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Student ID \_\_\_\_\_

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University of California  
Los Angeles  
Computer Science Department

CSM51A/EEM16 Midterm Exam #1

Winter Quarter 2019

February 6th 2019

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (Possible Points)	Points
1(20)	20
2(20)	20
3(20)	15
4(20)	14
5(20)	20
Total (100)	89

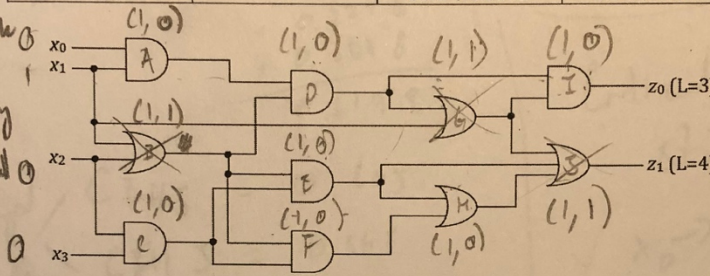
20

**Problem 1 (20 points):**

Given the network below, find the critical path and calculate critical path delay. Assume the values of  $(x_3, x_2, x_1, x_0)$  are initially  $(1,1,1,1)$  and they change to  $(0,0,1,0)$  in the next clock cycle. Now, choose a gate on the critical path which maximally decreases overall delay when the gate decreases its delay by 20%. Finally, find the critical path in the new network and its length.

Gate	Fan-in	$t_{pLH}$	$t_{pHL}$
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

Part 2: Gate J has the largest time delay both on the whole network and on the critical path, so decreasing its delay by 20% maximally decreases overall delay.  
(Continue on Back)



Part 1  
Low = 1  
High = 0

Critical path is  $x_0 \rightarrow A \rightarrow D \rightarrow G \rightarrow J \rightarrow z_0$   
or  
 $x_0 \rightarrow A \rightarrow D \rightarrow I \rightarrow z_0$   
both have time delay 0.672

- $B = 0, G = 0, J = 0$
- A - Fanin 2, L=1, AND  
 $0.15 + 0.037 = 0.187$
- C - Fanin 2, L=2, AND  
 $0.15 + 2(0.037) = 0.224$
- D - Fanin 2, L=2, AND  
 $= 0.224$
- E - AND, Fanin 2, L=2  
 $= 0.224$
- F - AND, Fanin 2, L=1  
 $= 0.187$
- H - OR, Fanin 2, L=1  
 $0.12 + (0.037) = 0.157$
- I - AND, Fanin 2, L=3  
 $0.15 + 3(0.037) = 0.261$

$x_0 \rightarrow A \rightarrow P$

$$\begin{array}{r} 0.187 \\ + 0.037 \\ \hline 0.224 \end{array}$$

$$\begin{array}{r} 0.224 \\ + 0.037 \\ \hline 0.261 \end{array}$$

$$\begin{array}{r} 0.261 \\ + 0.157 \\ \hline 0.418 \end{array}$$

$$\begin{array}{r} 0.418 \\ + 0.157 \\ \hline 0.575 \end{array}$$

$$\begin{array}{r} 0.575 \\ + 0.097 \\ \hline 0.672 \end{array}$$

Possible Paths

~~A D G I~~ 4 > 3  
~~A D G J~~  
~~A D H J~~  
~~C E H J~~ 4 > 3  
~~C E H I~~ J=0

Some are ADGI  
ADGI =  $0.187 + 0.224 + 0.261 = 0.672$

CEHS =  $0.224 + 0.224 + 0.157 + 0 = 0.605$

CFHS =  $0.224 + 0.187 + 0.157 + 0 = 0.568$

$$\begin{array}{r} 0.261 \\ \times 0.8 \\ \hline 2088 \end{array}$$

Problem 1 (Extra Page):

Continued from Front

-20% decrease of I  $\Rightarrow 0.8I = 0.8(0.261)$   
 $= 0.2088$

New Critical Path calculations

$$\begin{array}{r} ADHS \& ADI = 0.1870 \\ + 0.2240 \\ + 0.2088 \\ \hline 0.6198 \end{array}$$

unchanged / from part 1  
 CEHS = 0.605  
 CFHS = 0.568

Critical path is still ~~ADHS~~

✓  $X_0 \rightarrow A \rightarrow D \rightarrow G \rightarrow J \rightarrow Z_0$   
 or

$X_0 \rightarrow A \rightarrow D \rightarrow J \rightarrow Z_0$

but time delay is now

0.6198

$$\text{AND} = (x' + y)'$$

**Problem 2 (20 points):**

You are given the following Boolean function.

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6 x_5 x_4 x_3 + x_6 x_5 x_2 + x_6 x_5 x_3' x_2' + x_6' x_1 x_0 + x_6' x_5 x_0'$$

Given the universal operation E as specified in the table, implement F using only the gates specified by E.

X	Y	E(X, Y)
0	0	1
0	1	1
1	0	0
1	1	1

$$E(x, y) = x' + y$$



NOT

$$E(x, 0) = (x') + (0) = x'$$

OR

$$E(E(x, 0), y) = ((x') + (0)) + y = x + y$$

AND

$$E(E(x, E(y, 0)), 0) = ((x') + ((y') + (0))) + (0) = (x' + y')$$

Demorgan's

$$xy = E(E(x, y), 0) = (x' + y)' + 0 = xy'$$

eqn on both

$$\bar{E}(E(xy, E(z, 0)), 0)$$

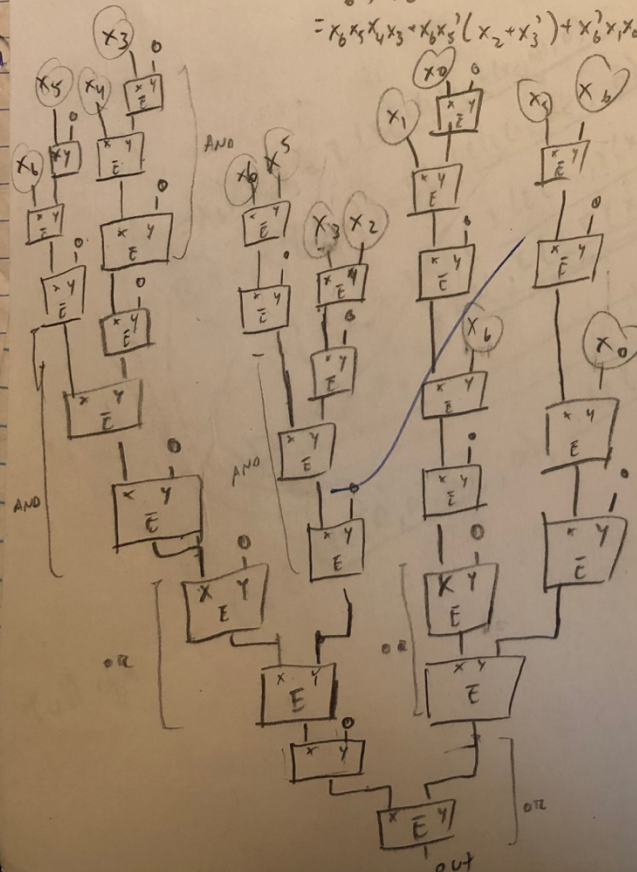
$$= ((xy)' + z)'$$

$$= (xy)z$$

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6 x_5 x_4 x_3 + x_6 x_5 x_2 + x_6 x_5 x_3' x_2' + x_6' x_1 x_0 + x_6' x_5 x_0'$$

$$= x_6 x_5 x_4 x_3 + x_6 x_5 (x_2 + x_3 x_2') + x_6' x_1 x_0 + x_6' x_5 x_0'$$

$$= x_6 x_5 x_4 x_3 + x_6 x_5 (x_2 + x_3) + x_6' x_1 x_0 + x_6' x_5 x_0'$$



Problem 2 (Extra Page):

Use the Tower of Hanoi to find the number of moves required to solve the puzzle. Assume the number of moves is a function of the number of disks. Let  $n$  be the number of disks. Let  $T(n)$  be the number of moves required to solve the puzzle with  $n$  disks.

For example, in the following 4-disk Tower of Hanoi puzzle:



$$\begin{aligned}
 A \rightarrow x_6 + x_5 + x_4 + x_3 &= E(E(E(E(x_6, E(x_5, 0)), 0), E(E(E(x_4, E(x_3, 0)), 0)), 0)), 0) \\
 B \rightarrow x_6 + x_5 + (x_2 + x_3) &= E(E(E(E(x_6, x_5), 0), E(E(E(x_2, E(x_3, 0)), 0)), 0)), 0) \\
 C \rightarrow x_6 + x_1 + x_0 &= E(E(E(E(x_6, E(x_1, E(x_0, 0))), 0), x_6), 0), 0) \\
 D \rightarrow x_6 + x_5 + x_0 &= E(E(E(E(x_6, x_5), 0), E(E(x_0, 0), 0)), 0), 0)
 \end{aligned}$$

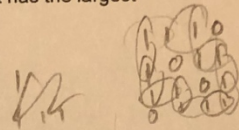
Full eqn  $E(E(E(E(A, 0), B), 0), E(E(C, 0), D))$

**Problem 3 (20 points):**

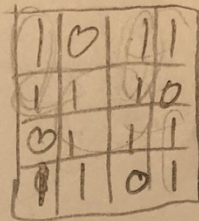
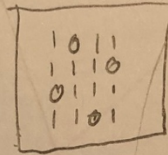
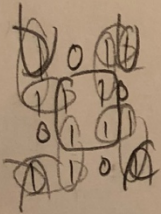
For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4x4 K-map that has the largest value of  $P=M-N$  among all the 4x4 K-maps.

For example, in the following 4x4 K-map,  $M=3$ ,  $N=2$ ,  $P=M-N=1$ .

	$x_0$				
	0	0	0	0	
	1	1	0	0	$x_2$
$x_3$	1	1	1	0	
	0	0	1	0	
	$x_1$				



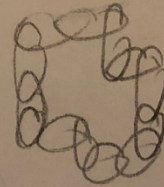
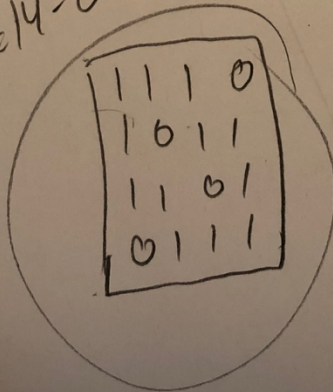
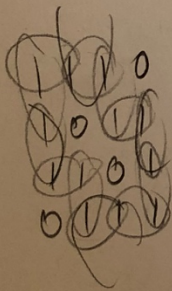
13 PI



13 PI, Essential

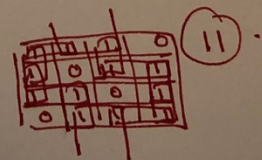
$$P = 13 - 0 = 13$$

$$P = 14 - 0 = 14$$



actually

actually



**Problem 4 (20 points):**

Given an input stream X, we want to recognize interchangably patterns A and B. We recognize A first, then B, followed by A again, then B again and so on.

For example,

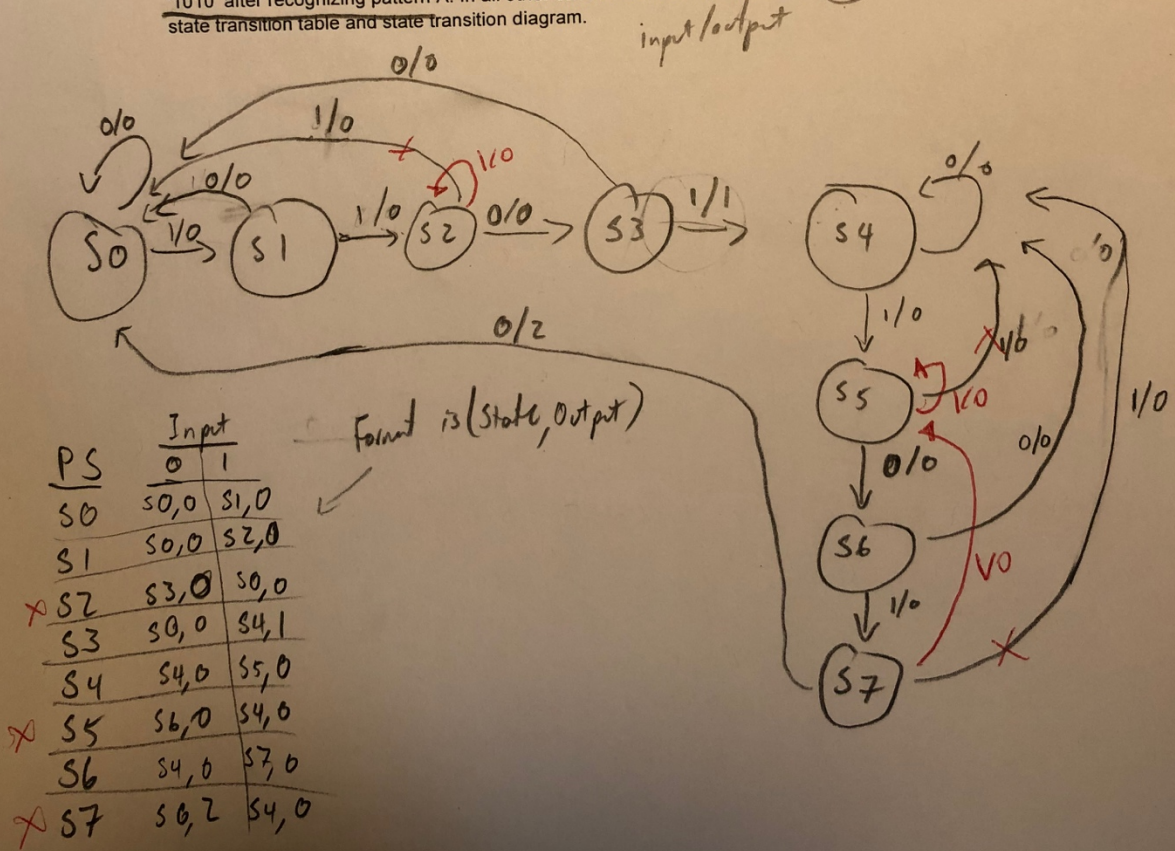
1. Assume  $X = 01011010010101$ ,  $A = 101$  and  $B = 001$ .

We will first recognize A, then look for B. Please note that we ignore the second '101' (A) in X and we only search for B once we have found A. After finding B, we again search for A.

2. Assume that we have  $X = 1011$ ,  $A = 101$  and  $B = 011$

We recognize A, but we do not recognize B as we only start looking for B once we have detected A. In other words, A and B do not overlap.

Now, you are given any input stream X. Design a finite state machine such that the system outputs 1 when it recognizes pattern A = '1101' and outputs 2 when it recognizes pattern B = '1010' after recognizing pattern A. In all other cases the machine should output 0. Show the state transition table and state transition diagram.

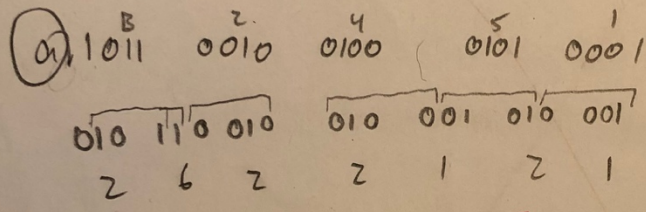


**Problem 5 (20 points):**

Perform the following conversions:

- a)  $(B2451)_{16} \rightarrow (x)_8$
- b)  $(354)_7 \rightarrow (y)_5$

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F



$(B2451)_{16} \rightarrow (2622121)_8$

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 147 \end{array}$$

b)  $(354)_7 = 3(7^2) + 5(7^1) + 4(7^0)$

$\downarrow$   
 $(186)_{10}$

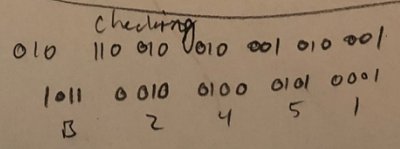
$$\begin{array}{r} 147 \\ + 39 \\ \hline 186 \end{array}$$

$$\begin{array}{r} 37 R1 \\ 5 \overline{)186} \\ \underline{15} \\ 36 \\ \underline{35} \\ 1 \\ 5 \overline{)137} \\ \underline{10} \\ 37 \\ \underline{35} \\ 2 \\ 5 \overline{)17} \\ \underline{15} \\ 2 \\ 5 \overline{)2} \\ \underline{0} \\ 2 \\ 5 \overline{)1} \\ \underline{0} \\ 1 \end{array}$$

$$\begin{array}{r} 5 \overline{)186} \\ \underline{5} \phantom{0} \\ 37 \\ \underline{5} \phantom{0} \\ 7 \\ \underline{5} \phantom{0} \\ 2 \\ \underline{0} \\ 1 \end{array}$$

$(1221)_5$

$$\begin{array}{r} 37 \\ 5 \overline{)186} \\ \underline{15} \\ 36 \\ \underline{35} \\ 1 \end{array}$$



$(1221)_5 = 1 \times 5^3 + 2 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$

$+ 125 + 50 + 10 + 1$

$$\begin{array}{r} 2624 \\ 7 \overline{)186} \\ \underline{14} \\ 46 \\ \underline{42} \\ 4 \end{array}$$

$$\begin{array}{r} 186 \\ 7 \overline{)186} \\ \underline{14} \\ 46 \\ \underline{42} \\ 4 \end{array}$$

$(354)_7$