Name

Student ID #

University of California

Los Angeles

Computer Science Department

CSM51A/EEM16 Midterm Exam

Winter Quarter 2016

February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	20
5 (20)	20
Total (100)	100

4:00 + lex? min = 5:40

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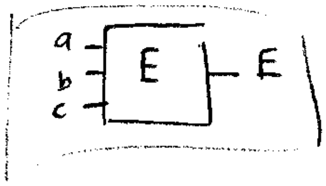
For E gate,



Problem 1 (20 points)

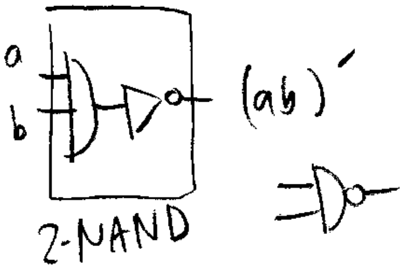
Use only the "E" gate defined below to implement Boolean function:
F=w'xy'+wxz+w'x'z+wx'y'z'
You may also use constants 0 and 1 as inputs.

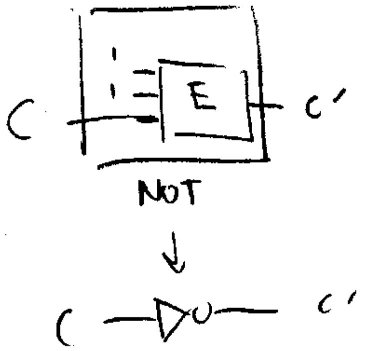
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

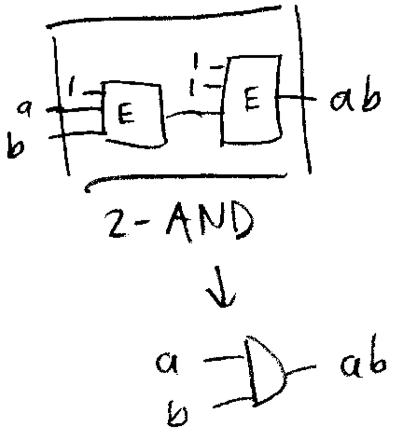
$E = (a+b+c)(a'+b'+c') = (a+b+c)(abc)'$

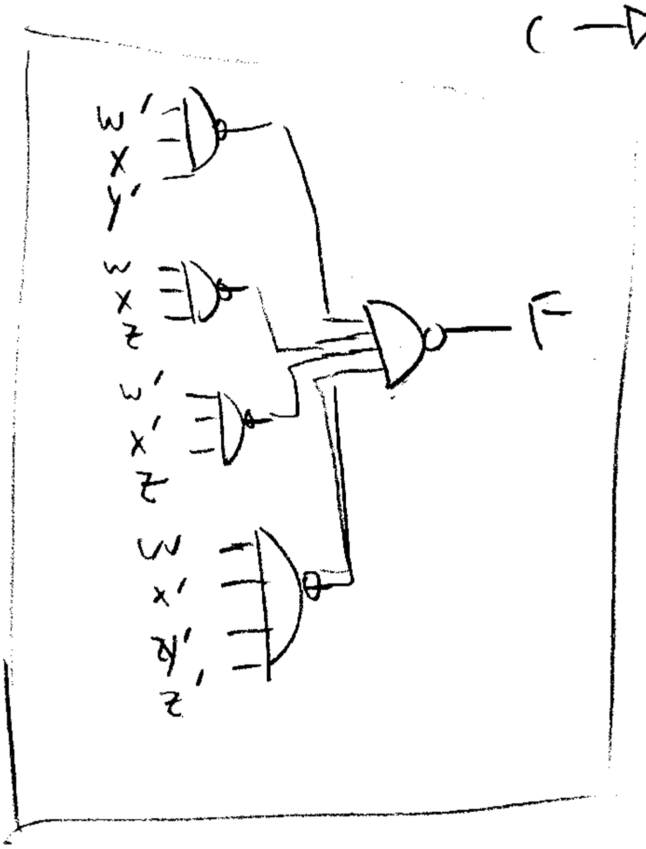
$E(1,1,c) = c'$

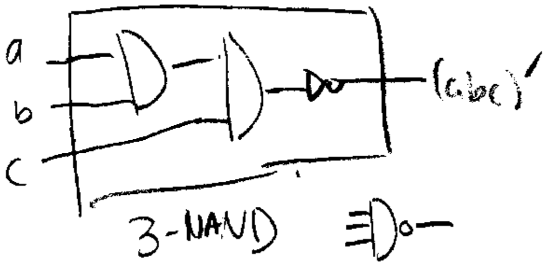
$E(1,a,b) = (ab)'$

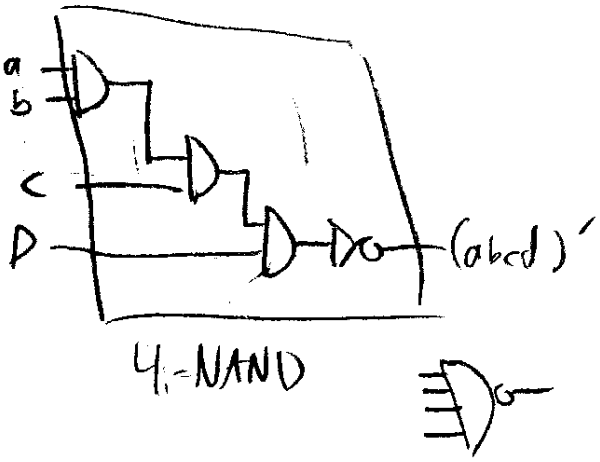












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Problem 2 (20 points)

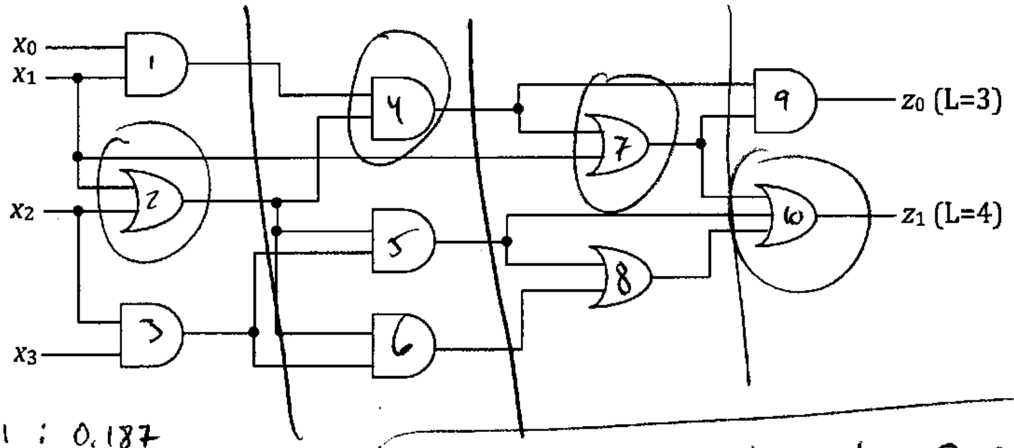
Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

$0.037 \times 3 = 0.111$

$0.038 \times 4 = 0.152$

$0.038 \times 4 = 0.152$



1 2 AND 1 : 0.187
2 2 OR 3 : 0.231
3 2 AND 2 : 0.224
4 2 AND 2 : 0.224
~~5 2 AND 2 : 0.224~~
6 2 AND 1 : 0.187
7 2 OR 2 : 0.194
8 2 OR 1 : 0.157
9 2 AND 3 : 0.261
10 3 OR 4 : 0.272

Critical path along gates 2 → 4 → 7 → 10

Delay tre =
$$\begin{array}{r} 0.231 \\ 0.224 \\ 0.194 \\ + \; 0.272 \\ \hline 0.921 \end{array}$$
→ $\boxed{D = 0.921}$ (ns)

2 AND 1	0.15 + 0.037	= 0.187
2 AND 2	0.15 + 0.074	= 0.224
2 AND 3	0.15 + 0.111	= 0.261
2 OR 1	0.12 + 0.037	= 0.157
2 OR 2	0.12 + 0.074	= 0.194
2 OR 3	0.12 + 0.111	= 0.231
3 OR 4	0.12 + 0.152	= 0.272

$A = a_3a_2a_1a_0$, $B = b_3b_2b_1b_0$, $C = c_3c_2c_1c_0$, $D = d_3d_2d_1d_0$

$E = e_4e_3e_2e_1e_0$ $F: f_4f_3f_2f_1f_0$ 20

Problem 3 (20 points)

Four 4-bit numbers A,B,C, and D are given as inputs. E=A+B, F=C+D. Design a system that outputs the larger number between E and F. If E=F, output either E or F. You can use any type of gates to implement your design.

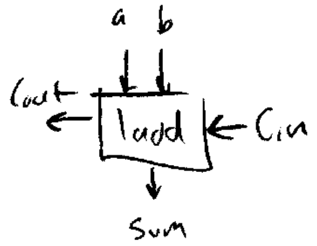
largest possible = 30

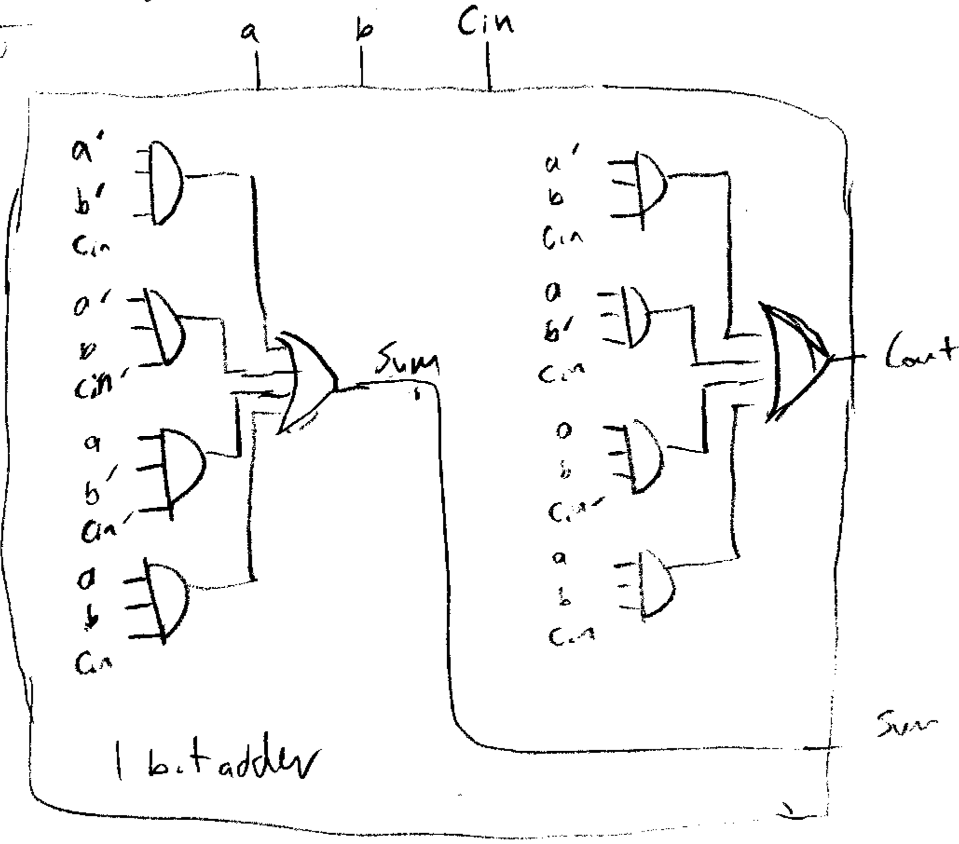
$$\begin{array}{r} 1111 \\ +\ 1111 \\ \hline 11110 \end{array}$$

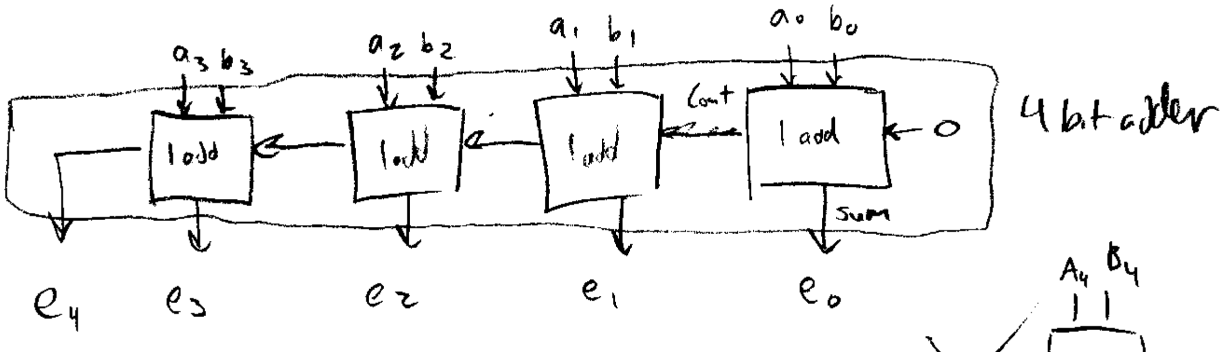
$2+4+8+16 = 30$

1 bit adder

a	b	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





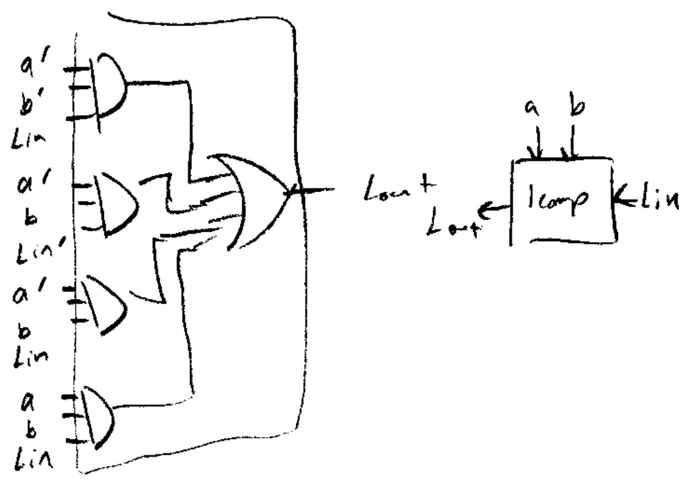


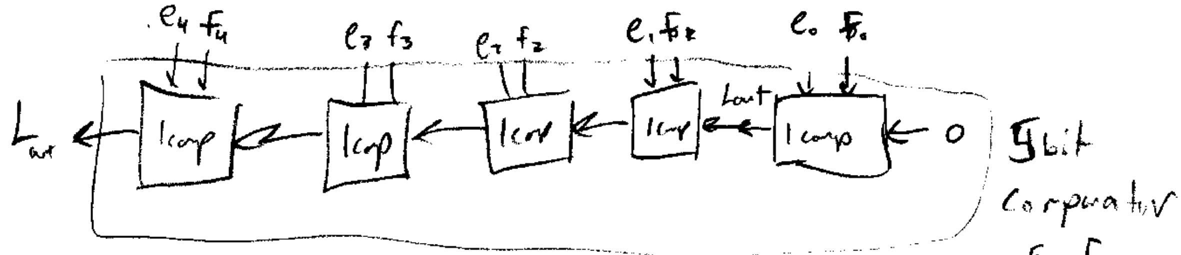
Problem 3) Extra Page

1 bit Comparator Define $L=0$ $a \geq b$
$L=1$ $a<b$

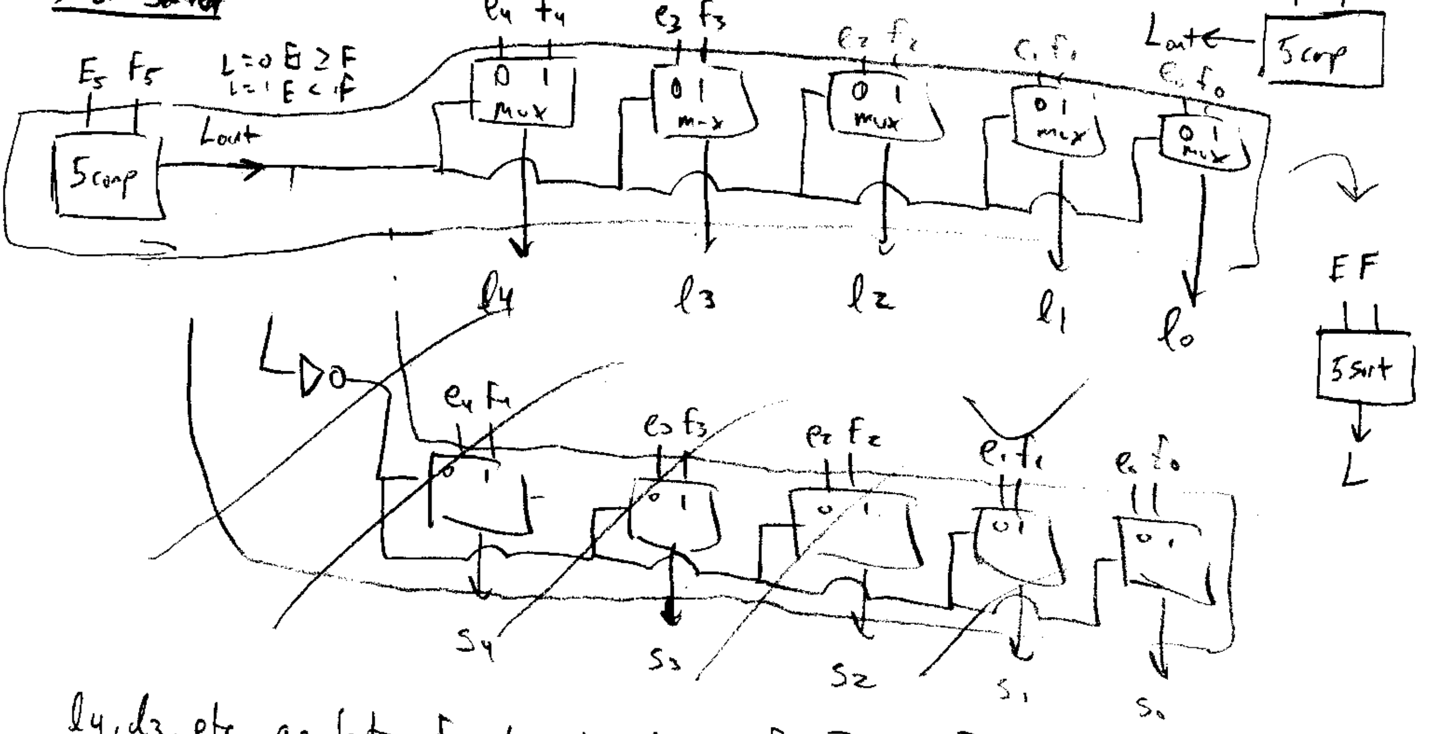
a	b	Lin	Lout
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

1 bit Comparator





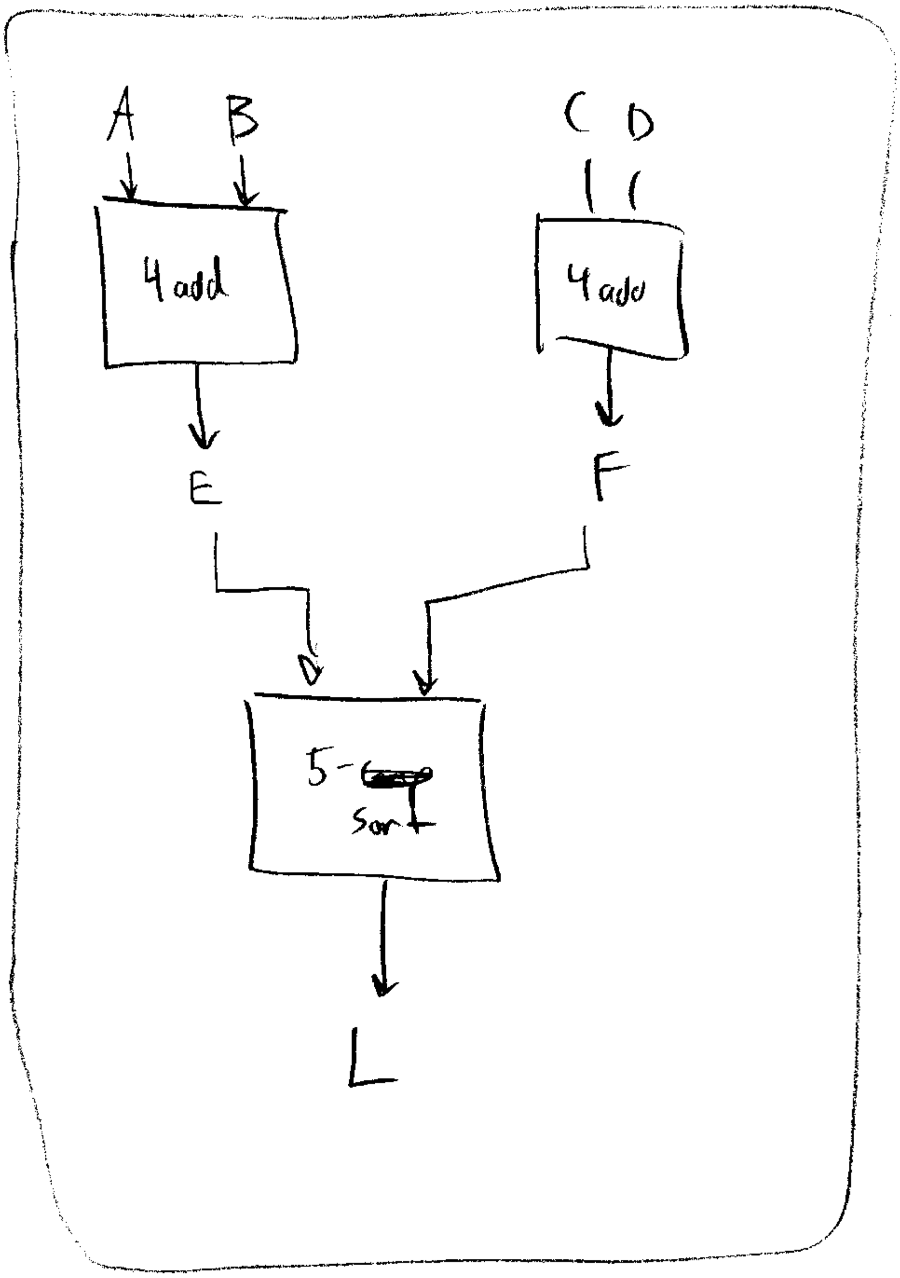
5 bit Sorter



l_4, l_3, etc are bits of L, the larger of E and F

~~s_4, s_3, etc are bits of S, the smaller of E and F~~

(see back) →



Answer to #3

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Problem 4 (20 points)

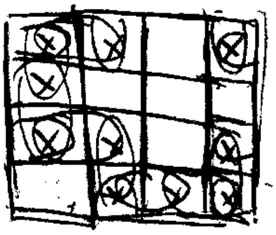
For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

X_0 (columns 2–3), X_1 (columns 3–4), X_2 (rows 2–3), X_3 (rows 3–4)

0	0	0	0
1	1	0	0
1	1	1	0
0	0	1	0

the most Prime implicants / fewest essential Prime implicants

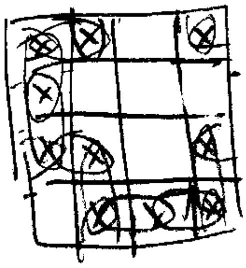


← 12 Prime

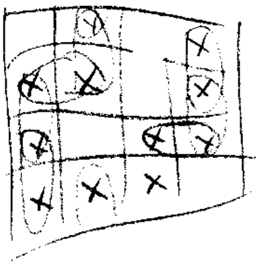
0 essential

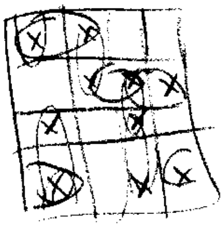
x_0, x_1, x_2, x_3

1	1	0	1
1	0	0	0
1	1	0	1
0	1	1	1



$P = 12 - 0 = 12$





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Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in (0,1,2,\dots 8)$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.
Note that the outputs y and z represent the two digits of a base-3 number.
For example, if x=7 ($x_3x_2x_1x_0$=0111), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ (y_1y_0=10) and $z = 1$ (z_1z_0=01).

X_3	X_2	X_1	X_0	Y_1	Y_0	Z_1	Z_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	1	0
1	0	0	1	DC			
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

Y_1 (X_3 top, X_0 right, X_1 left, X_2 bottom)

0	1	-	0
0	-	-	0
0	-	-	1
0	-	-	1

Y_0 (X_3 top, X_0 right, X_1 left, X_2 bottom)

0	0	-	1
0	-	-	1
1	-	-	0
0	-	-	0

Z_1 (X_3 top, X_0 right, X_1 left, X_2 bottom)

0	1	-	0
0	-	-	1
0	-	-	0
1	-	-	0

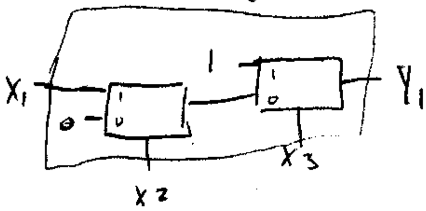
Z_0 (X_3 top, X_0 right, X_1 left, X_2 bottom)

0	0	-	1
1	-	-	0
0	-	-	1
0	-	-	0

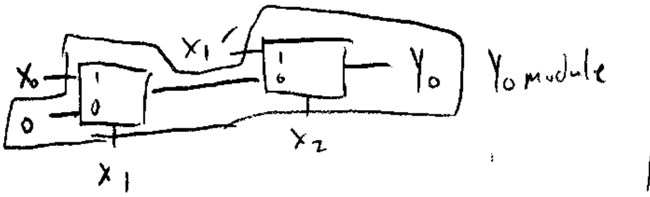
$$Y_1 = X_3 + X_2X_1$$
$$Y_0 = X_2X_1' + X_2'X_1X_0$$
$$Z_1 = X_3 + X_2X_1'X_0 + X_2'X_1X_0'$$
$$Z_0 = X_2X_1X_0 + X_2'X_1'X_0 + X_2X_1'X_0'$$

$Y_{1X_3} = 1$
$Y_{1X_3'} = X_2X_1$
$Y_{1X_3'X_2} = X_1$
$Y_{1X_3'X_2'} = 0$

Y_1 Module

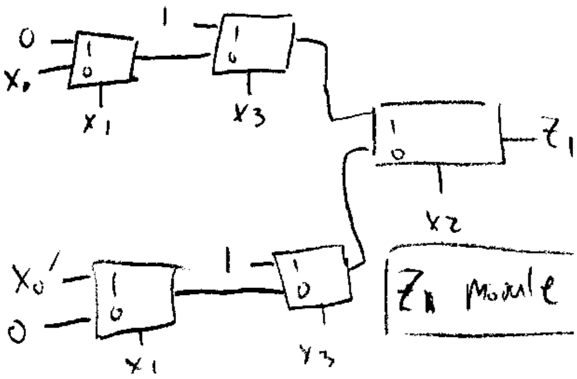


$Y_{0X_2} = X_1'$
$Y_{0X_2'} = X_1X_0$
$Y_{0X_2'X_1} = X_0$
$Y_{0X_2'X_1'} = 0$



Y_0 module

$Z_{1X_2} = X_3 + X_1'X_0$
$Z_{1X_2'} = X_3 + X_1X_0'$
$Z_{1X_2X_3} = 1$
$Z_{1X_2X_3'} = X_1'X_0$
$Z_{1X_2X_3'X_1} = 0$
$Z_{1X_2X_3'X_1'} = X_0$
$Z_{1X_2'X_3} = 1$
$Z_{1X_2'X_3'} = X_1X_0'$
$Z_{1X_2'X_3'X_1} = X_0'$
$Z_{1X_2'X_3'X_1'} = 0$



Z_1 Module

Problem 5) Extra Page

$Z_0 = X_2 X_1 X_0 + X_2' X_1' X_0 + X_2 X_1' X_0'$

$Z_{0X_2} = X_1 X_0 + X_1' X_0'$

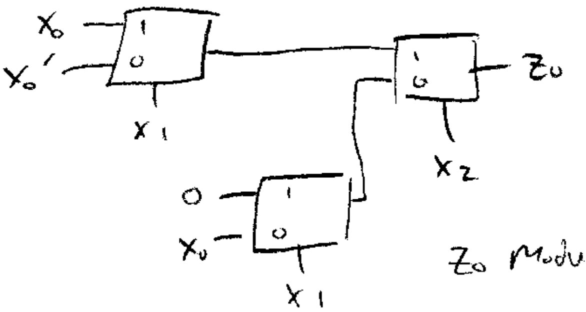
$Z_{0X_2'} = X_1' X_0$

$Z_{0X_2'X_1} = 0$

$Z_{0X_2'X_1'} = X_0$

$Z_{0X_2X_1} = X_0$

$Z_{0X_2X_1'} = X_0'$



Z_0 Module