1

a) Decimal digits in BCD

16,000,000 4 bits per single decimal digit, 8 decimal digits

4 4 4 4 4 4 4 4

$4 \times 8 = \boxed{32 \text{ bits}}$

b) Hexadecimal representation

$(16{,}000{,}000)_{10} \rightarrow (F42400)_{16}$

$\boxed{6 \text{ bits}}$

Hex is more efficient because it uses less bits than the BCD representation. In BCD, each decimal number corresponds to 4 bits, while in the hex representation, digits 0-9 as well as A-F are used, so fewer total bits are required for the hexadecimal system.

2

Radix	Value	Value in Decimal
16	(517)	1303
8	(517)	335

$(517)_{16} \rightarrow$ decimal:

$7 \times 16^0 + 1 \times 16^1 + 5 \times 16^2$

$= 7 + 16 + 1280$

$= \boxed{1303}$

$(517)_8 \rightarrow$ decimal

$7 \times 8^0 + 1 \times 8^1 + 5 \times 8^2$

$= 7 + 8 + 320$

$= \boxed{335}$

a) $\boxed{\text{False}}$; bubble on output means active LOW

b) $\boxed{\text{True}}$

c) $\boxed{\text{True}}$

③ $E(a,b,c,d) = \overline{(ab+c)}\,(ac + \overline{(\bar b + \bar c + \bar a c d)}) + a(\overline{(b+c)(b+d)+c})$

simplify:

$= ((\overline{a+b})\cdot \bar c)(ac + \overline{(\bar b + \bar c + \bar a c d)}) + a(\overline{(b+c)(b+d)+c})$

$= ((\overline{a+b})\cdot \bar c)(ac + \overline{\bar b \bar c}\,(\overline{\bar a + \bar c \bar d})) + a(\overline{(b+c)(b+d)+c})$

$= ((\overline{a+b})\cdot \bar c)(ac + bc(a+\bar c+\bar d)) + a(\overline{(b+c)(b+d)+c})$

$= ((\overline{a+b})\bar c)(ac + bc(a+\bar c+\bar d)) + a((\overline{b+c}) + (\overline{b+d})\cdot \bar c)$

$= ((\overline{a+b})\bar c)(ac + bc(a+\bar c+\bar d)) + a(\overline{bc} + \overline{bd})\bar c$

$= \bar a \bar c[ac + bc(a+\bar c+\bar d)] + \bar b \bar c[ac + bc(a+\bar c+\bar d)] + a(\overline{bc} + \overline{bd})\bar c$

$= \underbrace{\bar a a \bar c c + \bar a \bar c bc(a+\bar c+\bar d) + \bar b \bar c ac + \bar b \bar c bc(a+\bar c+\bar d)}_{\text{all cancel out}} + a(\bar b \bar c + a\bar b \bar d)\bar c$

$= a\bar c \bar b \bar c + a \bar c \bar b \bar d$

$= a\bar c \bar b + a \bar c \bar b \bar d$

$= \boxed{a\bar c \bar b}$

$\boxed{\#5}$

(4) a) Gray code is unique because it is unweighted, allowing it to have a cyclical property. Essentially, the hamming distance between adjacent numbers is 1, so as you progress through each cycle, each number only changes by 1 bit. Once you reach the end of the cycle, the final number will be 1 bit different than the starting number (e.g. for n=4 bits, 15 = 1000 & 0 = 0000.

end   beginning

For converting GC to binary, the MSB stays the same, and you add the digits from left to right, neglecting the carry with each addition.

For example:

Gray Code: 1011   1100

⇓

Binary: 1101   1000

b) X-3 code is essentially the same as BCD but shifted by 3. You derive X-3 code from BCD by adding 3, but because you add 3, it isn't valid to represent -3 to -1 or 10 to 12 in x-3 code. Since x-3 code is only valid from 0 to 9, it's self-complementary, meaning two x-3 codes whose sum is 9 (the max value) are complements of each other. In other words, 1's complement of x-3 code is 9's complement of the corresponding decimal.

For example:

Decimal	X-3 code
2	0101
7	1010
+ 9	

complements

(4) c) $(25)_{10} - (-25)_8$

Convert to binary:

$(25)_{10} \Rightarrow 25/2 = 12 \text{ R}1$

$12/2 = 6 \text{ R}0$

$6/2 = 3 \text{ R}0$

$3/2 = 1 \text{ R}1$

$1/2 = 0 \text{ R}1$

$= (11001)_2$

from table

$(25)_8 = (10101)_2$

sign bit

2's complement: $00010101 \rightarrow \underline{1}1101010$

$+ \quad 1$

$(11101011)_2$ 2's complement of $(25)_8$

00011001

$- 11101011 \rightarrow$ 2's complement again:

$\underline{1}1101011 \rightarrow \underline{0}0010100$

$+ \quad 1$

00010101

00011001

$+ 00010101$

$\underline{1}000101110)_2 = (46)_{10}$

sign bit

$\boxed{(\underline{0}00101110)_2 = (46)_{10}}$

d)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

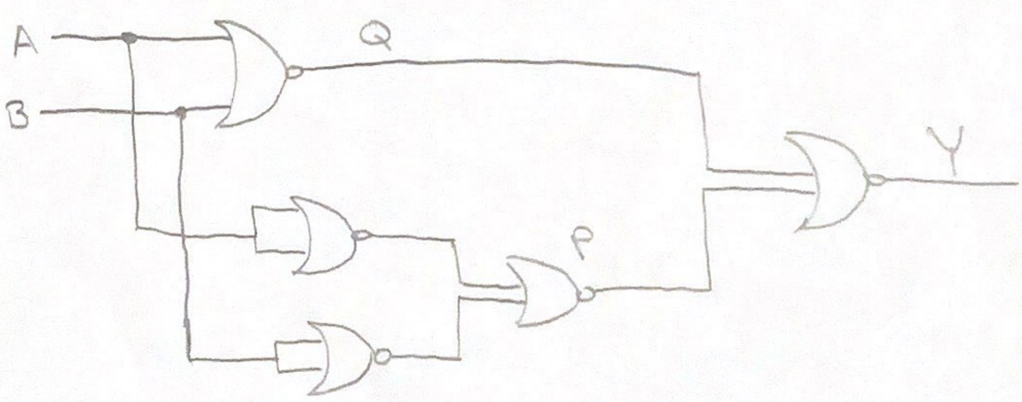
$\rightarrow$ XOR gate

$Y = \bar{A}B + A\bar{B} + (\overset{0}{A\bar{A}} + \overset{0}{B\bar{B}})$

$= (A+B)(\bar{A}+\bar{B})$

$= \overline{\overline{(A+B)(\bar{A}+\bar{B})}}$

$= \overline{\underbrace{\overline{(A+B)}}_{Q} + \underbrace{\overline{(\bar{A}+\bar{B})}}_{P}}$



(5) a) mistake (1) → (2)
incorrect DeMorgan's Law.

b) $\overline{(a\bar{b}+\bar{c})}(\bar{b}+c)(a+b\bar{c})$

$= (\bar{a}+b)c(\bar{b}+c)(a+b\bar{c})$

$= c\bar{a}(\bar{b}+c)(a+b\bar{c}) + cb(\bar{b}+c)(a+b\bar{c})$

$= c\bar{a}\bar{b}(a+b\bar{c}) + c\bar{a}c(a+b\bar{c}) + cb(\bar{b}+c)(a+b\bar{c})$

$= c\bar{a}\bar{b}(a+b\bar{c}) + c\bar{a}(a+b\bar{c}) + cb(\bar{b}+c)(a+b\bar{c})$

$= c\bar{a}(a+b\bar{c}) + cb(\bar{b}+c)(a+b\bar{c})$

$= c\underbrace{\bar{a}a}_{0} + c\bar{a}\underbrace{b\bar{c}}_{0} + cb(\bar{b}+c)(a+b\bar{c})$

$= cb(\bar{b}+c)(a+b\bar{c})$

$= c\underbrace{b\bar{b}}_{0}(a+b\bar{c}) + cbc(a+b\bar{c})$

$= cb(a+b\bar{c})$

$= cba + cb\underbrace{b}_{0}\bar{c}$

$= \boxed{cba}$

(6) a) $F = xy + \bar{x}z$

x	y	z	M
0	0	0	7
0	0	1	6
0	1	0	5
0	1	1	4
1	0	0	3
1	0	1	2
1	1	0	1
1	1	1	0

POS:

$(xy+\bar{x}z)$

$= (x+\bar{x})(y+\bar{x})(x+z)(y+z)$

$= 1\cdot(y+\bar{x})(x+z)(y+z)$

$= (y+\bar{x}+z\bar{z})(x+z+y\bar{y})(y+z+x\bar{x})$

$= (y+\bar{x}+z)(y+\bar{x}+\bar{z})(x+z+y)(x+z+\bar{y})(y+z+x)(y+z+\bar{x})$

equal

equal

$= \boxed{(y+\bar{x}+z)(y+\bar{x}+\bar{z})(x+z+y)(x+z+\bar{y})}$

$= M_4 \cdot M_5 \cdot M_0 \cdot M_2$

$= \boxed{\Pi(5,4,2,0)}$

6 b) $F=(A,B,C,D)= \bar{A}\bar{B}\bar{D} + \bar{A}CD + \bar{A}BC$, $d(A,B,C,D)= \bar{A}B\bar{C}D + ACD + A\bar{B}\bar{D}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	1
$\bar{A}B$	0	X	1	1
AB	0	0	X	0
$A\bar{B}$	X	0	X	X

$\Rightarrow$

$$\boxed{\begin{array}{l} F(A,B,C,D) = \sum(0,2,3,6,7) \\ d(A,B,C,D) = \sum(5,8,11,10,15) \end{array}}$$

c) $F = \sum(0,1,2,5,8,9,10)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	0	1	0	0
AB	0	0	0	0
$A\bar{B}$	1	1	0	1

SOP

from k-map (in dotted pencil)

$\Rightarrow F(A,B,C,D) = \boxed{(\bar{A}\bar{C}D)+(\bar{B}\bar{C})+(\bar{B}\bar{D})}$

POS

from k-map (in red pen)

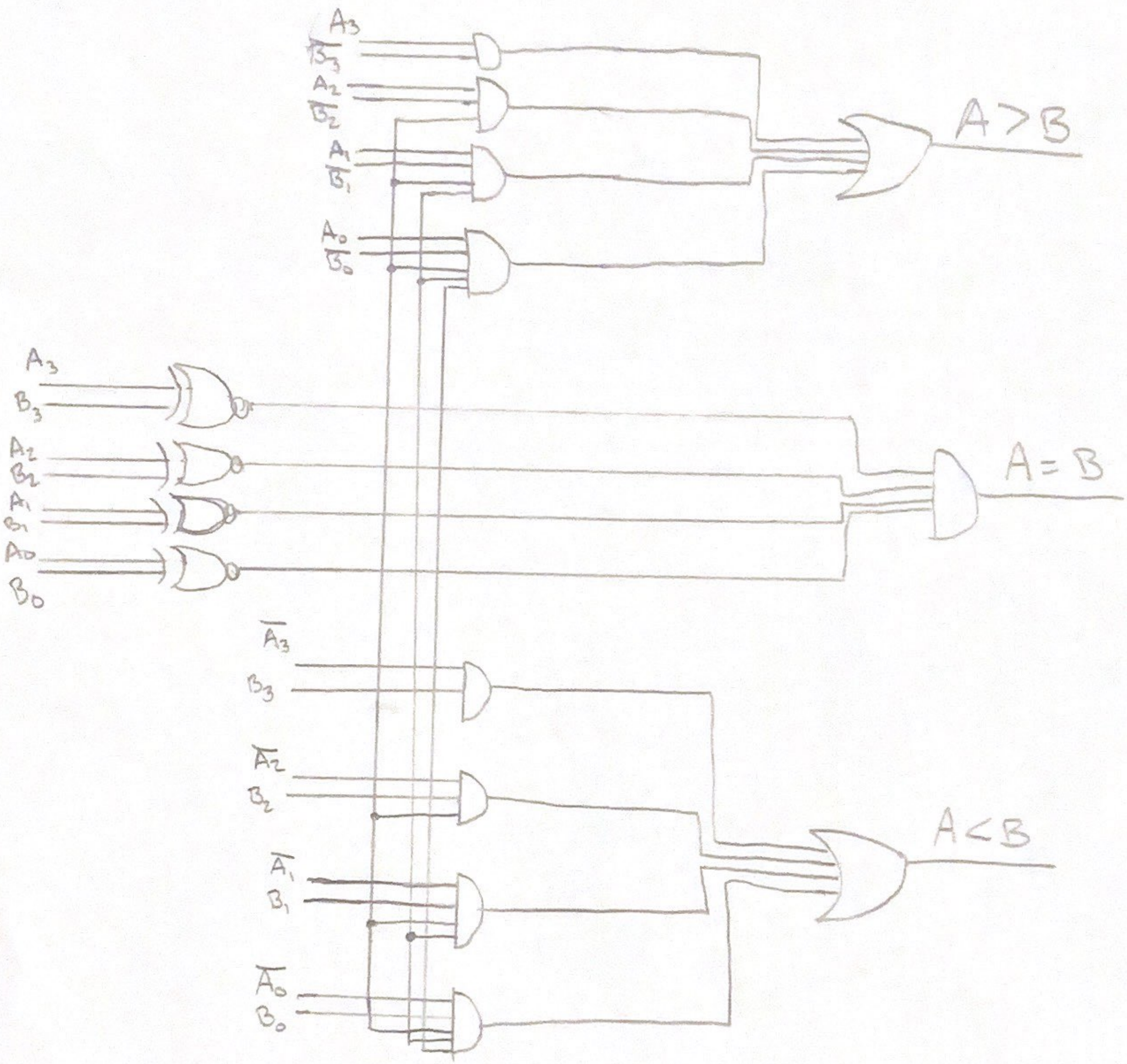
$\Rightarrow \bar{Y} = B\bar{D} + AB + CD$

$Y = \bar{\bar{Y}} = \overline{B\bar{D} + AB + CD}$

$= (\overline{B\bar{D}})(\overline{AB})(\overline{CD})$

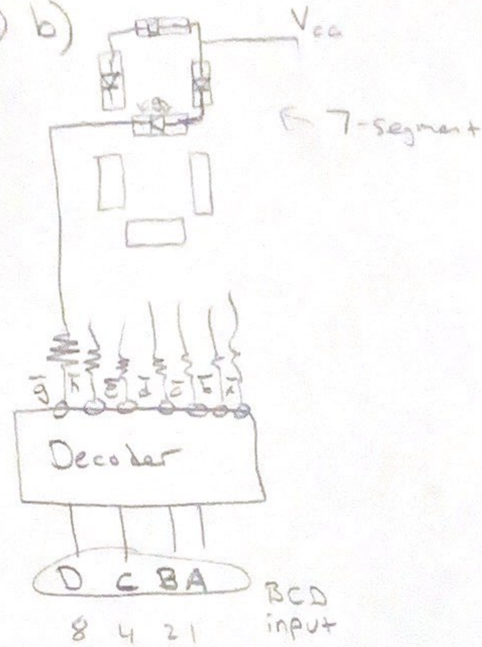
$= \boxed{(\bar{B}+D)(\bar{A}+\bar{B})(\bar{C}+\bar{D})}$

⑦ a)



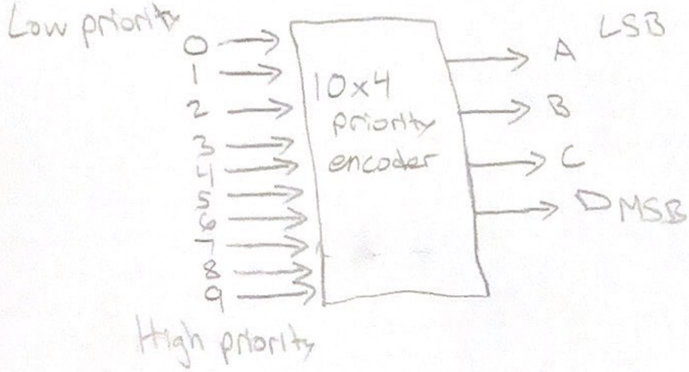
If $A \neq B$, the comparator sequentially checks the bits from MSB and cascades down to the LSB. In the > portion, the bits for B are inverted so that the and-gate will only be satisfied if A>B, otherwise the output will be 0. In the diagram, if $A_3 = B_3$, then A_2 & B_2 are compared next, and if those are equal, then A_1 & B_1, all the way to LSB. The extra lines are inhibitors in case a more significant bit is evaluated to be greater or less so you don't have to go through every single bit.

⑦ b)



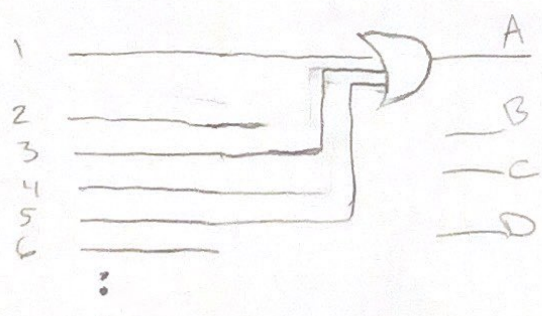
Within the decoder, the logic is constructed so that all numbers that utilize segment g (2,3,4,5,6,7,8,9) produce an active low output for g. This allows current to flow through the diodes in segment g from the Vcc due to the voltage potential created. The BCD inputs determine whether or not g is high or low, which in turn causes segment g to be either turned on or off.

c)



Input		Output			
6	3	D	C	B	A
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	0	1	0
0	1	0	0	1	1
0	X	0	1	0	0
0	X	0	1	0	1
1	X	0	1	1	0
X	X		⋮		

To prevent multiple inputs from being read at the same time, you can prioritize inputs, which in this problem is prioritizing from highest decimal to lowest. If a higher priority input is used, then the rest of the inputs with lower priority are disregarded, hence the 'x' in the truth table.



$A = 1+3+5+7+9$

$B = 2+3+6+7$

$C = 4+5+6+7$

$D = 8+9$

(8)

M	S_1S_0	Expression for F_i	Expression for C_{i+1}
0	00	$0 \oplus [(0 \oplus A_i) \oplus (0)]$	$[(0)(0 \oplus A_i)] \cdot [(0)((0 \oplus A_i) \oplus (0))]$
0	01	$0 \oplus [(1 \oplus A_i) \oplus (0)]$	$[(0)(1 \oplus A_i)] \cdot [(0)((1 \oplus A_i) \oplus (0))]$
0	10	$B_i \oplus [(0 \oplus A_i) \oplus (0)]$	$[(0)(0 \oplus A_i)] \cdot [(1)((0 \oplus A_i) \oplus (0))]$
1	10	$B_i \oplus [(0 \oplus A_i) \oplus (C_i)]$	$[(C_i)(0 \oplus A_i)] \cdot [(1)((0 \oplus A_i) \oplus (C_i))]$
1	11	$B_i \oplus [(1 \oplus A_i) \oplus (C_i)]$	$[(C_i)(1 \oplus A_i)] \cdot [(1)((1 \oplus A_i) \oplus (C_i))]$

general expression for F_i: $S_1B_i \oplus [(S_0 \oplus A_i) \oplus (MC_i)]$

" " " C_{i+1}: $[(MC_i)(S_0 \oplus A_i)] \cdot [(S_1B_i)((S_0 \oplus A_i) \oplus (MC_i))]$