

[CS M51A W18] MIDTERM EXAM

Date: 02/12/18

- The midterm is closed book, and up to 2 sheets (= 4 pages) of summary notes are allowed.
 - For multiple choice questions, wrong answers may have a negative score value so choose carefully. Some questions can have multiple answers.
 - Please show all your work and write legibly, otherwise no partial credit will be given.
 - This should strictly be your own work; any form of collaboration will be penalized.
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Name :

Student ID :

Problem	Points	Score
1	20	19
2	20	19
3	20	18
4	8	8
5	12	9
6	20	20
Total	100	93

Problem 1 (20 points)

Digit Value	BCD 8421	2421	Excess-3	2-out-of-5
0	0000	0000	0011	00011
1	0001	0001	0100	11000
2	0010	0010	0101	10100
3	0011	0011	0110	01100
4	0100	0100	0111	10010
5	0101	1011	1000	01010
6	0110	1100	1001	00110
7	0111	1101	1010	10001
8	1000	1110	1011	01001
9	1001	1111	1100	00101

1. (4 points) What are the decimal numbers, x and y , represented in the specified code?

4

$$x = (\overset{6}{1}\overset{3}{1}\overset{5}{0}\overset{0}{0}\overset{1}{1}\overset{1}{1}\overset{0}{1}\overset{1}{1})_{2421} = 635$$

$$y = (\overset{12}{1}\overset{7}{1}\overset{3}{0}\overset{3}{0}\overset{11}{1}\overset{3}{1}\overset{1}{0}\overset{1}{1})_{\text{EXCESS-3}} = 908$$

2. (5 points) When the length of a bit vector is 20, how many different integers can be represented by using

- 4
- (a) BCD code $5^{10} = 100,000$
- (b) 2421 code $100,000$
- (c) Excess-3 code $100,000$
- (d) 2-out-of-5 code $100,000$ $10,000$
- (e) Binary $2^{20} = 1,048,576$
- $20/4 = 5$

3. (3 points) Is the 2421 code is unique? If not, list all digit values that have more than one 2421 code and show codes different from 2421 code shown in the above table for these digit values.

3

No

2: 1000 5: 0101

3: 1001 6: 0110

4: 1010 7: 0111

4. (5 points) In general, how many different 2421 codes are there? Explain briefly.

5

There are only 8 complementing 2421 codes; There are 6 digits with 2 forms, but only by choosing the correct pairs of 2:3, 3:6, and 4:5 will the 9's complement rule hold. So there are 2 choices for 3 complementary pairs and $2^3 = 8$.

5. (3 points) (True/False) Are all 2421 codes are self-complementing? If not, how many self-complementing codes are there?

3

There are 64 2421 codes because there are 6 digits with 2 forms, and $2^6 = 64$

Problem 2 (20 points)

Design a combinational system that has two inputs x and y in $\{0, 1, 2, 3\}$ represented in the binary code. The output is the product $p = (x \times (3y + 1)) \bmod 8$ also in the binary code.

1. (4 points) Show a high-level description of the system.

		y				
		0	1	2	3	
x	0	0	0	0	0	$+3$
	1	1	4	7	2	
	2	2	0	6	4	
	3	3	4	5	6	

Output set

2. (4 points) Show a binary-level description of the system as truth tables of the output switching functions

z_2, z_1, z_0			x		y				
z_2	z_1	z_0	x_1	x_0	y_1	y_0			
0	0	0	0	0	0	0			
0	0	0	0	1	0	0			
0	0	0	0	1	1	0			
0	0	0	1	0	0	0			
0	0	0	1	0	1	0			
0	0	0	1	1	0	0			
0	0	0	1	1	1	0			
0	0	0	0	0	0	1			
0	0	0	0	0	1	1			
0	0	0	0	1	0	1			
0	0	0	0	1	1	1			
0	0	0	1	0	0	1			
0	0	0	1	0	1	1			
0	0	0	1	1	0	1			
0	0	0	1	1	1	1			

$+4$

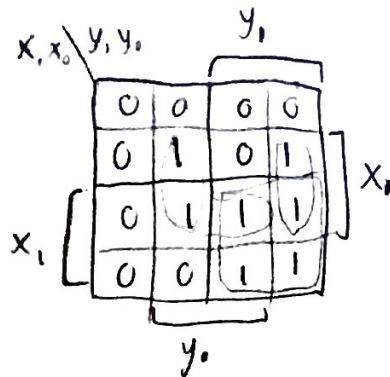
3. (4 points) Derive a sum of minterms in m-notation for the output z_2 .

$$z_2 = \sum m(5, 6, 10, 11, 13, 14, 15) \quad +4$$

(11)

4. (4 points) Derive a minimal sum of products (SOP) for the output z_2 .

5, 6, 10, 11, 13, 14, 15

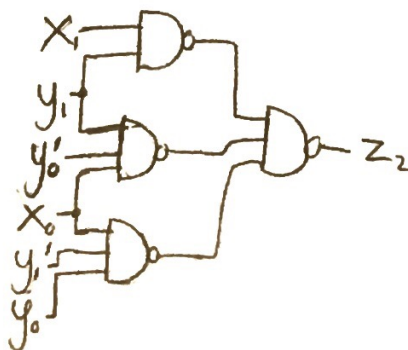


$$z_2 = x_1 y_1 + x_0 y_1 y_0' + x_0 y_1' y_0$$

+4

5. (4 points) Draw a NAND-NAND network that implements the minimal SOP for z_2 . Assume that the complements are available.

$$z_2 = ((x_1 y_1)' (x_0 y_1 y_0')' (x_0 y_1' y_0)')'$$



+4

Problem 3 (20 points)

$a + b = b + a$	$ab = ba$	Commutativity
$a + (bc) = (a + b)(a + c)$	$a(b + c) = (ab) + (ac)$	Distributivity
$a + (b + c) = (a + b) + c = a + b + c$	$a(bc) = (ab)c = abc$	Associativity
$a + a = a$	$aa = a$	Idempotency
$a + a' = 1$	$aa' = 0$	Complement
$1 + a = 1$	$0a = 0$	
$0 + a = a$	$1a = a$	Identity
$(a')' = a$		Involution
$a + ab = a$	$a(a + b) = a$	Absorption
$a + a'b = a + b$	$a(a' + b) = ab$	Simplification
$(a + b)' = a'b'$	$(ab)' = a' + b'$	DeMorgan's law

For the function $E(w, x, y, z)$, we are given the following boolean expression

$$E(w, x, y, z) = w'x'y'z + w'xy'z' + w'xy'z + w'xyz' + wx'y'z$$

1. (4 points) Simplify the given boolean expression. Indicate at each simplification step the identity used.

$$w'x'y'z + w'xy'z' + w'xy'z + w'xyz' + wx'y'z$$

$$w'xy'(z' + z) + w'x'y'z + w'xyz' + wx'y'z \quad \text{distribution } \} \text{ commutativity}$$

$$w'xy'(z' + z) + x'y'z(w' + w) + w'xyz' \quad \text{distribution } \} \text{ commutativity}$$

$$w'xy' \cdot 1 + x'y'z \cdot 1 + w'xyz' \quad \text{complement}$$

$$w'xy' + x'y'z + w'xyz' \quad \text{identity}$$

$$? \quad y'(w'x + x'z) + w'xyz' \quad \text{distribution}$$

$$y'((w'x)'(x'z)') + w'xyz' \quad \text{DeMorgan}$$

$$y'((w + x')(x + z')) + w'xyz' \quad \text{DeMorgan}$$

$$y'(wx + wz' + x'x + x'z') + w'xyz' \quad \text{distribution}$$

$$y'(wx + wz' + x'z') + w'xyz' \quad \text{complement, identity}$$

$$\begin{aligned} & w'x(y' + yz') + x'y'z \\ &= w'x(y' + z') + x'y'z \\ &= w'xy' + w'xz' + x'y'z \end{aligned}$$

3.

can be simplified
further!

2. (4 points) Complete the K-map for the given expression.

$$w'xy + x'y'z + w'xyz'$$

		yz			
		00	01	$\overbrace{11 \quad 10}^y$	
wx	00	0	1	0	0
	01	1	1	0	1
	11	1	0	0	0
	10	0	1	0	0

Handwritten annotations: A bracket labeled 'x' spans the last two columns (yz=11, 10). A bracket labeled 'z' spans the last two rows (wx=11, 10). A red checkmark is present to the right of the K-map.

min SOP: $w'xy + x'y'z + w'xz'$ min POS: $(x+z)(z'+y')(w'+x')$

3. (4 points) Which of the following are the essential prime implicants for the above expression:

(a) $w'y'z$

(b) $x'y'z$

(c) $w'xyz'$

(d) $w'xz'$

4. (2 points) Which of the following are the minimal SOP expression for the above function:

(a) $w'y'z + x'y'z + w'xy' + w'xz'$

(b) $w'y'z + w'xy' + w'xyz' + w'xz'$

(c) $w'y'z + x'y'z + w'xz'$

(d) $x'y'z + w'xy' + w'xz'$

5. (4 points) Which of the following are the essential prime implicants for the above expression:

(a) $w' + x'$

(b) $w' + y'$

(c) $y' + z'$

(d) $x + z$

6. (2 points) Which of the following are the minimal POS expression for the above function:

(a) $(w' + x').(w' + y').(y' + z').(x + y')$

(b) $(w' + x').(w' + y').(y' + z').(x + z)$

(c) $(w' + x').(y' + z').(x + z)$

(d) $(w' + x').(y' + z').(x + y')$

Problem 4 (8 points)

Implement a single complex CMOS gate for the XOR operation.

1. (2 points) Write the expression for the pull-down network.

$$z = ab' + a'b$$

$$z' = (ab' + a'b)'$$

$$z' = (ab')'(a'b)'$$

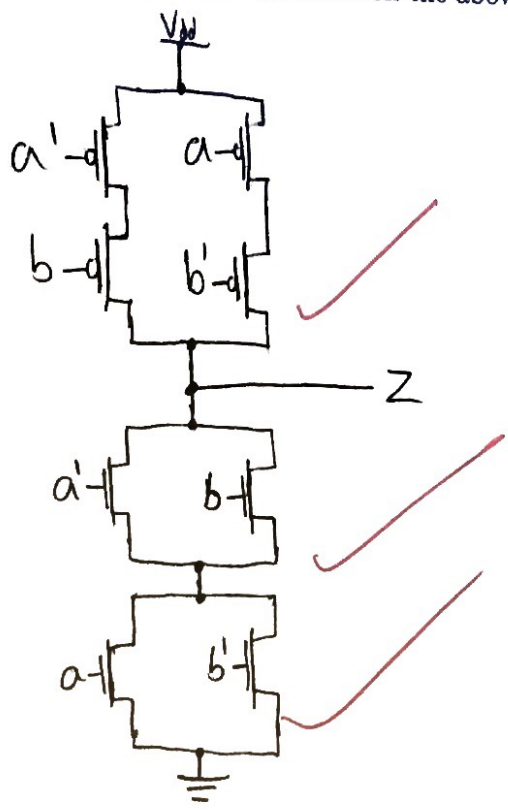
$$z' = (a'+b)(a+b')$$

2. (2 points) Write the expression for the pull-up network.

$$z = ab' + a'b$$

$$z = (a')'b' + a'(b)'$$

3. (4 points) Draw the CMOS network based on the above expressions.



Problem 5 (12 points)

9

$$E(a, b, c) = ac' + a'bc + ab'c$$

1. (6 points) Prove that $E(a, b, c)$ is a universal function. You can use constant 1 or 0 as an input.

✓ $E(1, 1, c) = 1c' + 0 \cdot 1c + 10c = c'$ NOT

→ $E(x, x, c) = xc' + x'xc + xx'c = xc'$

*not a correct way of proof
can't use different input*

→ $E(x, x, E(1, 1, c)) = x \cdot (E(1, 1, c))' = x \cdot (c')' = x \cdot c$ AND

{AND, NOT} is universal

(+3)

2. (6 points) Implement the following boolean function using only "E" gates. You can use constant 0 or 1 as inputs.

$$F(w, x, y, z) = w'x'yz + y'x$$

✓

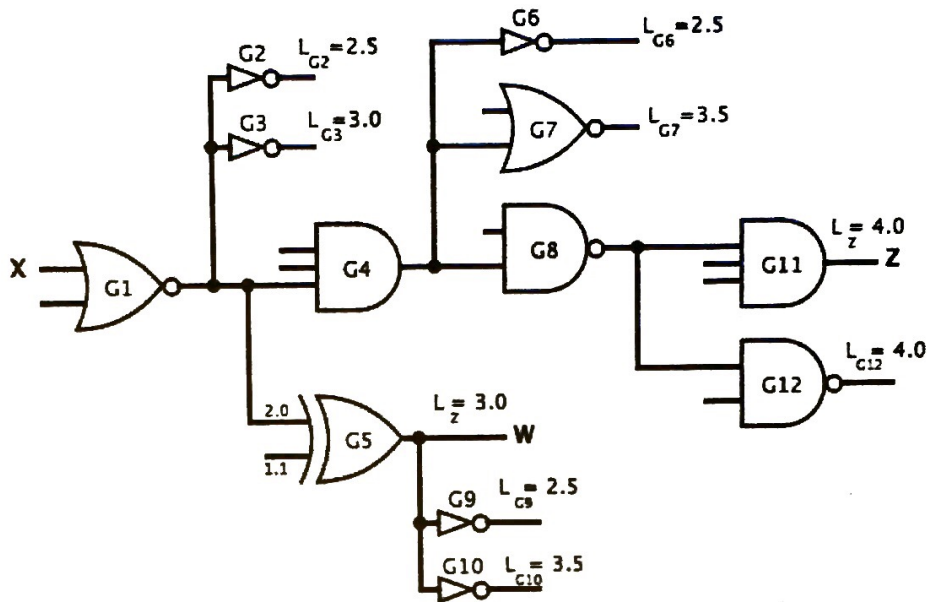
$$= ((w'x'yz)'(y'x)')$$

$$= E(1, 1, E(E(1, 1, E(x, x, y)), E(1, 1, E(x, x, y)), E(E(y, y, w), E(y, y, w), E(1, 1, E(z, z, x))))))$$

Problem 6 (20 points)

Determine the propagation delay of the gate network shown. The outputs are z and w , and the input signal that we are interested in is x . The related gate characteristics are given in the table below.

Gate Type	Fan-in	Propagation Delays (ns)		Load Factor I
		t_{pLH}	t_{pHL}	
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$	1.0
XOR	2	$0.30 + 0.036L$	$0.30 + 0.021L$	1.1
		$0.16 + 0.036L$	$0.15 + 0.020L$	2.0



1. (10 points) Find the worst case value of $t_{pLH}(x \rightarrow z)$. Fill in the blanks below with the appropriate values.

Gate type & fan-in
LH / HL
Total load L
Delay
 $t_{pLH}(x \rightarrow z)$

G1: NOR 2 → G4: AND 3 → G8: NAND 2 → G11: AND 3
 G1: HL → G4: HL → G8: LH → G11: LH
 G1: 5 → G4: 3 → G8: 2 → G11: 4
 G1: 0.15 → G4: 0.234 → G8: 0.126 → G11: 0.352
0.862 (ns)

2. (8 points) Unlike other gates, a low to high input for an XOR gate can cause the output to transition both from low to high and from high to low, depending on the value of the other input gate.

x	y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

From the table, when $x = 0$, a low to high ($0 \rightarrow 1$) transition at input y will cause the output to move from low to high, but when $x = 1$, the same low to high transition at y will cause the output to move from high to low.

Taking this into consideration, find the worst case value of $t_{pLH}(x \rightarrow w)$.

Because G5 is an XOR gate, we need to consider both low to high and high to low transitions at the output of G1, and select the worst case. Fill in the blanks below with the appropriate values.

Gate type & fan-in	G1: <u>NOR 2</u> → G5: <u>XOR 2</u>
Total load L	G1: <u>5</u> → G5: <u>5</u>
LH / HL	G5: <u>LH</u>
Propagational delay of G5	G5: <u>0.34 ns</u>
Delay of G1	G1(LH): <u>0.435 ns</u> or G1(HL): <u>0.15 ns</u>

3. (2 points) What is the worst case value of $t_{pLH}(x \rightarrow w)$?

0.775 ns (when G1 goes from Low to High)