Formula Sheet for EE101 midterm:

1 Know telegraphers equation, expressions for Z_0, γ

2 $\Gamma = \dfrac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta_r}$

3 $Z(d) = \dfrac{\tilde{V}(z)}{\tilde{I}(z)} = Z_0 \left[\dfrac{e^{-j\beta z} + \Gamma e^{+j\beta z}}{e^{-j\beta z} - \Gamma e^{-j\beta z}} \right]$

Put $z = -d$

4 S = VSWR or SWR $= \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

5 $Y = 1/Z$

$y = g + jb$

$z = r + jx$

EE 101 Midterm.   Name

15 points   Student ID:

1 when an unknown Z_L is connected to a

lossless "slotted - air" transmission line the VSWR or SWR of 2 is

lossless. recorded and the voltage two consecutive minima are found

in z direction at 11 cm and 19 cm. When the load is replaced by

next to each other a short-circuit the minima are recorded at 16 cm and

24 cm.

If $Z_0 = 50\,\Omega$ calculate f and Z_L.

What is Γ when Z_L is connected

(Use Smith-chart) Hint: Sketch the standing-wave pattern.

15 points

60 Ω

2 A lossless, $Z_0 = 50\Omega$ transmission line is terminated by a load $Z_L = 60 + j60\ \Omega$

Find Γ and S

If $Z_{in} = 120 - j60\ \Omega$ how far (in terms of wavelengths) is the load from the generator?

How far is the "first" voltage maximum from the load?

How far is the "third" voltage minimum from the load?

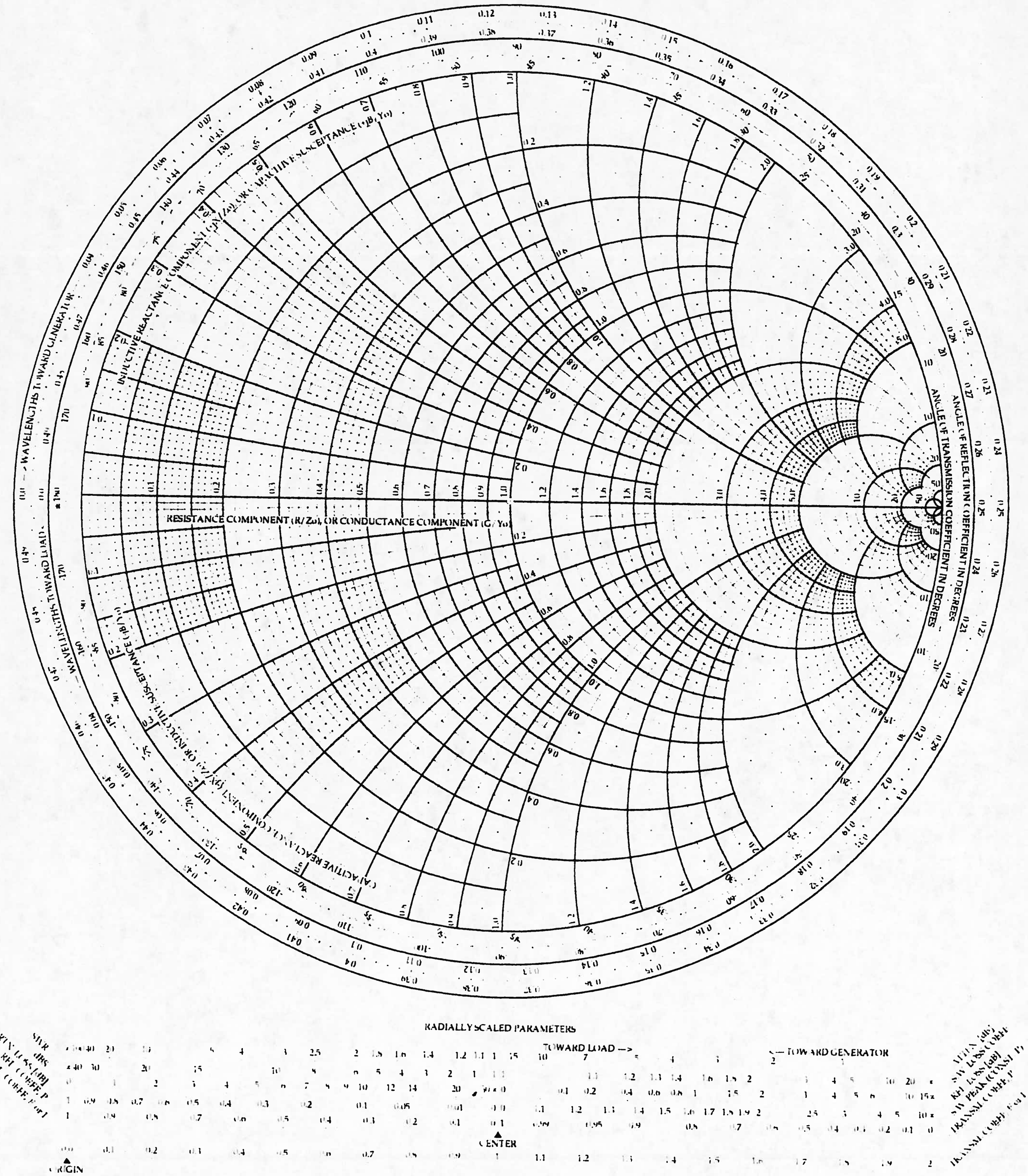
25

3 A 75 Ω transmission line is to be matched to a load $Z_L = 100 - j80\ \Omega$.

Calculate ~~at~~ If the matching is done with a shorted stub, the stub length, the distance between the load and the stub, admittance $y(d_1)$, $y(d_2)$ at the stub looking towards the load.

The Complete Smith Chart

Black Magic Design



4. 10'

A lossless transmission line with $Z_0 = 50\,\Omega$ is 30m long and operates at 2 MHz. The line is terminated with a load $Z_L = 60 + j\,40\,\Omega$. If the phase velocity $u_p = 0.6\,c$ find

a) Γ

b) S

c) Z_{in} Hint $Z_{in} = Z_0 \left[\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$

— Do not use a Smith Chart. $l = 30\text{ m}$

10'

5 A laser beam of light propagating through space has an electric field given by

$$E(y,t) = 75\, e^{-0.05y} \cos(6\times10^{15} t - 2\times10^{7} y) \;\; V/m$$

where y is the distance from the source in meters.

Calculate

a) The direction of wave travel

b) The phase velocity of the wave

c) The wave amplitude at a distance of 400 m.

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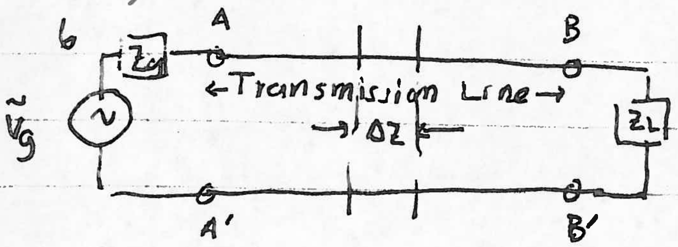


A transmission line has following four parameters R', G', L' and C' per unit length

a) Draw an equivalent circuit representation for a section length Δz of the transmission line.

b) Then using Kirchoff's voltage and current laws, derive a differential equation for voltage and current as a function of z.

6 (cont) Then show that we can infer the existance of voltage and current waves on the transmission line

Hint: Derive the wave equation

Find expressions for γ and Z_0.

Assume time harmonic voltage and current.

Points.

15 points total

Solution to 1.

Distance between successive voltage minima is $\lambda/2$ 1

With Z_L connected $\lambda/2 = 19-11 = 8$ cm 1

with short $\lambda/2 = 24-16 = 8$ cm. 1

$\lambda = 16$ cm

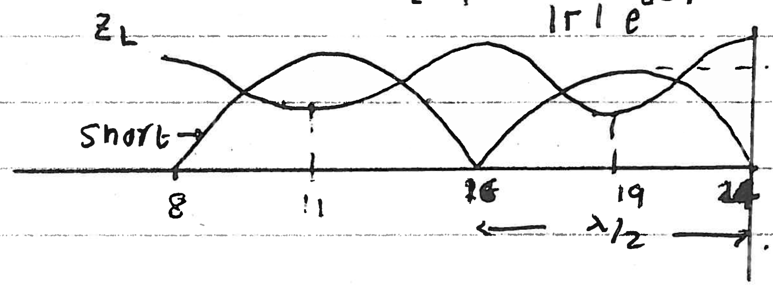
"Air line" so $u_p = c = 3\times10^{10}$ cm s^{-1}

$f = u_p/\lambda = \dfrac{3\times10^{10}}{16} = 1.875\times10^{9}$ Hz. 1

With short circuit $Z_L = 0$ and $|\Gamma| = 1$ 1

$\Gamma = \dfrac{z_L - 1}{z_L + 1} = -1$ $\quad \theta_r = \pm\pi$ rad.

$|\Gamma| e^{j\theta_r}$ $\quad |\ \theta_r = \pm\pi$. 1



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When z_L is replaced with short the minimum moves from 19 to 24 cm or 5 cm towards the load.
But Z_L is located at 24 cm or $5/8\times\lambda$ towards the load $= 0.3125\lambda$. 1

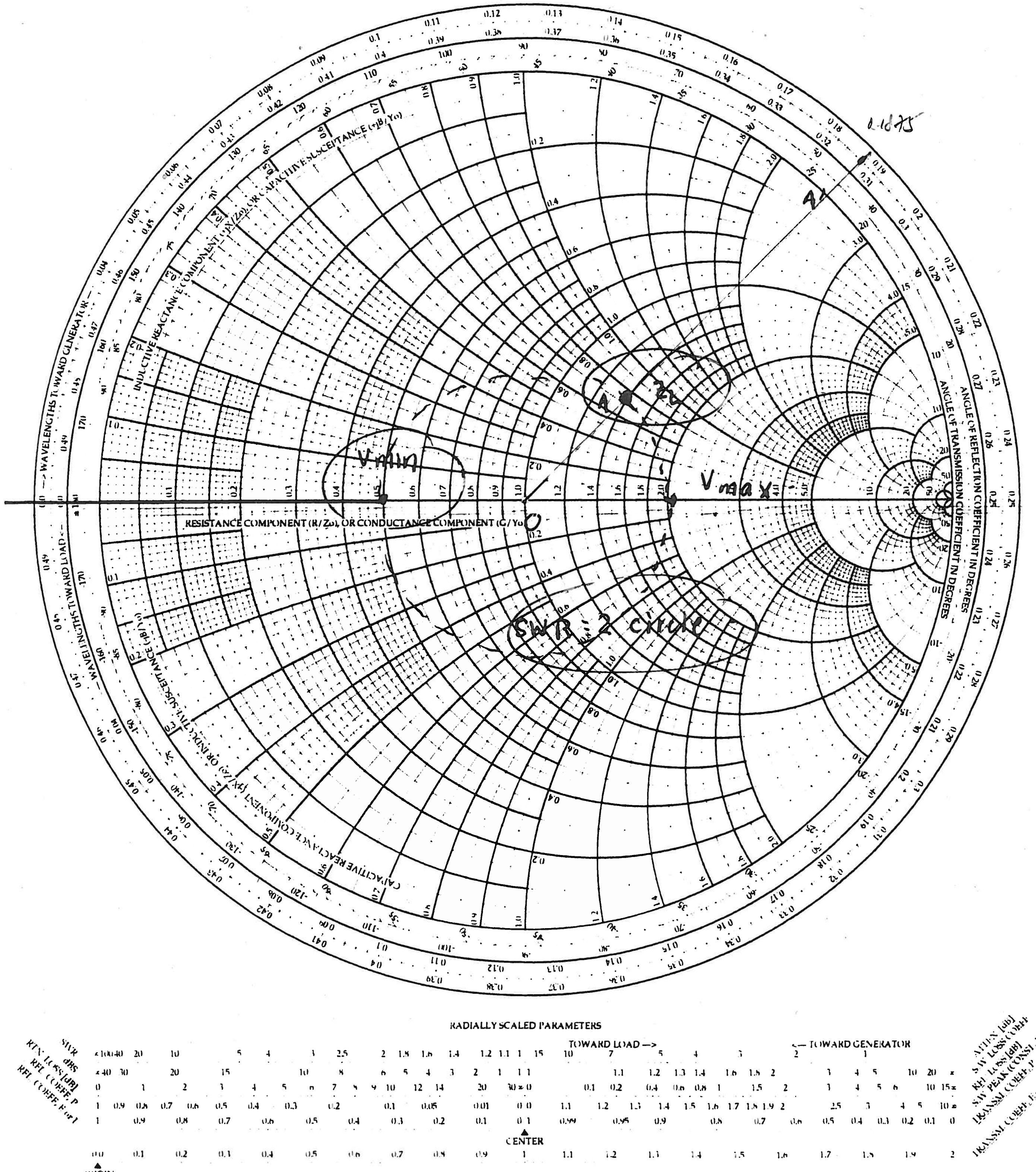
So Z_L is at $0.5 - 0.3125 = 0.1875\lambda$. on the SWR circle. 1

Here $Z_L = (1.4 + j\,0.75)\,Z_0$ 1

Solution to 1

The Complete Smith Chart

Black Magic Design



(15 points)

Solution to 2. $Z_L = 60 + j60\ \Omega$

$z_L = 1 + j1$ 1

$|\Gamma| = \frac{OP}{OP'} = 0.45$ and $\theta_r = 64^0$. 2

$\Gamma = 0.45 \angle 64^0$.

$S = 2.6$ 1

$z_{in} = \frac{Z_{in}}{Z_0} = \frac{120 - j60}{60} = 2 - j1$ 1

This is point R at -26^0 1

The angle between OP' and OR is $64^0 - (-26^0) = 90^0$ 1

or $l = 90/2 \times 360^0 = \lambda/8$. 1

The first voltage maximum is at S = or at 0^0 1

distance $d = 64/720 = 0.085\lambda$.

The first voltage minimum is at $(0.085 + 0.25)$

$= 0.335\lambda$ 1

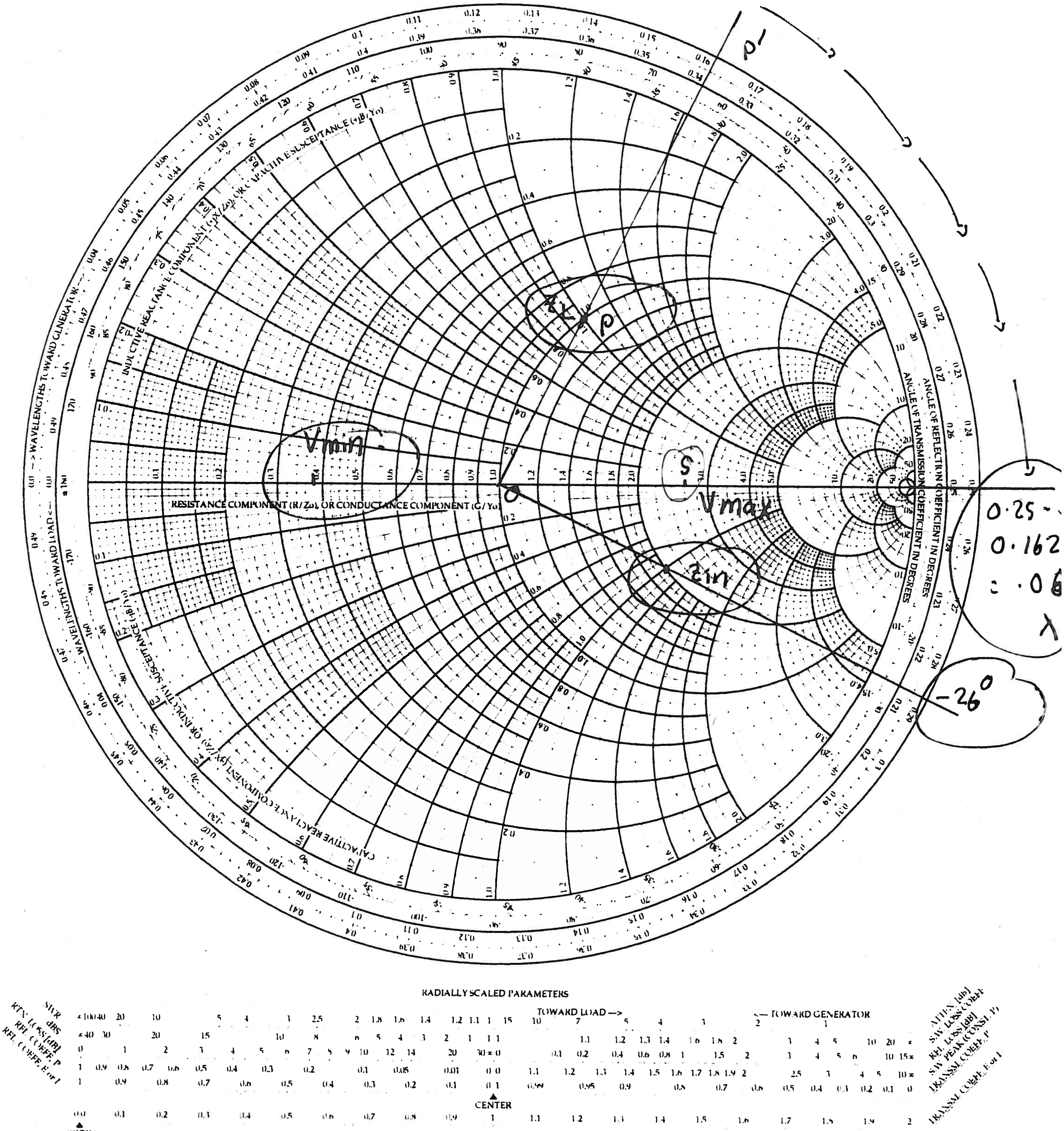
The third voltage minimum is $0.335\lambda + 1\lambda$

$= 1.335\lambda$ from load. 1

Solution to 2.

The Complete Smith Chart

Black Magic Design

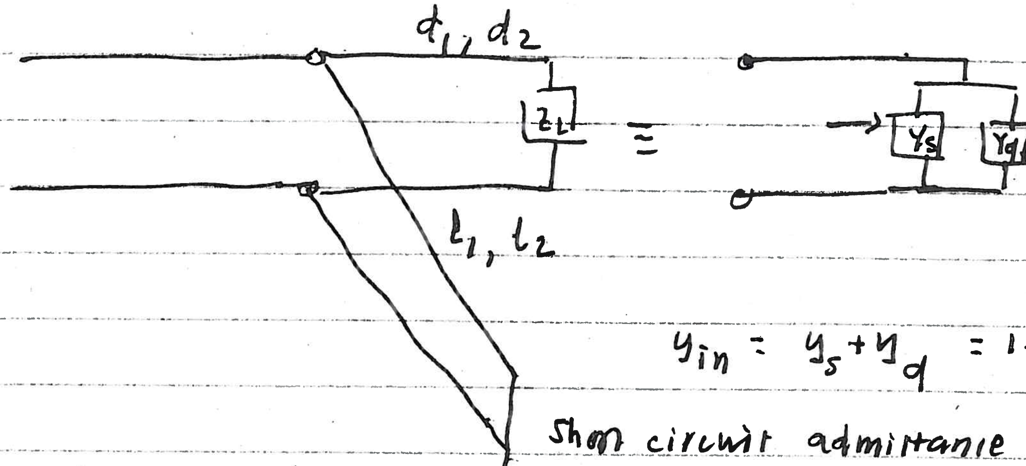


25 points

Solution to 3. 1) $z_L = \frac{Z_L}{Z_0} = \frac{100 - j80}{75} = 1.33 - j\,1.067$ (

(A)

2) Two solutions.



(3)

$y_{in} = y_s + y_q = 1.0 + j0.$ (1

Short circuit admittance ∞.

distance d_1 between load + stub

(1) d_1 is at R, $d_1 = (0.159\lambda - 0.069\lambda) = 0.09\lambda$. (1)

$y_{d_1} = 1 + j1$ (1)

$y_s = 0 - j1$. This is point U (1)

Stub-length distance $l_1 = (0.8750\lambda - 0.25\lambda) = 0.125\lambda$ (1

distance d_2 between load + stub.

(2) d_2 is at S $d_2 (0.3385\lambda - 0.069\lambda) = 0.2695\lambda$

$\simeq 0.27\lambda$ (1

and $y_{d_2} = 1 - j1$ (1)

$y_s = 0 + j1$ (1)

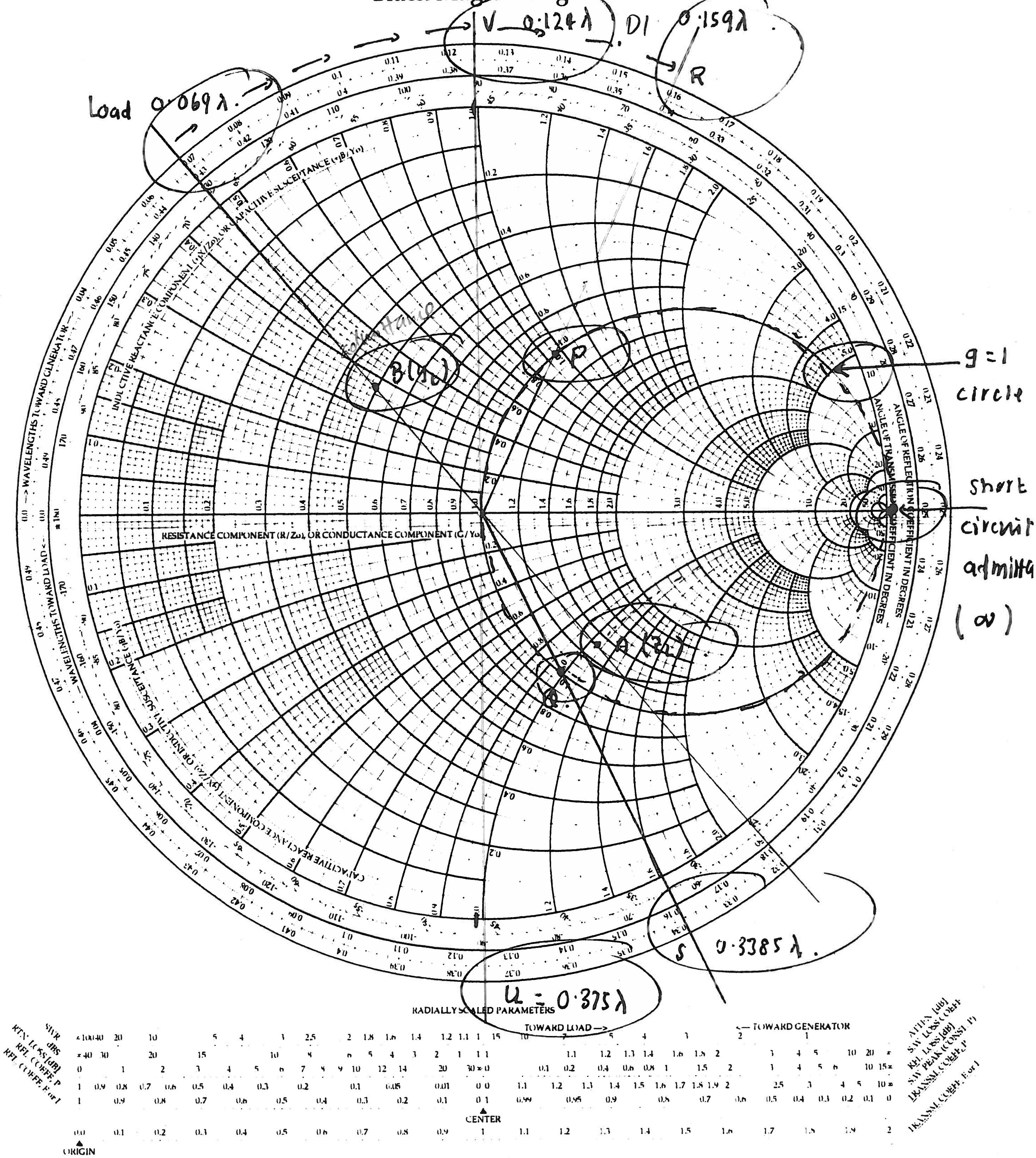
This is point V (1)

$l_2 = (0.25\lambda + 0.125\lambda) = 0.375\lambda$ (1)

Solution to 3.

The Complete Smith Chart

Black Magic Design



10 points

solution to 4. a) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{60 + j40 - 50}{60 + j40 + 50} = \dfrac{10 + j40}{110 + j40}$

$= 0.3523 \angle 56^0$ $\quad 0.1971 + j0.2920$

b) $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.3523}{1 - 0.3523} = 2.088$

c) Since $u_p = \omega / \beta$, $\beta = \omega / u_p$

$\beta l = \dfrac{\omega l}{u_p} = \dfrac{2\pi (2 \times 10^6)(30)}{0.6 (3 \times 10^8)} = \dfrac{2\pi}{3} = 120^0$.

$Z_{in} = Z_0 \left[\dfrac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$

$= 50 \left(\dfrac{60 + j40 + j50 \tan 120^0}{50 + j(60 + j40) \tan 120^0} \right)$

$= 50 \left(\dfrac{6 + j4 - j5\sqrt{3}}{5 + 4\sqrt{3} - j6\sqrt{3}} \right) = 24.01 \angle 3.22^0$.

$= 23.97 + j\, 1.35\ \Omega$.

(10 points)

Solution to 5. a) Since the coefficients of t and y have opposite signs, the wave must be propagating in

3 the y direction

3 b) $u_p = \omega/\beta = \frac{6\times10^{15}}{2\times10^{7}} = 3\times10^{8}$ m/s

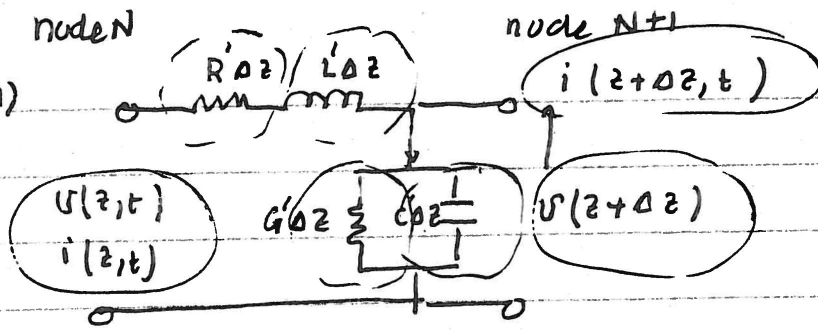
c) At y = 400 m

4 E(y,t) is $75\, e^{-0.05(400)} = 75\, e^{-20} = $ 10.1 (V/m)

1.546×10^{-7}

20 points

Solution to 6 a)



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b) Using Kirchhoff's law

1 $$v(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{di(z,t)}{dt} - v(z+\Delta z, t) = 0$$

1 $$-\left[\frac{v(z+\Delta z, t) - v(z)}{\Delta z}\right] = R' i(z,t) + L' \frac{\partial i}{\partial t}(z,t)$$

1 $$-\frac{\partial v}{\partial z} = R' i(z,t) + L' \frac{\partial i}{\partial t}(z,t)$$

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Similarly for the current

1 $$i(z,t) - G'\Delta z\, v(z+\Delta z, t) - C'\Delta z \frac{\partial v}{\partial t}(z+\Delta z, t) - i(z+\Delta z, t) = 0$$

1 $$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v}{\partial t}(z,t).$$

c) Use time harmonic solutions for $v(z,t)$ and $i(z,T)$

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1 $$v(z,t) = Re\left(\tilde{V}(z)\, e^{j\omega t}\right)$$ $\tilde{V}$ and $\tilde{I}$ are phasors

1 $$i(z,t) = Re\left(\tilde{I}(z)\, e^{j\omega t}\right)$$

Substituting

$$-\frac{\partial \tilde{V}}{\partial z} = (R' + j\omega L')\, \tilde{I}(z)$$

and $$-\frac{\partial \tilde{I}}{\partial z} = (G' + j\omega C')\, \tilde{V}(z)$$

Taking derivative w.r.t. z.

$$\frac{\partial^2 \tilde{V}}{\partial z^2} = +(R' + j\omega L')(G' + j\omega C')\, \tilde{V}(z)$$

$$\frac{\partial^2 \tilde{V}}{\partial z^2} = +\gamma^2 \tilde{V}(z) \quad , \quad \gamma = \left((R' + j\omega L')(G' + j\omega C')\right)^{1/2}$$

and $$\frac{\partial^2 \tilde{I}}{\partial z^2} = +\gamma^2 \tilde{I}(z)$$

Solution is $$\tilde{V}, \tilde{I} = (V_0^+, I_0^+)\, e^{-\gamma z} + (V_0^-, I_0^-)\, e^{+\gamma z}.$$

incident wave Reflected wave.