Problem 1.

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a) Find the dimensions of a hollow rectangular metallic waveguide for single mode operation within 1-1.5 GHz.

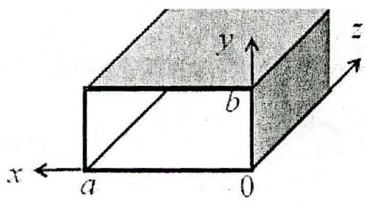
b) Determine the frequency range over which the waveguide supports dual modes.

Only two modes

c) Determine the waveguide guided modes at 4.5 GHz.

d) Find the group velocity, phase velocity, wave impedance, and wavelength of a 1.25 GHz wave propagating along the waveguide.

e) If the metallic waveguide in part (a) is filled with glass ($\varepsilon_r = 2.25$), find the frequency range for waveguide single mode operation.



(a<b)

a). the lowest cutoff frequency:

assuming $a > b$

$$f_{10} = \frac{c}{2a}$$

next mode:

$$f_{01} = \frac{c}{2b}$$

$$\Rightarrow \begin{cases} f_{10} = 1\,GHz \\ f_{01} = 1.5\,GHz \end{cases} \Rightarrow \begin{cases} \frac{c}{2a} = 1\,GHz & a = 15\,cm \\ \frac{c}{2b} = 1.5\,GHz & b = 10\,cm \end{cases}$$

b). the third mode is $f_{11} = 1.8\,GHz$.

For the waveguide to have only two modes:

f should be in between 1.5 GHz and 1.8 GHz.

(c).

$$f_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$f_{10} = 1\,GHz$ TE

$f_{01} = 1.5\,GHz$ TE

$f_{11} = 1.8\,GHz$ TE & TM.

$f_{12} = 3.16\,GHz$ TE & TM

$f_{21} = 2.5\,GHz$ TE & TM.

$f_{02} = 3\,GHz$ TE $f_{22} = 3.6\,GHz$ TE & TM.

$f_{20} = 2\,GHz$ TE

$f_{03} = 4.5\,GHz$ not $< 4.5\,GHz$

$f_{30} = 3\,GHz$ ~~TM~~ TE

$f_{31} = 3.35\,GHz$ TE & TM. $f_{40} = 4\,GHz$ TE

$f_{32} = 4.24\,GHz$ TE & TM. $f_{41} = 4.27\,GHz$ TE & TM

Good!

$\Rightarrow$ The modes are listed above excepted for f_{03} since

$f_{03} = 4.5\,GHz$ not $< 4.5\,GHz$.

(d) $f = 1.25\,GHz$

only one mode: TE_{10}.

$f_{mn} = f_{10} = 1\,GHz$.

$$u_g = c\sqrt{1-\left(\frac{f_{mn}}{f}\right)^2}$$

$$= 3\times10^8\,m/s\sqrt{1-\left(\frac{1}{1.25}\right)^2}$$

$$= 1.8\times10^8\,m/s.$$

$$u_p = \frac{c}{\sqrt{1-\left(\frac{f_{mn}}{f}\right)^2}}$$

$$= \frac{3\times10^8}{\sqrt{1-\frac{1}{(1.25)^2}}}\,m/s$$

$$= 5\times10^8\,m/s$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_{mn}}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{1}{1.25}\right)^2}}\,\Omega \cong 628\,\Omega$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8\,m/s}{1.25\,GHz} = 0.24\,m$$ −1

(e) $f_{mn} = \frac{u_{po}}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{10} = \frac{3\times10^8}{2\times1.5}\sqrt{\left(\frac{1}{0.15}\right)^2}\ Hz = 6.67\times10^8\ Hz$$

$$f_{01} = \frac{3\times10^8}{2\times1.5}\sqrt{\left(\frac{1}{0.1}\right)^2} = 10^9\ Hz$$

f should be in between

6.67×10^8 Hz and 10^9 Hz

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$\frac{\quad}{\varepsilon_0 \varepsilon_r}$

Problem 2.

An x-polarized optical beam is propagating in *z* direction inside a semi-infinite dielectric slab (ε_r = 12.5) placed in free space. The dielectric slab has rectangular sidewalls (as shown below) with dimensions much larger than the optical beam size.

a) If the optical beam is normally incident on one of the slab sidewalls in the xy-plane, determine the propagation direction and portion of the optical power that is transmitted from the slab to air.

$\varepsilon_r = 12.5$

direction: $\hat{z}$.

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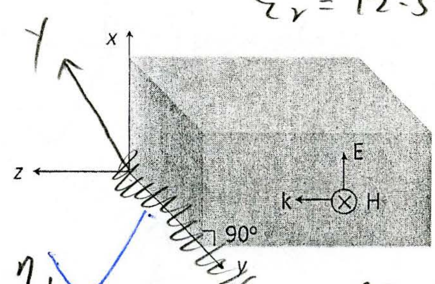


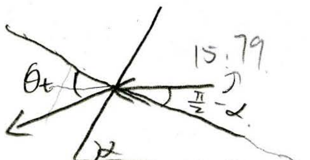
$\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{1}{\varepsilon_r}} = \frac{1}{\sqrt{12.5}}\eta_0$

$\eta_2 = \eta_0$,

$\tau = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2}{\frac{1}{\sqrt{12.5}}+1} = 1.56$

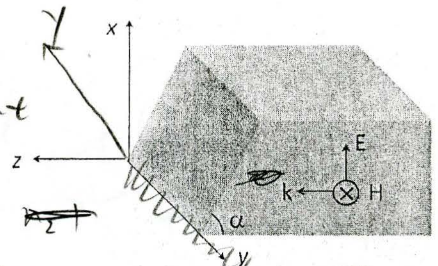
$T = |\tau|^2 \cdot \frac{\eta_1}{\eta_2} = 0.687 \Rightarrow 68.7\%$ of power is transmitted

b) If there is a possibility of polishing the slab sidewalls, find the slab sidewall angel, α, at which maximum optical power is transmitted from the slab to air.



This is parallel polarization since $\vec{H}$ is normal to incident plane. the incidence angle should equal to brewster angle which leads to a maximum power transmittance: $\frac{\pi}{2} - \alpha = \tan^{-1}\sqrt{\frac{\varepsilon}{\varepsilon_0}}$

$\alpha = 74.21$



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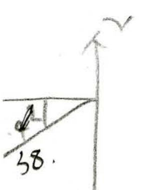
c) For part (b), determine the propagation direction and portion of the optical power that is transmitted from the slab to air.



d) How can we maximize optical power transmission from the slab to air, if a y-polarized optical beam (instead of an x-polarized optical beam) is propagating in z direction.

e) For the solution you offer for part (d), determine the propagation direction and portion of the optical power that is transmitted from the slab to air.

c). $n_1 = \sqrt{\varepsilon_r \mu_r} = \sqrt{12.5}$.

$n_2 = 1$

$n_2 \sin\theta_t = n_1 \sin\theta_i$

$1 \cdot \sin\theta_t = \sqrt{12.5}\,\sin(90° - 74.21°)$

$\theta_t \doteq 74.17°$

So: relative to $\hat{z}$, transmitted beam is $\theta_t - \frac{\pi}{2} + \alpha = 58.38°$

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$\Rightarrow$ propagation direction: $\hat{x}(-0.852) + \hat{z}\cdot(0.524)$

power portion transmitted: 100% at Brewster angle.

(d). if the wave become perpendicular polarized.,

brewster angle doesn't exist for nonmagn materials.

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~~[illegible]~~ In this case, $|\Gamma|$ is smallest when θ_i approaches 0, $\Rightarrow$ normal incidance.

(e) propagation direction: $\hat{z}$

$$\tau = \frac{2\eta_2}{\eta_1+\eta_2} = 1.56$$

$$T = |\tau|^2 \frac{\eta_1}{\eta_2} = 0.687$$

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$\Rightarrow$ 68.7% of power is transmitted.

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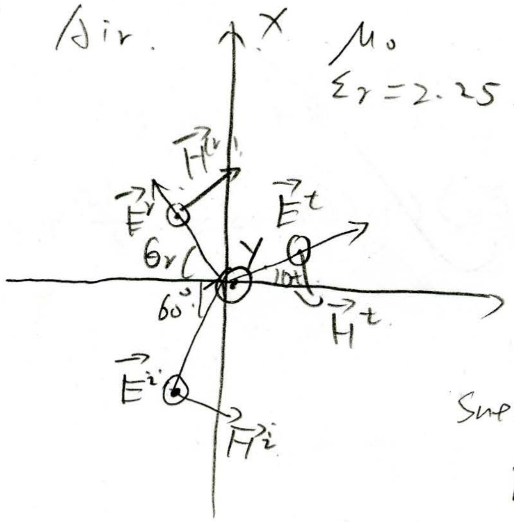
Problem 3.

$E_0 \rightarrow E_i \rightarrow$ magnitude for

A 1GHz, y-polarized plane-wave is propagating in air along $\hat{k} = \frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}$ direction. The wave is incident on a semi-infinite dielectric medium with $\mu = \mu_0$ and $\varepsilon_r = 2.25$. The interface between the dielectric medium and air is the z = 0 plane.

a) Write the phasor domain and time domain equations for the incident and reflected wave in air and the transmitted wave into the dielectric medium.

b) Determine the portion of the wave power that is transmitted into the dielectric medium.



This is perpendicular polarization. Since $\vec{E^i}$ is perpendicular to the incidence plane:

$\eta_1 = 120\pi = 376.7\Omega$, $n_1 = 1$ $\quad n_2 = \sqrt{\mu_r \varepsilon_r} = 1.5$

$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0 \sqrt{\frac{1}{\varepsilon_r}} = \frac{120\pi}{1.5} = 80\pi = 251.33$

Snell's law: $\theta_r = \theta_i = 60°$.

$n_1 \sin\theta_i = n_2 \sin\theta_t$ $\quad \begin{cases} \sin\theta_t = \frac{\sqrt{3}}{3} \\ \cos\theta_t = \frac{\sqrt{6}}{3} \end{cases}$

$1 \cdot \sin 60° = 1.5 \sin\theta_t \Rightarrow \theta_t = 35.264°$

$k_1 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{2\pi f}{c} = \frac{2\pi \times 10^9\, Hz}{3\times 10^8\, m/s} = \frac{20}{3}\pi\ m^{-1}$

$k_2 = \frac{2\pi f \sqrt{\varepsilon_r}}{c} = \frac{2\pi \times 10^9\, Hz \times 1.5}{3\times 10^8\, m/s} = 10\pi\ m^{-1}$ $\quad \omega = 2\pi f = 2\pi \times 10^9$

Incident wave:

$\tilde{E}^i = \hat{y} E_0 e^{-j\frac{20}{3}\pi\left(x\frac{\sqrt{3}}{2} + z\frac{1}{2}\right)}$

$\tilde{H}^i = \left(-\hat{x}\frac{1}{2} + \hat{z}\frac{\sqrt{3}}{2}\right)\frac{E_0}{376.7} e^{-j\frac{20}{3}\pi\left(x\frac{\sqrt{3}}{2} + z\frac{1}{2}\right)}$

$\vec{E}(x,y,z,t) = \hat{y} E_0 \cos\left(2\pi\times 10^9 t - \frac{10\sqrt{3}}{3}\pi x - \frac{10}{3}\pi z\right)$

$\vec{H}(x,y,z,t) = \left(-\hat{x}\frac{1}{2} + \hat{z}\frac{\sqrt{3}}{2}\right)\frac{E_0}{376.7}\cos\left(2\pi\times 10^9 t - \frac{10}{3}\sqrt{3}\pi x - \frac{10}{3}\pi z\right)$

Reflected Wave:

$$\Gamma = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{80\pi \times \frac{1}{2} - 120\pi \times \frac{\sqrt{6}}{3}}{80\pi \times \frac{1}{2} + 120\pi \times \frac{\sqrt{6}}{3}} = -0.42$$

$$E^r = \Gamma E_0 = -0.42 E_0$$

$$\tilde{E}^r = \hat{y}(-0.42 E_0) e^{-j\frac{20}{3}\pi\left(x\frac{\sqrt{3}}{2} - z\frac{1}{2}\right)}$$

$$\tilde{H} = \left(\hat{x}\frac{1}{2} + \hat{z}\frac{\sqrt{3}}{2}\right)(-1.11 \times 10^{-3} E_0) e^{-j\frac{20}{3}\pi\left(x\frac{\sqrt{3}}{2} - z\frac{1}{2}\right)}$$

$$\vec{E}^r(x,y,z,t) = \hat{y}(-0.42 E_0)\cos\left(2\pi \times 10^9 t - \frac{10\sqrt{3}}{3}\pi x + \frac{10}{3}\pi z\right)$$

$$\vec{H}^r(x,y,z,t) = \left(\hat{x}\cdot\frac{1}{2} + \hat{z}\cdot\frac{\sqrt{3}}{2}\right)(-1.11 \times 10^{-3} E_0)\cos\left(2\pi \times 10^9 t - \frac{10\sqrt{3}}{3}\pi x + \frac{10}{3}\pi z\right)$$

Transmitted Wave:

$$\tau = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{2 \times 80\pi \times \frac{1}{2}}{80\pi \times \frac{1}{2} + 120\pi \times \frac{\sqrt{6}}{3}} = 0.58$$

$$E^t = \tau E_0 = 0.58 E_0$$

$$\tilde{E}^t = \hat{y}\, 0.58 E_0 e^{-j10\pi\left(x\frac{\sqrt{3}}{3} + z\frac{\sqrt{6}}{3}\right)}$$

$$\tilde{H}^t = \left(-\hat{x}\frac{\sqrt{6}}{3} + \hat{z}\frac{\sqrt{3}}{3}\right)(2.31 \times 10^{-3} E_0) e^{-j10\pi\left(x\frac{\sqrt{3}}{3} + z\frac{\sqrt{6}}{3}\right)}$$

$$\vec{E}(x,y,z,t) = \hat{y}\, 0.58 E_0 \cos\left(2\pi \times 10^9 t - \frac{10\sqrt{3}}{3}\pi x - \frac{10\sqrt{6}}{3}\pi z\right)$$

$$\vec{H}(x,y,z,t) = \left(-\hat{x}\frac{\sqrt{6}}{3} + \hat{z}\frac{\sqrt{3}}{3}\right)(2.31 \times 10^{-3} E_0)$$

$$\cos\left(2\pi \times 10^9 t - \frac{10\sqrt{3}}{3}\pi x - \frac{10\sqrt{6}}{3}\pi z\right)$$

c) $T = |\tau|^2 \frac{\eta_1}{\eta_2}$ ✗

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$= (0.58)^2 \cdot \frac{120\pi}{80\pi} = 50.46\%$

$\Rightarrow$ portion transmitted is 50.46%