EE100 Midterm

Winter 2012

T.A. E.A.L

Name

SOLUTIONS

Signature

Student ID

Problem 1 (18 pts):

Problem 2 (10 pts):

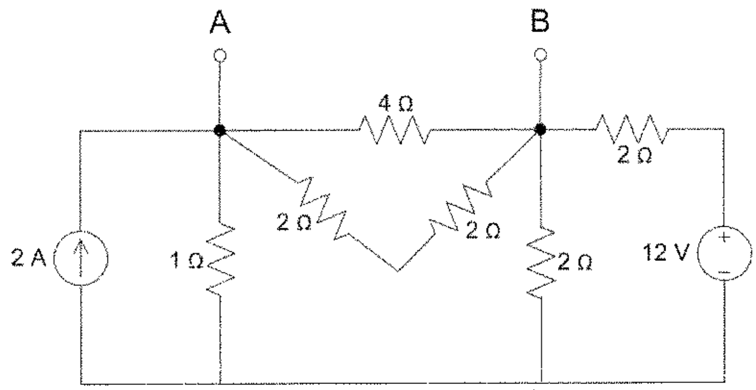
Problem 3 (10 pts):

Problem 4 (12 pts):

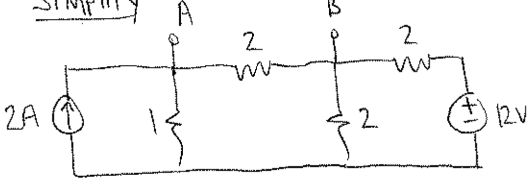
Total (50 pts):

There are 7 pages total.

1a) Find and draw the Thevenin and Norton equivalent circuits as seen from terminals A and B for the circuit below. (**10 pts**)



Simplify



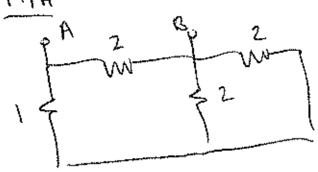
Node to find $V_{TH} = V_{AB}$

$$\frac{V_A - V_B}{2} + \frac{V_A}{1} = 2 \quad \Rightarrow \quad 1.5V_A - 0.5V_B = 2$$

$$\frac{V_B - V_A}{2} + \frac{V_B}{2} + \frac{V_B - 12}{2} = 0 \quad \Rightarrow \quad -0.5V_A + 1.5V_B = 6$$

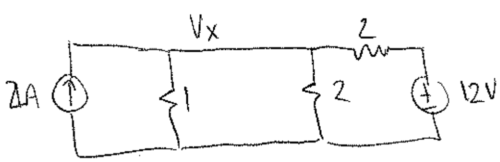
$\Rightarrow \quad V_A = 3V \quad V_B = 5V \quad V_{AB} = -2V$

R_{TH}



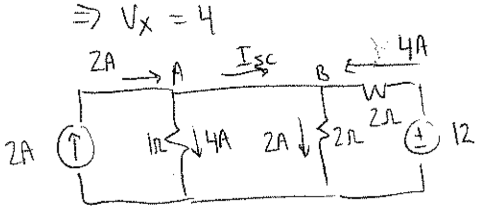
$$R_{TH} = \left[(2\Omega \| 2\Omega) + 1\Omega\right] \| 2\Omega = 1\Omega$$

I_{SC} (Already have V_{TH} and R_{TH}, but let's find this anyways)

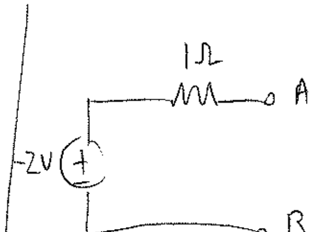


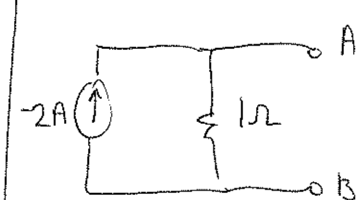
$$\frac{V_x}{1} + \frac{V_x}{2} + \frac{V_x - 12}{2} = 2$$

$\Rightarrow V_x = 4$



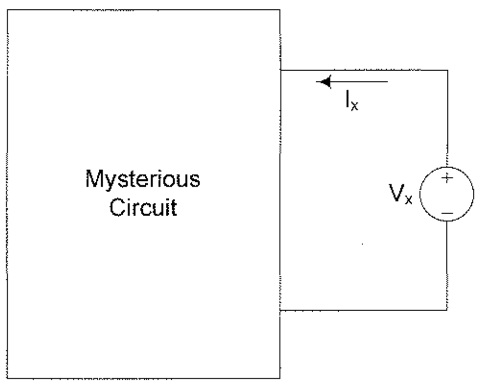
$I_{SC} = -2A$

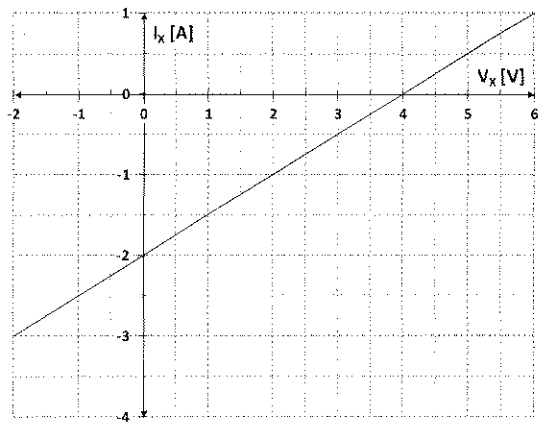




1b) You are given a mysterious two terminal circuit but you have no idea what's inside it! Cleverly, you decide to measure the I-V characteristics by applying a voltage V_X across the terminals and measuring the resulting current I_X that passes through the circuit as shown in the graph.

From the data given, find and draw the Thevenin and Norton equivalent representations of this mysterious circuit. **(8 pts)**





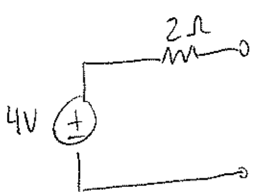
When $V_X = 4V$ $\quad I_X = 0$

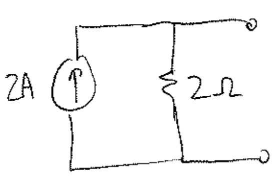
$\Rightarrow V_{TH} = 4V$

When $V_X = 0V$ $\quad I_X = -2A$

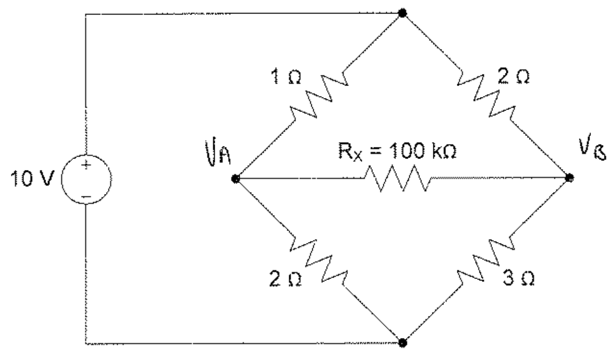
$\Rightarrow I_{SC} = 2A$

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = 2\Omega$$





2) Is the Wheatstone bridge shown below balanced or unbalanced? Find the power delivered to R_X. Hint: You may use the fact that R_X is much greater than the other resistors to simplify the analysis. **(10 pts)**



Bridge is NOT balanced $\frac{1\Omega}{2\Omega} \neq \frac{2\Omega}{3\Omega}$!

Since R_x is large compared to the other resistors, we can omit it in finding V_A and V_B.

$V_A = \frac{2\Omega}{2\Omega + 1\Omega} 10V = 6.66V$

$V_B = \frac{3\Omega}{3\Omega + 2\Omega} 10V = 6V$

Now, find power across R_x

6.66V —— 100kΩ —— 6V

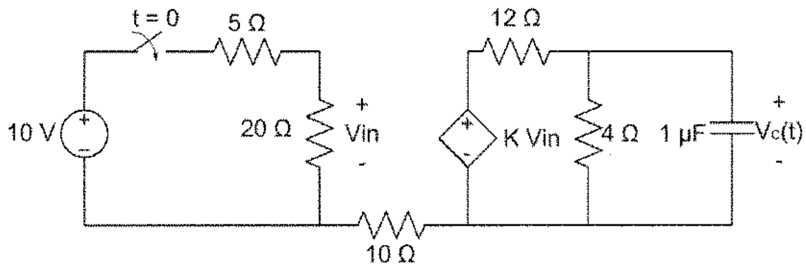
$P = \frac{V^2}{R} = \frac{(6.66-6)^2}{100k\Omega} = \boxed{4.44\ \mu W}$

If we want to be really precise and include R_x to find V_A V_B

$P = 4.44427\ \mu W$

$\Rightarrow$ Our simplification is reasonable

3) For this circuit, the switch has been open for a long time. Then the switch closes at t = 0. Assume K = 2. **(10 pts)**



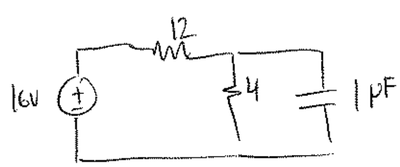
a) Can we determine $V_C(0+)$ or do we need more information? Explain. **(2 pts)**

Yes, before the switch closes, $V_{in} = 0V \Rightarrow KV_{in} = 0V$. The capacitor would be fully discharged. $V_c(0^-) = 0V$. This is true because we have a complete circuit when $V_{in} = 0$.

b) Assuming $V_C(0\text{-}) = 0V$, find $V_C(t)$. **(8 pts)**

$$V_{in} = \frac{20}{20+5} 10 = 8V$$

$$KV_{in} = 16V$$



Final voltage of $V_c(\infty) = \frac{4}{12+4} 16 = 4V$

Thus $V_c(t) = 4 - 4e^{-t/RC}$ V

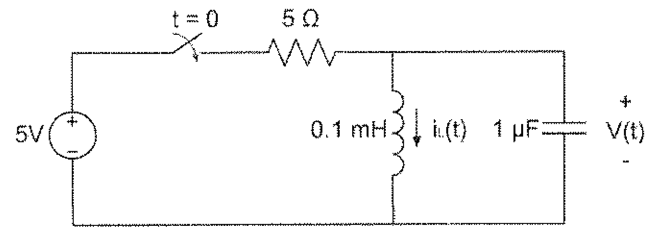
$R = 4 \| 12 = 3\Omega$

$C = 1\mu F$

$RC = \tau = 3 \times 10^{-6}$

Notice the 10Ω resistor has no effect on the circuit. Current has no return path thus no current will flow through the 10Ω resistor! Thus we can treat it as an open.

4) For this circuit, the switch has been open for a long time. Then the switch closes at t = 0. Assume V(0-) = 0V and i_L(0-) = 0A. **(12 pts)**

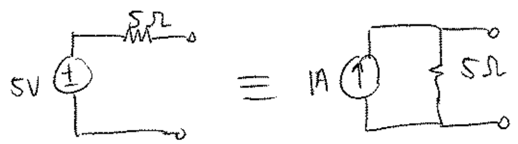


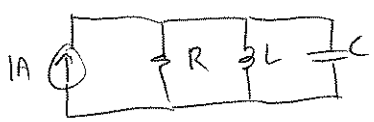
a) Write the differential equation for V(t) in the form.

$$\frac{d^2V(t)}{dt^2} + 2\alpha\frac{dV(t)}{dt} + \omega_0^2 V(t) = f(t)$$

Define α and ω_0 in terms of R, L, and C. **(6 pts)**

We can do a source transformation





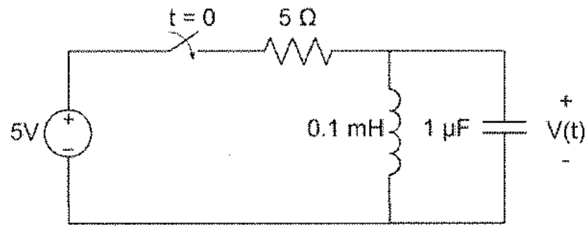
Now we just have a parallel R, L, C

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = 10^5 \qquad \omega_0 = 10^5$$

4) Continued.



b) Is the response V(t), underdamped, critically damped, or overdamped? **(1 pts)**

$\frac{\alpha}{\omega_o} = 1$ critically damped

c) What is the particular solution of V(t)? **(2 pts)**

Look at steady state. Inductor is a short circuit.

Thus $V_p(t) = 0$

d) What is the complementary solution of V(t)? Hint: Find dV/dt to help solve for initial conditions. **(3 pts)**

Since $V_p(t) = 0$ the full solution $V(t) = V_p(t) + V_c(t) = V_c(t)$

We are critically damped, $V(t) = K_1 e^{s_1 t} + K_2 t e^{s_2 t}$ $s_1 = s_2 = -\alpha = -10^5$

$\underline{K_1}$

$V(0^+) = 0 = K_1 e^{s_1(0)} + K_2(0) e^{s_2(0)} \Rightarrow K_1 = 0$

$\underline{K_2}$

We know (deduce) current through cap (i_{cap}) at $t=0$ is 1A

$C \frac{dV_{cap}(0)}{dt} = 1A = C\frac{dV(0)}{dt} \Rightarrow \frac{dV(0)}{dt} = \frac{1A}{C} = 10^6$

$\frac{dV(t)}{dt} = s_2 K_2 t e^{s_2 t} + K_2 e^{s_2 t}$

$\frac{dV(0)}{dt} = 10^6 = K_2 e^{s_2(0)} \Rightarrow K_2 = 10^6$

$V(t) = 10^6 t e^{-10^5 t}$