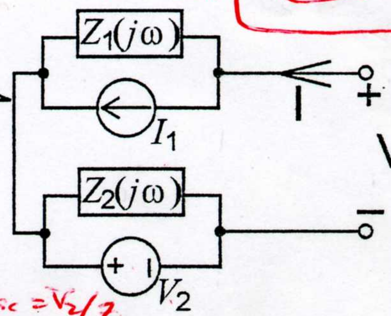

Superpos $\Rightarrow V_{oc} = 3-2 = 1$

$\Rightarrow$

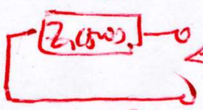
Thev. eq. ckt




1C. (10 points) Find the Norton's equivalent circuit for this one-port.

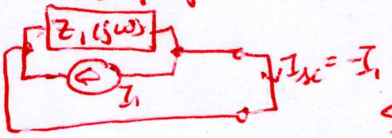



Set $I_1 = 0$ & $V_2 = 0$ to get $Z_{Norton}$:

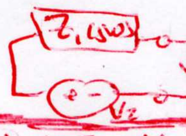



$\leftarrow Z_{Norton} = Z_1(j\omega)$

Use superposition to get $I_{Norton}$: ① Turn on $I_1$ only ($V_2 = 0$)



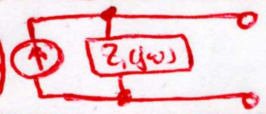

② Turn on $V_2$ only ($I_1 = 0$)




So! $I_{sc} = \frac{V_2}{Z_1} - I_1$

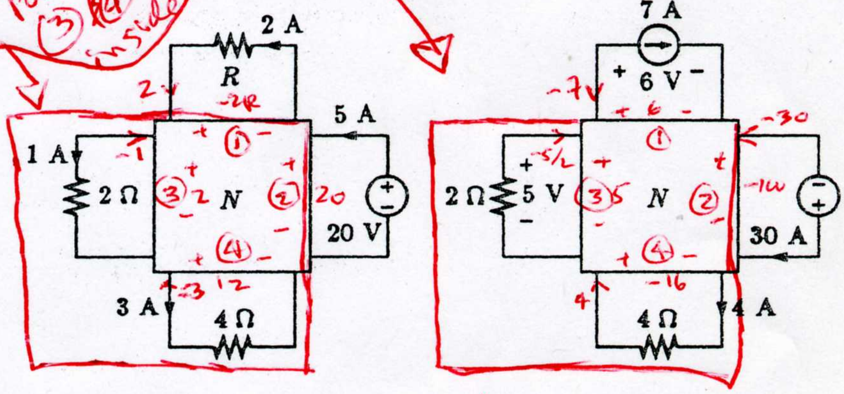
i.e. Norton's Eq. Ckt. is:

$\left(\frac{V_2(j\omega)}{Z_1(j\omega)} - I_1(j\omega)\right)$




1D. (5 points) Network *N* contains only linear resistors. Determine the value of *R*.

leave ports ③ & ④ inside




| | ① | ② | ③ | ④ |
|---|---|---|---|---|
| V | -2R | 20 | 2 | 12 |
| I | 2 | 5 | -1 | -3 |
| $\tilde{V}$ | 6 | -100 | 5 | -16 |
| $\tilde{I}$ | -7 | -30 | -5/2 | 4 |

$V \times \tilde{I}$

$(-7)(-2R)$ -600 | ~~-5 48~~

$\tilde{V} \times I$

12 -500 | ~~-5 48~~

ports ③ & ④ are unnecessary!

$(V, I)$ $(\tilde{V}, \tilde{I})$

$(14)R - 600 = 12 - 500 \Rightarrow 14R = 112 \Rightarrow R = \frac{112}{14} = 8$

$\boxed{R=8}$

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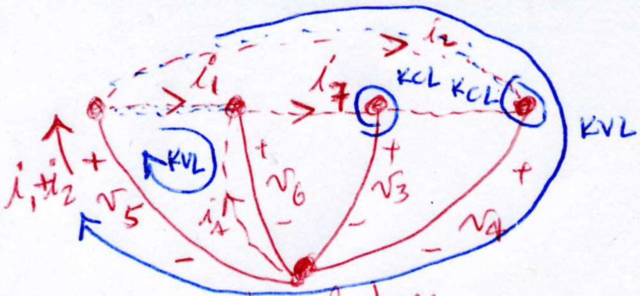
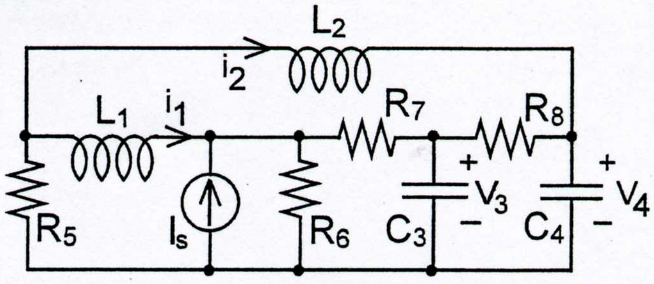
2. (25 points) Write state equations for this circuit.



Normal tree




$$L_1\frac{di_1}{dt} = -R_5(i_1+i_2) - \underline{v_6}$$
$$L_2\frac{di_2}{dt} = -R_5(i_1+i_2) - v_4$$
$$C_3\frac{dv_3}{dt} = \underline{i_7} + \frac{1}{R_8}(v_4-v_3)$$
$$C_4\frac{dv_4}{dt} = i_2 + \frac{1}{R_8}(v_3-v_4)$$

We (just!) need to express $v_6$ & $i_7$ in terms of the state variables ($v_3, v_4, i_1, i_2$).

One more KCL:
$$i_7 + \frac{1}{R_6}v_6 = i_1 + i_s \quad (A)$$
One more KVL:
$$i_7R_7 + v_3 = v_6 \quad (B)$$

Substituting $v_6$ from (B) into (A) gives:
$$i_7 + \frac{R_7}{R_6}i_7 + \frac{1}{R_6}v_3 = i_1 + i_s$$
$$\Rightarrow i_7 = \frac{R_6}{R_6+R_7}i_1 - \frac{1}{R_6+R_7}v_3 + \frac{R_6}{R_6+R_7}i_s \quad (C)$$

Then, putting (C) into (B) gives:
$$v_6 = v_3 + \frac{R_6R_7}{R_6+R_7}i_1 - \frac{R_7}{R_6+R_7}v_3 + \frac{R_6R_7}{R_6+R_7}i_s = \frac{R_6}{R_6+R_7}v_3 + \frac{R_6R_7}{R_6+R_7}i_1 + \frac{R_6R_7}{R_6+R_7}i_s$$

$$\therefore \begin{cases} L_1\frac{di_1}{dt} = -R_5 i_1 - R_5 i_2 - \frac{R_6}{R_6+R_7}v_3 - \frac{R_6R_7}{R_6+R_7}i_1 - \frac{R_6R_7}{R_6+R_7}i_s \\ L_2\frac{di_2}{dt} = -R_5 i_1 - R_5 i_2 - v_4 \\ C_3\frac{dv_3}{dt} = \frac{R_6}{R_6+R_7}i_1 - \frac{1}{R_6+R_7}v_3 - \frac{1}{R_8}v_3 + \frac{1}{R_8}v_4 + \frac{R_6}{R_6+R_7}i_s \\ C_4\frac{dv_4}{dt} = i_2 + \frac{1}{R_8}v_3 - \frac{1}{R_8}v_4 \end{cases}$$

$$\Downarrow$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{bmatrix} -\frac{(R_5 + R_6\|R_7)}{L_1} & -\frac{R_5}{L_1} & -\frac{R_6}{L_1(R_6+R_7)} & 0 \\ -\frac{R_5}{L_2} & -\frac{R_5}{L_2} & 0 & -\frac{1}{L_2} \\ \frac{R_6}{C_3(R_6+R_7)} & 0 & -\frac{1}{C_3}\left(\frac{1}{R_6+R_7}+\frac{1}{R_8}\right) & \frac{1}{C_3R_8} \\ 0 & \frac{1}{C_4} & \frac{1}{C_4R_8} & -\frac{1}{C_4R_8} \end{bmatrix}\begin{pmatrix} i_1 \\ i_2 \\ v_3 \\ v_4 \end{pmatrix} + \begin{bmatrix} -\frac{R_6\|R_7}{L_1} \\ 0 \\ \frac{R_6}{C_3(R_6+R_7)} \\ 0 \end{bmatrix} i_s}$$

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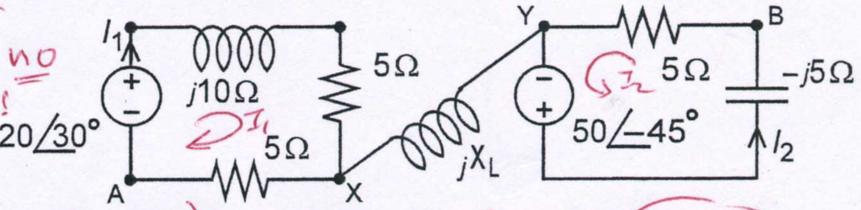
3A. (5 points) This circuit is in the sinusoidal steady state. Find the voltage (phasor) $V_{AB}$ (i.e., $V_A - V_B$).

Notice that the inductor $jX_L$ is irrelevant because no current flows through it!
Thus, its $\frac{di}{dt} = 0 \Rightarrow V_{XY} = 0$




Now, $20\angle 30° = (10+j10)I_1 \Rightarrow I_1 = (20\angle 30°)/(10\sqrt{2}\angle 45°) = \frac{2}{\sqrt{2}}\angle -15° = \sqrt{2}\angle -15°$

Also, $50\angle -45° = I_2(5-j5) \Rightarrow I_2 = (50\angle -45°)/(5\sqrt{2}\angle -45°) = \frac{10}{\sqrt{2}}$

And $V_{AB} = (V_A - V_X) + \underbrace{(V_X - V_Y)}_{=0} + (V_Y - V_B) = (-5I_1) + 0 + (-5I_2) = -5(I_1+I_2) = -5\left(\frac{10}{\sqrt{2}} + \sqrt{2}\angle -15°\right)$

$= -\left(\frac{50}{\sqrt{2}} + 5\sqrt{2}[\cos 15° - j\sin 15°]\right) = -(35.3553 + 6.8301 - j1.8301) = \boxed{-42.1855 + j1.8301}$

$\boxed{V_{AB} = 42.225\angle +177.52°}$

3B. (20 points)

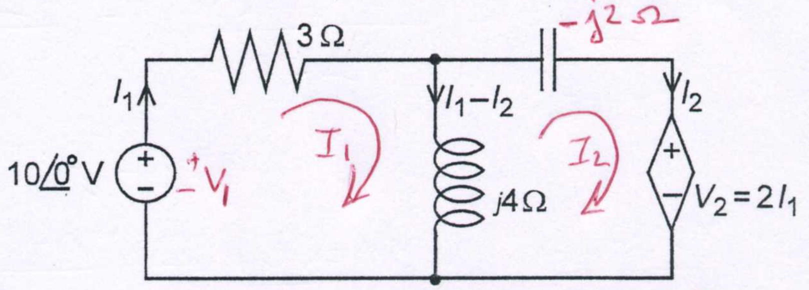
a) In this circuit, calculate the complex power supplied by each source.

b) Calculate the total complex power absorbed by the circuit elements. (Hence, verify the *part a* answer.)

a) Writing mesh equations:

$3I_1 + j4(I_1 - I_2) = 10\angle 0°$

and $j4(I_2 - I_1) - j2I_2 + 2I_1 = 0$




$(3+j4)I_1 - j4I_2 = 10$

$(2-j4)I_1 + j2I_2 = 0$

Adding twice 2nd row to 1st row $\Rightarrow (7-j4)I_1 = 10 \Rightarrow I_1 = \frac{10}{7-j4}$

$= \frac{10(7+j4)}{49+16} = \frac{70+j40}{65} = \frac{14+j8}{13}$. That is $\boxed{I_1 = \frac{14+j8}{13}} = \boxed{1.240\angle 29.7° \text{A}}$

$\therefore$ From 2nd mesh eqn.: $j2I_2 = -(2-j4)I_1 \Rightarrow I_2 = \frac{-(2-j4)}{j2}I_1 = \frac{j4-2}{j2}I_1 = (2+j)I_1$

$\therefore I_2 = (2+j)(14+j8)/13 = (28+j14+j16-8)/13 \Rightarrow \boxed{I_2 = \frac{20+j30}{13}} = \boxed{2.774\angle 56.31° \text{A}}$

Complex Power Supplied by $V_1$ source:

$S_1 = \frac{1}{2}V_1 I_1^* = \frac{1}{2}(10\angle 0°)(1.240\angle -29.7°) = (\frac{1}{2})12.40\angle -29.7° = \boxed{5.383 - j3.077 \text{ VA}}$

Answer to (a)

An complex Power Supplied by controlled source:

$S_2 = \frac{1}{2}(V_2)(-I_2^*) = -\frac{1}{2}2I_1I_2^* = -(1.240\angle 29.7°)(2.774\angle -56.31°) = -3.440\angle -26.565°$

$= \boxed{-3.077 + j1.538 \text{ VA}}$

b) $P_{3\Omega} = \frac{1}{2}|I_1|^2 \times 3 = 2.308$ W

$Q_{j4\Omega} = \frac{1}{2}(j|I_1 - I_2|^2 4) = j6.154$ VAR

$Q_{-j2\Omega} = \frac{1}{2}(-j|I_2|^2 \times 2) = -j7.692$ VAR

$\Rightarrow$ Complex power absorbed is: $S = 2.308 + j6.154 - j7.692$

$= \boxed{2.308 - j1.538 \text{ VA}}$ Answer to (b)

and, from (a): $S_1 + S_2 = 2.306 - 1.539$ VA $\approx$

7.692
− 6.15
1.53

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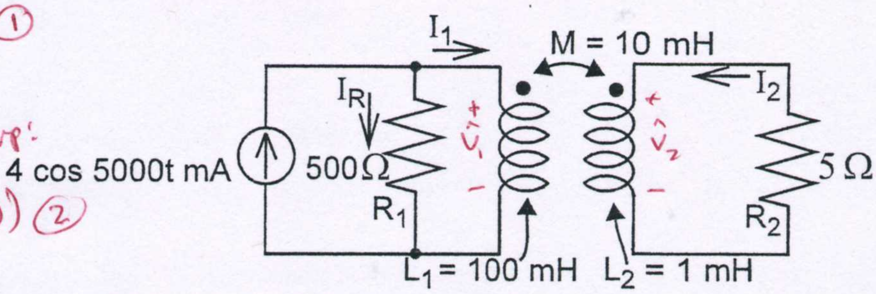
4. (25 points)

In this circuit, find the complex phasors: (a) $I_2$, (b) $I_1$, and (c) $I_R$.




(a) Writing voltage/current relations for the coupled coils:

$$\hat{V}_1 = \hat{I}_1(j\omega(0.1)) + \hat{I}_2(j\omega(0.01)) \quad (1)$$
$$\hat{V}_2 = \hat{I}_2(j\omega(0.001)) + \hat{I}_1(j\omega(0.01))$$

Using KVL in the secondary loop:

$$0 = \hat{I}_2(5 + j\omega(0.001)) + \hat{I}_1(j\omega(0.01)) \quad (2)$$

& $\hat{V}_1 = 500\,\hat{I}_R$ (3)

$$\hat{I}_1 + \hat{I}_R = 4\angle 0°\,\text{mA} = 0.004\angle 0°\,\text{A} \quad (4)$$

$\Rightarrow \hat{I}_R = 0.004 - \hat{I}_1$ (5) ← Amps

Putting (5) into (3) gives $\hat{V}_1$ as a function of $\hat{I}_1$:

$$\hat{V}_1 = 500(0.004 - \hat{I}_1) = 2 - 500\hat{I}_1 \quad (6)$$

substituting $\hat{V}_1$ in (1) by using (6): $2 - 500\hat{I}_1 = j\omega(.1)\hat{I}_1 + j\omega(0.01)\hat{I}_2$

$$\Rightarrow \hat{I}_1 = [2 - \hat{I}_2(j\omega(0.01))]/[500 + (0.1)j\omega] \quad (7)$$

Substituting for $\hat{I}_1$ in (2), using (7):

$$0 = (5 + j\omega 0.001)\hat{I}_2 + j\omega 0.01(2 - j\omega 0.01\,\hat{I}_2)/[500 + j\omega(0.1)] \Rightarrow \hat{I}_2 = \frac{-j\omega(0.02)}{2500 + j\omega}$$

Since the current source is $4\cos\omega t = 4\cos 5000t$, we have $\omega = 5000$

$$\therefore \hat{I}_2 = \frac{-j100}{2500 + j5000} = \frac{-j}{25 + j50} = \frac{1\angle -90°}{56\angle 63.43°} = \boxed{17.9\angle -153.4°\text{ mA}}$$

(b) Putting $\hat{I}_2$ into (2): $0 = \hat{I}_2(5 + j5) + j50\,\hat{I}_1 \Rightarrow \hat{I}_1 = \frac{-(5+j5)\hat{I}_2}{j50}$

$$\Rightarrow \hat{I}_1 = \left(\frac{j}{25 + j50}\right)\left(\frac{5 + j5}{j50}\right) = \frac{-5 + j5}{-2500 + j1250} = \frac{-1 + j1}{-500 + j250} = \frac{\sqrt{2}\angle 135°}{(250)(-2 + j1)}$$

$$= \frac{\sqrt{2}\angle 135°}{(250)\sqrt{5}\angle 153.44°} = \boxed{2.53\angle -18.4°}\text{ mA}$$

(c) Putting $\hat{I}_1$ into (5): $\hat{I}_R = (4 - 2.53\angle -18.4°)$ mA

$$= 4 - 2.53\cos(18.4°) + j2.53\sin(18.4°)$$
$$= 1.6 + j0.8 = \boxed{1.79\angle 26.6°\text{ mA}}$$