[CS M51A W13] Solution to Quiz 1B

Date: 01/25/13

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Quiz Problems (50 points total)

Problem 1 (10 points)

The XNOR operation is defined as $x \odot y = xy + x'y'$ and the XOR operation is defined as $x \oplus y = x'y + xy'$. Show that $(x \odot y)' = x \oplus y$ using the postulates and theorems of Boolean algebra. **Do not use a table**.

Solution

$$\begin{aligned}
(x \odot y)' &= (x'y + xy')' \\
&= (xy)'(x'y')' \\
&= (x' + y')(x + y) \\
&= (xx' + x'y + xy' + yy') \\
&= x'y + xy' \\
&= x \oplus y
\end{aligned}$$

Problem 2 (25 points)

We are given the following switching function.

$$E_1(a,b,c,d) = bcd + a'b'd + bc + ac'd' + a'b'c' + ac'd + a'bcd'$$

1. **(10 points)** Simplify expression E_1 using the postulates of Boolean algebra.

   Solution

$$\begin{aligned}
E_1(a,b,c,d) &= bcd + a'b'd + bc + ac'd' + a'b'c' + ac'd + a'bcd' \\
&= bc(d + 1 + a'd') + a'b'd + ac'(d' + d) + a'b'c' \\
&= bc + a'b'd + ac' + a'b'c' \\
&= bc + a'b'd + c'(a + a'b') \\
&= bc + a'b'd + c'(a + b') \\
&= bc + a'b'd + ac' + b'c'
\end{aligned}$$

2. **(10 points)** E_2 is given as the function below.

$$E_2(a,b,c,d) = E_1 + a'cd + ab$$

   Given these two expressions, fill in the table below.

   Solution

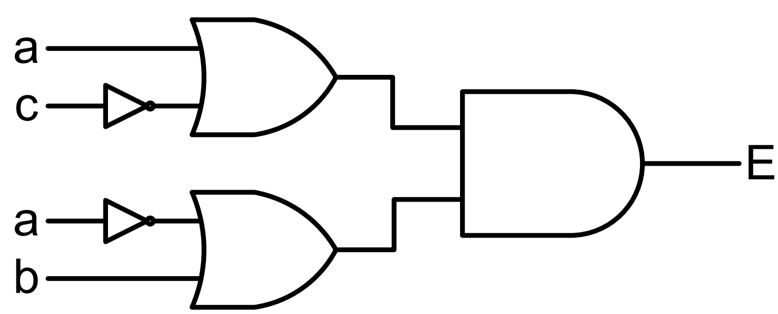
a	b	c	d	E_1	E_2
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

3. **(5 points)** Write the switching function in the form specified below.

   Solution From the table, we can write

$$\begin{aligned} E_1(x,y,z,w) &= \sum m(0,1,3,6,7,8,9,12,13,14,15) \\ E_2(x,y,z,w) &= \prod M(2,4,5,10,11) \end{aligned}$$

Problem 3 (15 points)



Given the above circuit:

1. **(5 points)** Obtain the most simplified sum of products form for $E(a,b,c)$.

   Solution We first obtain the expression from the gate network, which is $E(a,b,c) = (a+c')(a'+b)$. From here we can derive:

$$\begin{aligned} E(a,b,c) &= (a+c')(a'+b) \\ &= aa' + ab + a'c' + bc' \\ &= ab + a'c' + bc' \end{aligned}$$

2. **(10 points)** Obtain the product of maxterms form for $E(a,b,c)$.

   Solution We first obtain the expression from the gate network, which is $E(a,b,c) = (a+c')(a'+b)$.

From here we can derive:

$$\begin{aligned}
E(a,b,c) &= (a+c')(a'+b) \\
&= (a+c'+bb')(a'+b+cc') \\
&= (a+b+c')(a+b'+c')(a'+b+c)(a'+b+c') \\
&= M_1 \cdot M_3 \cdot M_4 \cdot M_5 \\
&= \prod M(1,3,4,5)
\end{aligned}$$