Formula Sheet for EE101 midterm :

1 Know telegraphers equation, expressions for Z_0, γ

2 $\Gamma = \dfrac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta_r}$

3 $Z(d) = \dfrac{\tilde{V}(z)}{\tilde{I}(z)} = Z_0 \left[\dfrac{e^{-j\beta z} + \Gamma e^{+j\beta z}}{e^{-j\beta z} - \Gamma e^{-j\beta z}} \right]$

Put $z = -d$

4 S = VSWR or SWR $= \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

5 $Y = 1/Z$

$y = g + jb$

$z = r + jx$

EE 101 Midterm.

1 When an unknown Z_L is connected [redacted] "Slotted - air" transmission line the VSWR or SWR of 2 is recorded and the voltage minima are found at 11 cm and 19 cm. When the load is replaced by a short-circuit the minima are recorded at 16 cm and 24 cm.

If $Z_0 = 50\,\Omega$ calculate f and Z_L.

What is Γ when Z_L is connected

(Use Smith-chart) Hint: Sketch the standing-wave pattern.

$\Delta d = \frac{\lambda}{2}$ $\lambda = 2\Delta d = 2(19cm - 11cm) = 16\,cm$.

$f = \frac{c}{\lambda} = \frac{c}{\lambda} = 1.875 \cdot 10^9\,Hz$

When the load is connected, the minima shift $3cm = \frac{5}{16}\lambda$ toward the [illegible]. So the angle between Z_{short} and Z_L is $\frac{5}{16}\lambda$.

$z_L = 1.4 + 0.75i$

$Z_L = Z_0 z_L = 70.5 + 37.5i$

$|\Gamma| = \frac{1.258cm}{3.775cm} = 0.333$, $\theta_r = 45°$.

$\therefore \Gamma = 0.333 \cdot e^{45° i} = 0.2355 + 0.2355i$

(15)

2 A lossless, $Z_0 = 60\Omega$ transmission line is terminated by a load $Z_L = 60 + j60\ \Omega$

Find Γ and S

If $Z_{in} = 120 - j60\ \Omega$ how far (in terms of wavelengths) is the load from the generator? How far is the "first" voltage maximum from the load? How far is the "third" voltage minimum from the load?

$z_L = \frac{Z_L}{Z_0} = (1+i)\ \Omega$

$|\Gamma| = \frac{3.8\,cm}{7.7\,cm} = 0.4581 \quad \theta_r = 64$

$\therefore \boxed{\Gamma = 0.4581\, e^{64°} = 0.2008 + 0.4117i}$

$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.6907}$

$z_{in} = (2 - i)\ \Omega$

$l = (0.287 - 0.162)\lambda + 0.5n\lambda$

$\boxed{l = (0.125 + 0.5n)\lambda}$

$d_{max,1} = (0.25 - 0.162)\lambda + 0.5n\lambda$

$\boxed{d_{max,1} = 0.088\lambda}$

$d_{min,3} = (0.5 - 0.162)\lambda + 0.5n\lambda$

$= 0.338\lambda + 0.5(3-1)\lambda$

$\boxed{d_{min,3} = 1.338\lambda}$

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3 A 75 Ω transmission line is to be matched to a load $Z_L = 100 - j80\ \Omega$.

Calculate a) If the matching is done with a shorted stub, the stub length, the distance between the load and the stub, admittance $y(d_1)$, $y(d_2)$ at the stub looking towards the load.

$z_L = \frac{Z_L}{Z_0} = 1.333 - 1.067i\ \Omega$, $Y_0 = \frac{1}{Z_0} = \frac{1}{75}\ \Omega$

Solution 1:

$d_{1,s1} = (0.338 - 0.066)\lambda = 0.279\lambda$

$y_{L,d_1} = (1 + 1.02i)$

$\therefore y(d_1) = -1.02i$, $Y(d_1) = Y_0 y(d_1) = -0.0136i\ \Omega$

$d_1 = 0.088\lambda$

Solution 2:

$d_{1,s2} = (0.338 - 0.316)\lambda = 0.022\lambda$

$y_{L,d_2} = 1 - 1.02i$

$\therefore y(d_2) = 1.02i$, $Y(d_2) = Y_0 y(d_2) = 0.0136i\ \Omega$

$d_2 = 0.412\lambda$ ✗

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4. A lossless transmission line with $Z_0 = 50\,\Omega$ is 30 m long and operates at 2 MHz. The line is terminated with a load $Z_L = 60 + j\,40\,\Omega$. If the phase velocity $u_p = 0.6c$ find

a) Γ

b) S

c) Z_{in} Hint $Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan\beta l}{Z_0 + j Z_L \tan\beta l} \right]$

– Do not use a Smith Chart. $l = 30$ m

$z_L = \frac{Z_L}{Z_0} = 1.2 + 0.8i$

$\Gamma = \frac{z_L - 1}{z_L + 1} = 0.1971 + 0.2920i = 0.3523 \cdot e^{0.9770i}$

$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.0878$

$\lambda = \frac{v}{f} = \frac{0.6c}{f} = 90\text{ m}$

$\beta = \frac{2\pi}{\lambda} = 0.0698$

$\therefore Z_{in} = Z_0 \left[\frac{Z_L + i Z_0 \tan\beta l}{Z_0 + i Z_L \tan\beta l} \right]$

$Z_{in} = 23.913 + 1.352$

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5 A laser beam of light propagating through space has an electric field given by

$$E(y,t) = 75\, e^{-0.05y} \cos\left(6\times10^{15}\, t - 2\times10^{7}\, y\right) \text{ V/m}$$

where y is the distance from the source in meters.

Calculate

a) The direction of wave travel

b) The phase velocity of the wave

c) The wave amplitude at a distance of 400m.

a) +y direction

b) $w = 6\cdot10^{15} \frac{rad}{s}$, $K = 2\cdot10^{7}$

$V = \frac{W}{K} = 3\cdot10^{8} \frac{m}{s}$

c) $75 \cdot e^{-0.05(400)} = 1.546\cdot10^{-7} \frac{V}{m}$

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$v = \frac{1}{c}$

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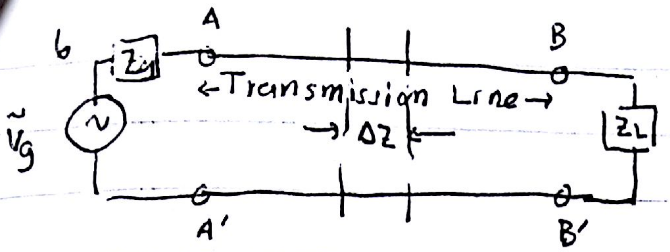
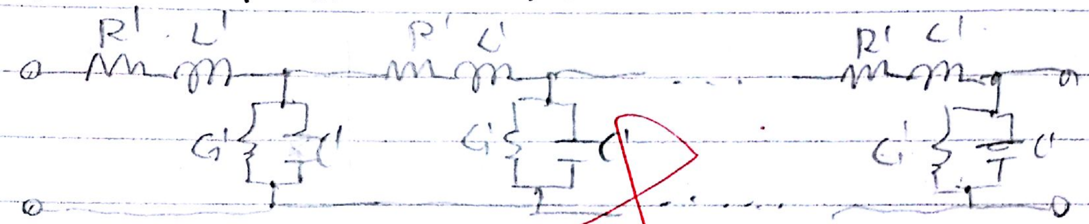
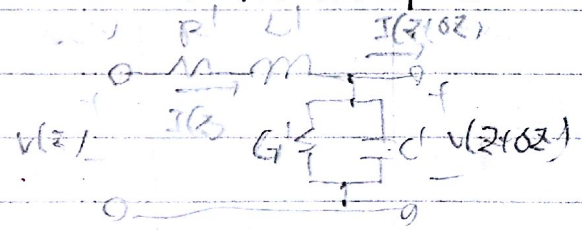


A transmission line has following four parameters R', G', L' and c' per unit length

a) Draw an equivalent circuit representation for a section length Δz of the transmission line.



b) Then using Kirchoff's voltage and current laws, derive a differential equation for voltage and current as a function of z.



$V(z) - V(z+\Delta z) = \left(I(z)R' + L'\frac{dI(z)}{dt}\right)\Delta z$.

$\therefore -\frac{V(z+\Delta z) - V(z)}{\Delta z} = I(z)R' + L'\frac{dI(z)}{dt}$

As $\Delta z \to 0$, we have

$-\frac{\partial V(z)}{\partial z} = I(z)R' + L'\frac{dI(z)}{dt}$

Take the phasor to get

$-\frac{\partial \tilde{V}}{\partial z} = \tilde{I}R' + i\omega L'\tilde{I} = (R' + i\omega L')\tilde{I}$ (1)

$-(I(z+\Delta z) - I(z)) = G'V(z+\Delta z) + C'\frac{dV(z+\Delta z)}{dt}$.

Divide by Δz and let $\Delta z \to 0$ to get

$-\frac{\partial I}{\partial z} = G'V + C'\frac{dV}{dt}$

Take the phasor and we have.

$-\frac{\partial \tilde{I}}{\partial z} = G'\tilde{V} + i\omega C'\tilde{V} = (G' + i\omega C')\tilde{V}$ (2)

6 (cont). Then show that we can infer the existance of voltage and current waves on the transmission line

Hint: Derive the wave equation

Find expressions for γ and Z_0.

Assume time harmonic voltage and current.

Differentiate (1) with respect to z to get

$$-\frac{\partial^2 \tilde{V}}{\partial z^2} = (R' + i\omega L')\frac{\partial \tilde{I}}{\partial z},$$ so we can substitute (2) in to get

$$-\frac{\partial^2 \tilde{V}}{\partial z^2} = (R' + i\omega L')(G' + i\omega C')\tilde{V}$$

let $\gamma = \sqrt{(R' + i\omega L')(G' + i\omega C')}$ so

$$\frac{\partial^2 \tilde{V}}{\partial z^2} = -\gamma^2 \tilde{V}$$

$$\therefore \tilde{V} = \hat{V}_0^+ e^{-i\gamma z} + \hat{V}_0^- e^{\gamma z} \quad (3)$$

Similarly, if we differentiate (2) and substitute (1) in we get

$$-\frac{\partial^2 \tilde{I}}{\partial z^2} = (R' + i\omega L')(G' + i\omega C')\hat{I}$$

$$\text{so } \frac{\partial^2 \tilde{I}}{\partial z^2} = -\gamma^2 \tilde{I}$$

$$\therefore \tilde{I}(z) = \hat{I}_0^+ e^{-i\gamma z} + \hat{I}_0^- e^{+i\gamma z}. \quad (4)$$

let $Z_0 = \frac{\hat{V}_0^+}{\hat{I}_0^+}$. If we plug the expression for $\hat{I}^+$ into (2) we find

$$\gamma \hat{I}_0^+ e^{-i\gamma z} = (G' + i\omega C')\hat{V}_0^+ e^{-i\gamma z}.$$

so we get $Z_0 = \frac{\hat{V}_0^+}{\hat{I}_0^+} = \frac{\gamma}{(G' + i\omega C')} = \sqrt{\frac{(R' + i\omega L')}{(G' + i\omega C')}}$

Multiply (3) and (4) by $e^{i\omega t}$ we have

$$V(z,t) = Re\left\{\hat{V}_0^+ e^{-i\gamma z + i\omega t} + \hat{V}_0^- e^{i\gamma z + i\omega t}\right\}$$

$$I(z,t) = Re\left\{\frac{\hat{V}_0^+}{Z_0} e^{-i\gamma z + i\omega t} - \frac{\hat{V}_0^-}{Z_0} e^{-i\gamma z - i\omega t}\right\}$$

which are the wave equations we are looking for.