EE100 Midterm 2

Winter 2012

Solutions

Name

Signature

Student ID

Problem 1 (pts):

Problem 2 (pts):

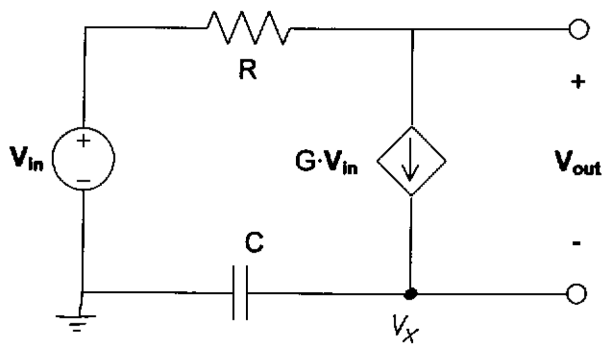
Problem 3 (pts):

Problem 4 (pts):

Total (pts):

There are 9 pages total.

1a) Find the transfer function $H(\omega) = V_{out}/V_{in}$ in terms of R, G, and ω for the circuit below.



KCL:

$$G\tilde{V}_{in} = \frac{\tilde{V}_x}{1/j\omega C} = j\omega C \tilde{V}_x$$

$$\tilde{V}_x = \tilde{V}_{in} \frac{G}{j\omega C}$$

KVL:

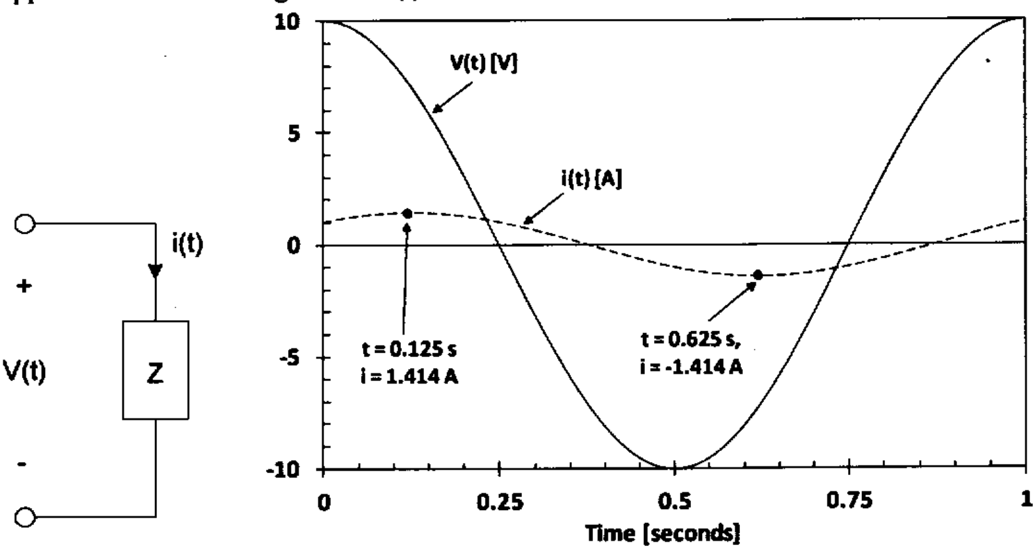
$$\tilde{V}_{in} = G\tilde{V}_{in} \cdot R + \tilde{V}_{out} + \tilde{V}_x$$

$$= GR\tilde{V}_{in} + \tilde{V}_{out} + \tilde{V}_{in}\frac{G}{j\omega C}$$

$$\tilde{V}_{out} = \tilde{V}_{in}\left(1 - G \cdot R - \frac{G}{j\omega C}\right)$$

$$\boxed{H(\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = 1 - G \cdot R - \frac{G}{j\omega C}}$$

1b) A certain two terminal load Z exhibits the following transient response when a sinusoidal voltage V(t) is applied and the resulting current i(t) is measured:



From the information given, reduce the load Z into its simplest possible configuration involving passive elements (i.e. resistors, inductors, capacitors).

$$v(t) = 10\cos(2\pi t)$$

$$i(t) = \sqrt{2}\cos(2\pi t - 0.125 \cdot 360^\circ)$$

$$= \sqrt{2}\cos(2\pi t - 45^\circ)$$

$$\tilde{V} = 10\angle 0^\circ$$

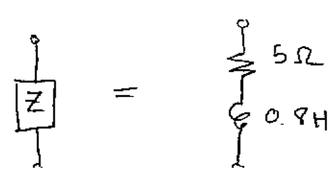
$$\tilde{I} = \sqrt{2}\angle -45^\circ$$

$$\frac{\tilde{V}}{\tilde{I}} = Z = \frac{10\angle 0^\circ}{\sqrt{2}\angle -45^\circ} = 7.07\angle 45^\circ = 5+5j = R+jX$$

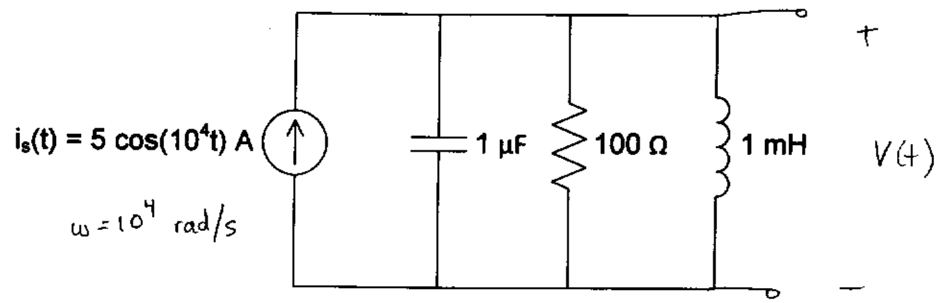
Since the reactance is positive (current lags voltage), it is inductive in nature. We can model the total impedance by a resistor and inductor in series:

$$Z = R + j\omega L = 5 + 5j$$

$$\boxed{\begin{array}{l} R = 5\,\Omega \\ L = \frac{5}{2\pi} = 0.8\,H \end{array}}$$



2a) Find the average power, reactive power, and apparent power delivered by the current source in the circuit below.



$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

$$= \frac{1}{0.01 + j(0.01 - 0.1)} = \frac{1}{0.01 - 0.09j}$$

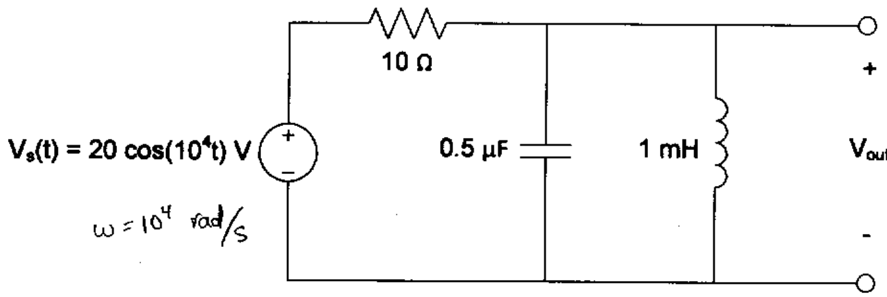
$$= \frac{1}{0.09055 \angle -83.66^\circ} = 11.04 \angle 83.66^\circ$$

$$\tilde{I}_s = 5\angle 0^\circ = \overbrace{3.536}^{I_{rms}}\sqrt{2} \angle 0^\circ$$

$$\tilde{V} = \tilde{I}_s Z = 5\angle 0^\circ \cdot 11.04\angle 83.66^\circ = 55.22 \angle 83.66^\circ = \underbrace{39.05}_{V_{rms}}\sqrt{2} \angle 83.66^\circ$$

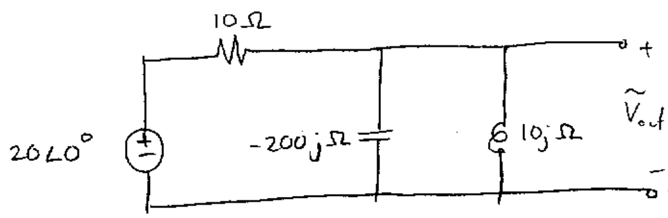
$$\boxed{\begin{aligned} \text{App. power} &= V_{rms} I_{rms} = 39.05\,(3.536) = 138.1 \text{ [VA]} \\ P_{avg} &= V_{rms} I_{rms} \cos\theta = 138.1 \cos(83.66^\circ) = 15.25 \text{ [W]} \\ Q &= V_{rms} I_{rms} \sin\theta = 138.1 \sin(83.66^\circ) = 137.25 \text{ [VAR]} \end{aligned}}$$

2b) **Find and draw the Thevenin and Norton equivalent circuits as seen from the port V_{out} in the circuit below.**



$Z_C = -\frac{j}{\omega C} = -200j\ \Omega$

$Z_L = j\omega L = 10j\ \Omega$



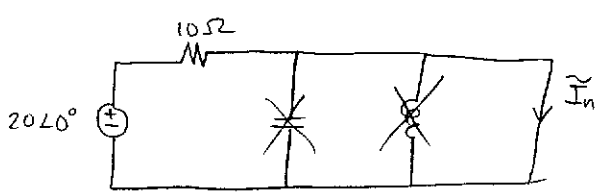
$$\tilde{V}_{out} = 20 \cdot \frac{-200j \parallel 10j}{10 + (-200j \parallel 10j)}$$

$$-200j \parallel 10j = \frac{-2000j^2}{-190j} = 10.53j$$

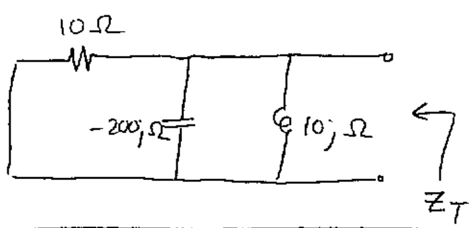
$$= 20 \cdot \frac{10.53j}{10 + 10.53j}$$

$$= \frac{210.53\angle 90^\circ}{14.52\angle +46.48^\circ}$$

$$\boxed{\tilde{V}_T = 14.5\angle 43.52^\circ}\ [V]$$



$$\boxed{\tilde{I}_n = \frac{20}{10} = 2\angle 0^\circ}\ [A]$$

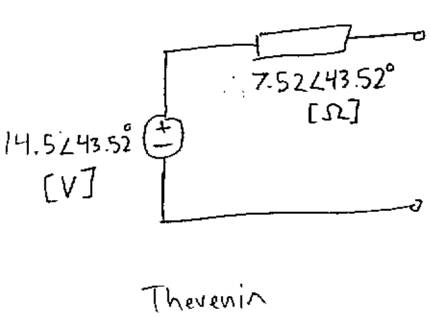


$$Z_T = 10 \parallel -200j \parallel 10j$$

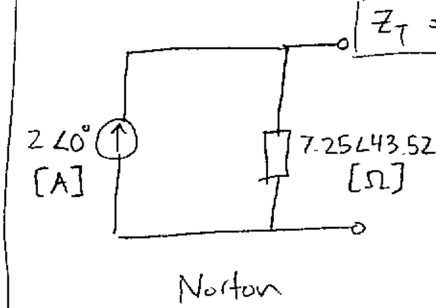
$$= 10 \parallel 10.53j$$

$$= \frac{105.3j}{10 + 10.53j} = \frac{105.3\angle 90^\circ}{14.52\angle 46.48^\circ}$$

$$\boxed{Z_T = 7.25\angle 43.52^\circ}\ [\Omega]$$



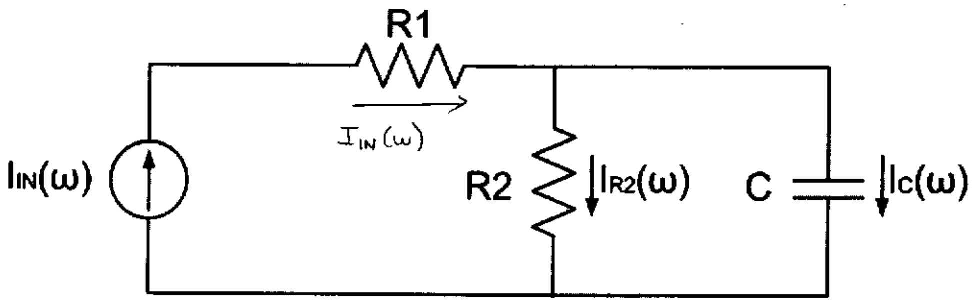
Thevenin



Norton

3a) Find the transfer functions in terms of R_1, R_2, and C.

$$H_1(\omega) = \frac{I_{R2}}{I_{IN}}(\omega) \qquad H_2(\omega) = \frac{I_C}{I_{IN}}(\omega)$$



$$\tilde{I}_{R2} = \tilde{I}_{IN}\frac{Z_c}{R_2 + Z_c} = \tilde{I}_{IN}\frac{1/j\omega C}{R_2 + 1/j\omega C} = \tilde{I}_{IN}\frac{1}{1+j\omega R_2 C}$$

$$\boxed{H_1(\omega) = \frac{\tilde{I}_{R_2}}{\tilde{I}_{IN}} = \frac{1}{1+j\omega R_2 C}} \leftarrow \text{LPF}$$

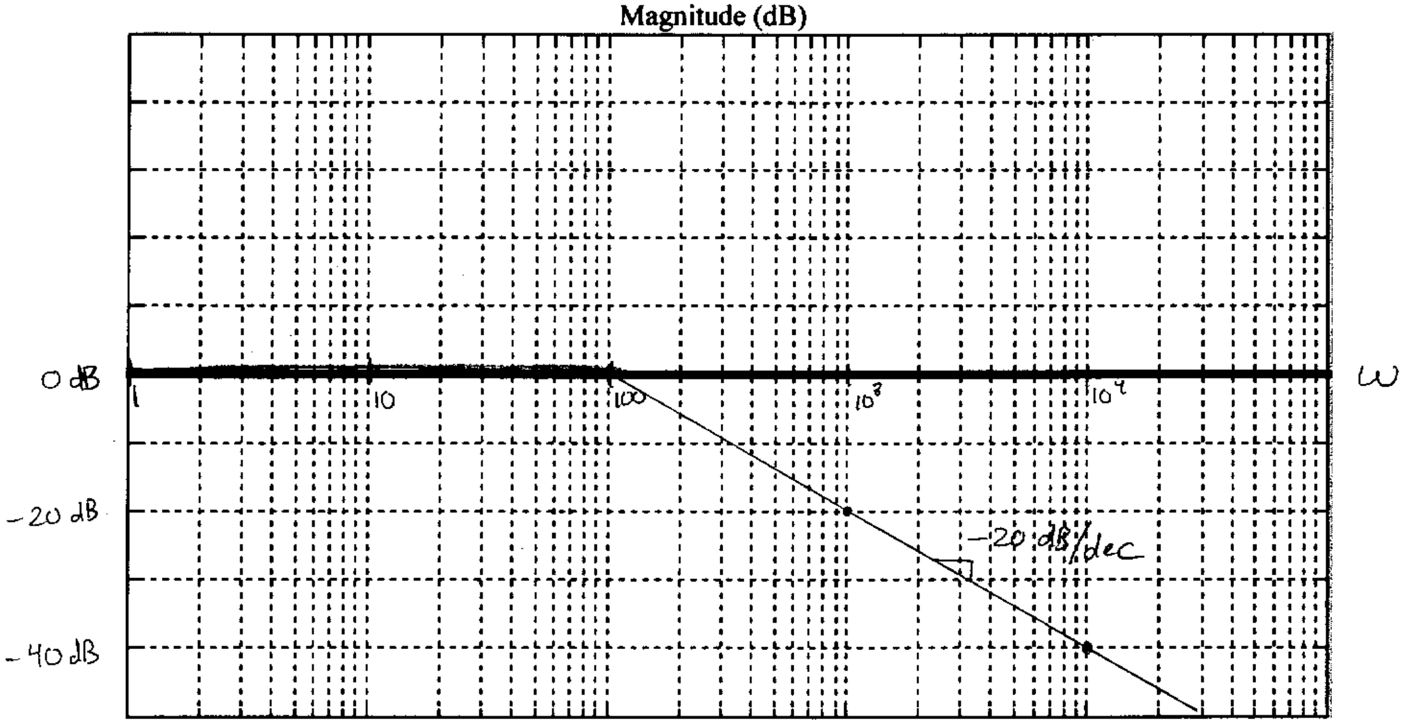
$$\tilde{I}_c = \tilde{I}_{IN}\frac{R_2}{R_2 + Z_c} = \tilde{I}_{IN}\frac{R_2}{R_2 + 1/j\omega C} = \tilde{I}_{IN}\frac{j\omega R_2 C}{1+j\omega R_2 C}$$

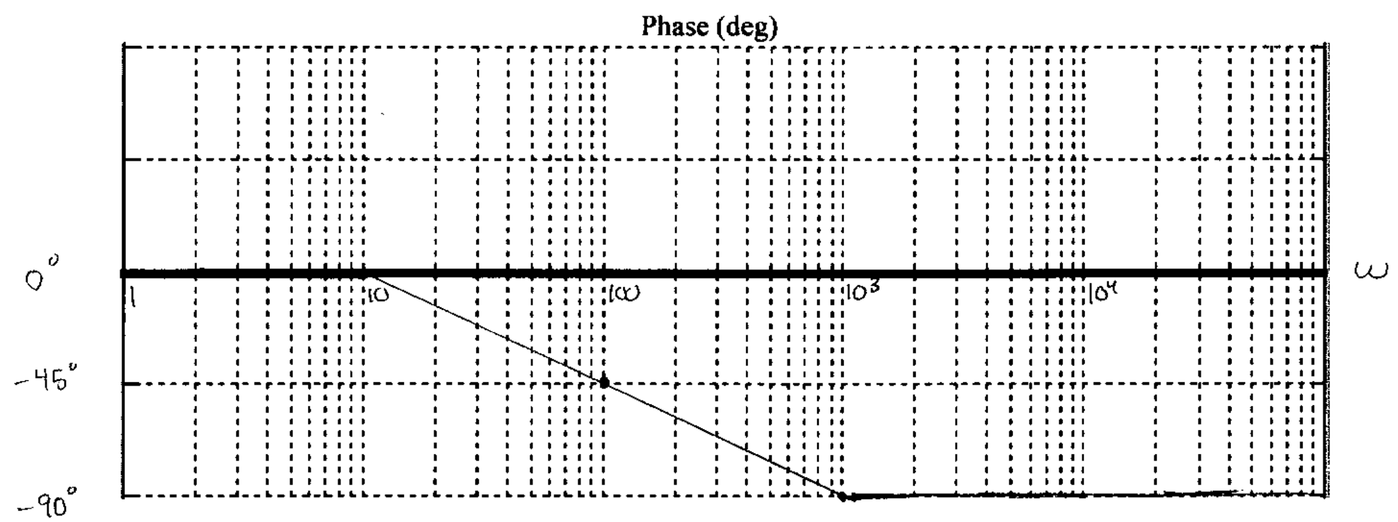
$$\boxed{H_2(\omega) = \frac{\tilde{I}_c}{\tilde{I}_{IN}} = \frac{j\omega R_2 C}{1+j\omega R_2 C}} \leftarrow \text{HPF}$$

For $\left.\begin{array}{l} R_1 = 1\,k\Omega \\ R_2 = 10\,k\Omega \\ C = 1\,\mu F \end{array}\right\}$ $\quad R_2 C = 10^4 \cdot 10^{-6} = 10^{-2}$

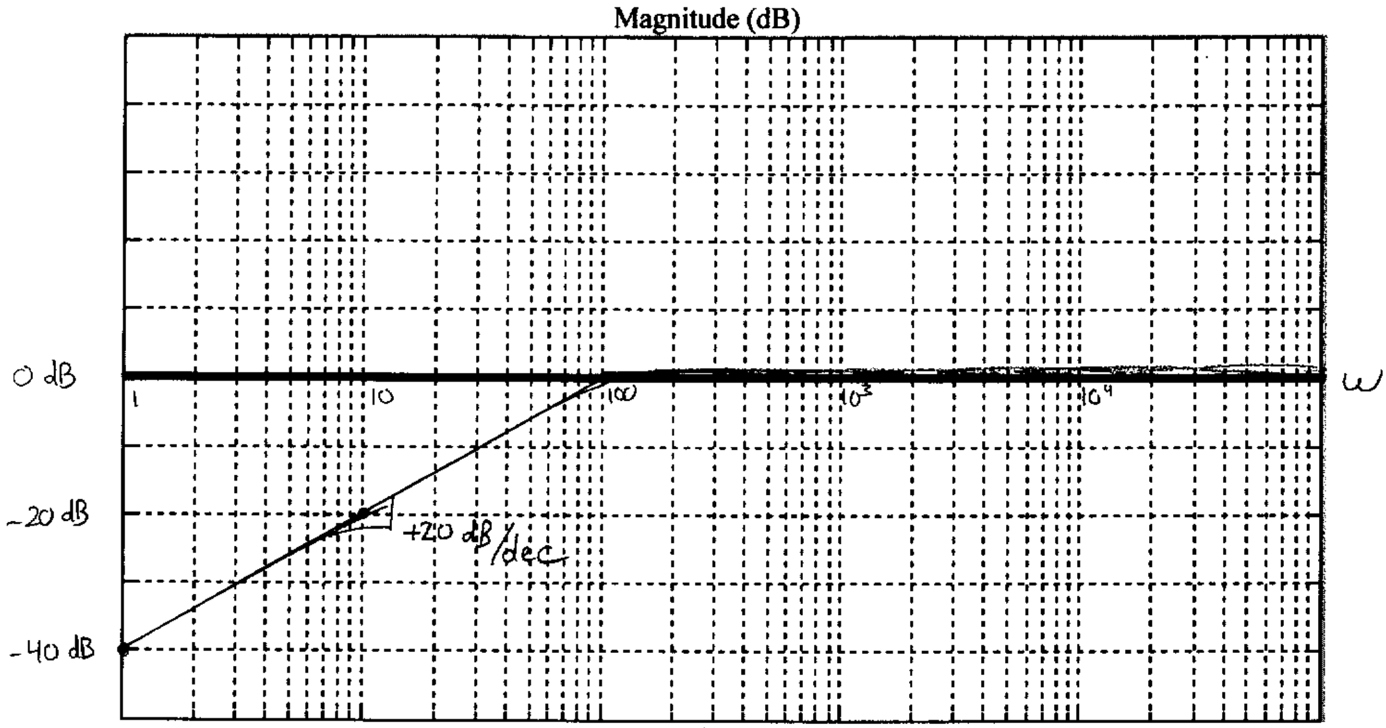
$\frac{1}{R_2 C} = 100 \text{ rad/s}$

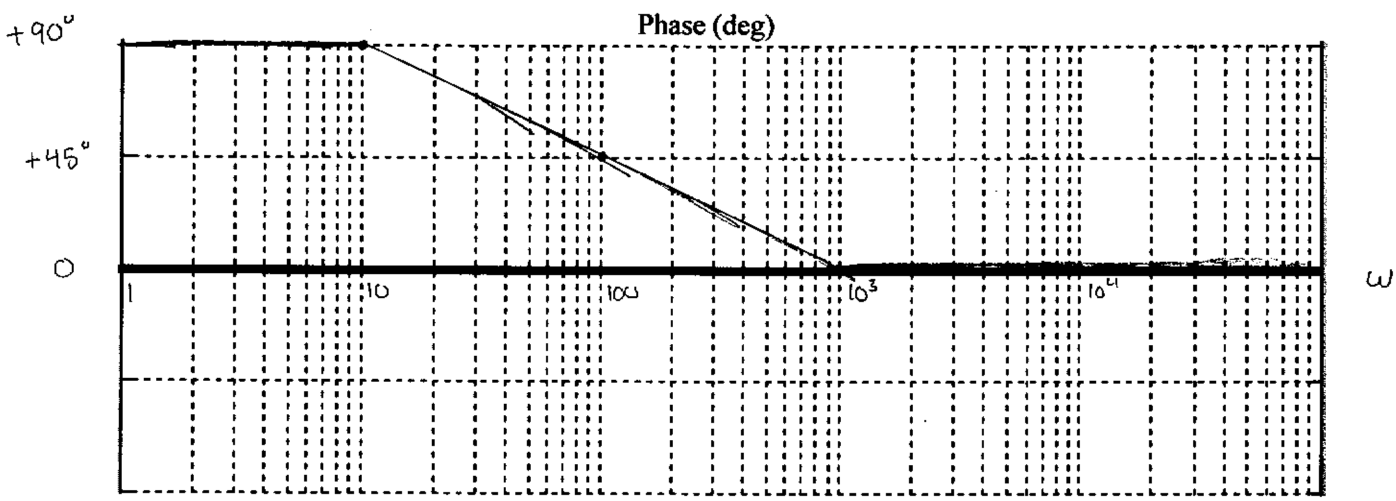
3b) Plot the magnitude and phase using Bode plots of $H_1(\omega) = \mathbf{I_{R2}/I_{IN}}\ (\omega)$.
Accurately depict and label the key features.
Assume $R_1 = 1K\Omega$, $R_2 = 10K\Omega$ and $C = 1\ \mu F$.



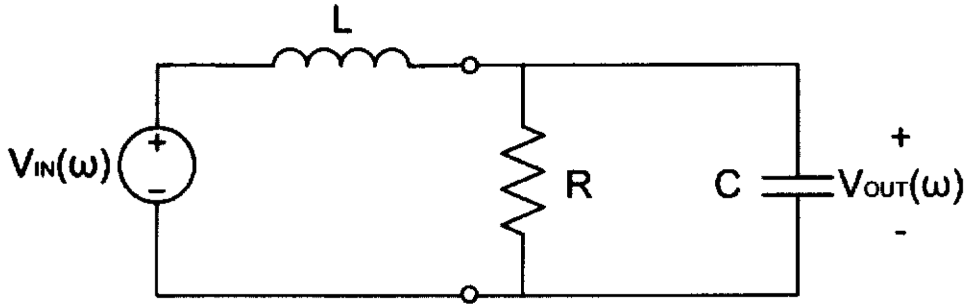


3c) Plot the magnitude and phase using Bode plots of $H_2(\omega) = I_C/I_{IN}(\omega)$.
Accurately depict and label the key features.
Assume $R_1 = 1K\Omega$, $R_2 = 10K\Omega$ and $C = 1\ \mu F$.

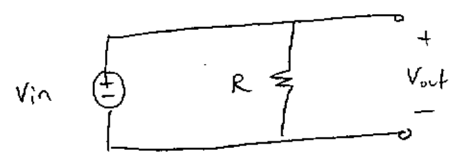




4a) Consider the circuit below. Given reasonable component values, what type of filter do we have? Low pass, high pass, band pass, band reject, etc.

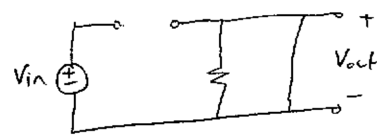


Very low frequencies



$\frac{V_{out}}{V_{in}} \to 1$

Very high frequencies



$\frac{V_{out}}{V_{in}} \to 0$

This represents a $\boxed{\text{low pass filter.}}$

4b) Find the resonant frequency of this circuit. Leave your answer in terms of R, L, and C.

$$Z = j\omega L + \frac{1}{\frac{1}{R} + j\omega C} = j\omega L + \frac{R}{1 + j\omega RC}$$

$$= j\omega L + \frac{R(1 - j\omega RC)}{1 + (\omega RC)^2} = \frac{R}{1+(\omega RC)^2} + j\underbrace{\left[\omega L - \frac{\omega R^2 C}{1 + (\omega RC)^2}\right]}_{=0 \text{ at } \omega = \omega_o}$$

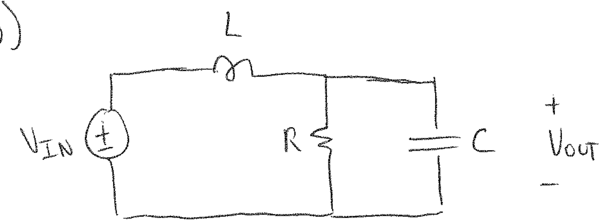
$$\cancel{\omega_o} L = \frac{\cancel{\omega_o} R^2 C}{1 + (\omega_o RC)^2}$$

$$1 + (\omega_o RC)^2 = \frac{R^2 C}{L}$$

$$\omega_o^2 = \left(\frac{R^2 C}{L} - 1\right)\frac{1}{(RC)^2} = \frac{1}{LC} - \frac{1}{(RC)^2}$$

$$\boxed{\omega_o = \sqrt{\frac{1}{LC} - \frac{1}{(RC)^2}}}$$

4b)



Alternate solution

Text mentions ω_0 occurs near the maximum of $|H(\omega)|$

This is only true with a large enough Q. However, full credit is still awarded if you followed this approach.

$$\frac{V_{OUT}}{V_{IN}} = H(\omega) = \frac{\left(\frac{1}{R} + j\omega C\right)^{-1}}{\left(\frac{1}{R} + j\omega C\right)^{-1} + j\omega L} = \frac{1}{1 + (j\omega L)\left(\frac{1}{R} + j\omega C\right)} = \frac{1}{1 - \omega^2 LC + j\omega \frac{L}{R}}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \left(\omega \frac{L}{R}\right)^2}} = \left(1 + \omega^4 L^2 C^2 - 2\omega^2 LC + \omega^2 \frac{L^2}{R^2}\right)^{-1/2}$$

$$\frac{d}{d\omega}|H(\omega)| = \frac{4\omega^3 L^2 C^2 - 4\omega LC + 2\omega \frac{L^2}{R^2}}{(\ldots)^{3/2}}$$

Assume ω_0 occurs when $\frac{d}{d\omega}|H(\omega)| = 0$

$$4\omega^3 L^2 C^2 - 4\omega LC + 2\omega \frac{L^2}{R^2} = 0$$

$$\omega^2 L^2 C^2 = LC - \frac{1}{2}\frac{L^2}{R^2}$$

$$\boxed{\omega = \sqrt{\frac{1}{LC} - \frac{1}{2}\frac{1}{R^2 C^2}} = \omega_0}$$