[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

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Problem	Points	Score
1	10	10
2	15	15
3	15	13
4	15	10
5	20	15
6	10	7
7	~~15~~ 9	3
Total	~~100~~ 94	73

20 bits = 10^5
5 chunks = 10^5

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   10^8 = 100,000,000

   a. Decimal digits in BCD

   $8 \cdot 4 = 32$

   32 bits

   b. Hexadecimal representation

   $16^5 \; 16^4 \; 16^3 \; 16^2 \; 16^1 \; 16^0$

   24 bits

   In binary, requires 24 bits

   $2^{24} > 16$ mil

   Which representation is more efficient? Why?

   Hexadecimal because it requires less bits

   $5 \times 16^2 + 1 \times 16^1 + 7 \times 16^0$

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	(4, 5, 3, 2)	1640

~~$5 \times 8^2 + 1 \times 16^1$~~

$5 \times 8^2 + 1 \times 8 + 7 \times 8^0$

23

x66

4 5 3 2
3 2 1 0

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$(ab+c)'(ac+(b'+c'+a'cd)')+[a((b+c)(b+d)+c)']$

$(a'+b')c'(ac+bc(a+c'+d'))+a((b+c)(b+d)+c)'$ De Morgan's Law

$(a'c'+b'c')(ac+bca+bcc'+bcd')+a(bb+bd+cb+cd+c)'$ distributivity, commutativity

$(a'c'+b'c')(ac+abc+bcc'+bcd')+a(bb+bd+bc+cd+c)'$

$(a'c'+b'c')(ac+abc+0+bcd')+a(bb+bd+bc+cd+c)'$ compliment

$(a'c'+b'c')(ac+abc+bcd')+a(b+bd+bc+cd+c)'$ idempotency & identity

$(a'c'+b'c')(ac+bcd')+a(b+bc+c)'$ absorption

$(a'c'ac+b'c'ac+a'c'bcd'+b'c'bcd')+a(b+bc+c)'$ distributivity

$(a'acc'+ab'c'c+a'bc'cd'+b'bc'cd')+a(b+bc+c)'$ commutativity

$(0+0+0+0)+a(b+bc+c)'$ compliment

$a(b+bc+c)'$

$a(b+c)'$ absorption

$\boxed{a(b'c')}$

3: 011
4: 100

$ab + ac' + b'b + b'c'$

6: 110
7: 111

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

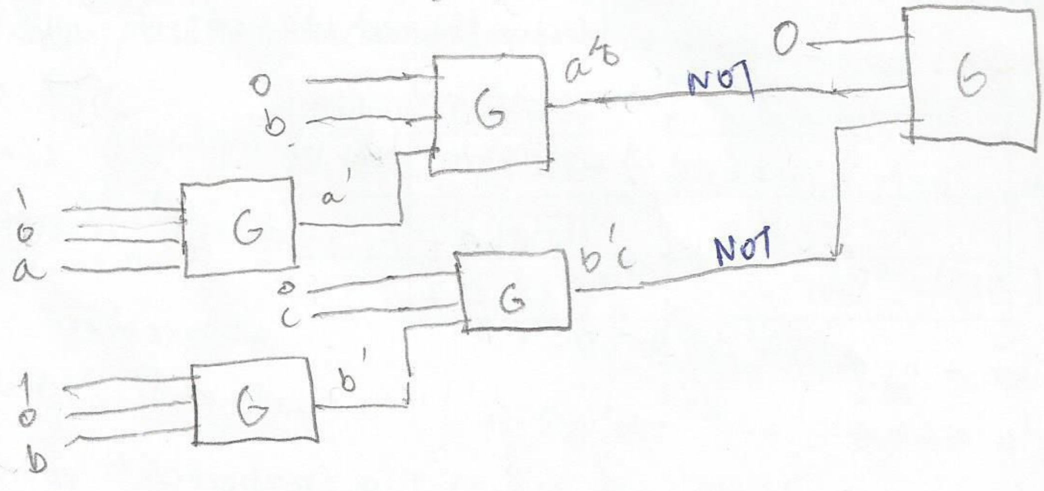
$(a'b)' \cdot (b'c)'$

$$E(a, b, c) = (a + b')(b + c')$$

$$G = x'yz + xy'z' + xyz' + xyz$$
$$= yz(x' + x) + xz'(y' + y)$$
$$G = yz + xz'$$

$$\boxed{\begin{aligned} AND(a,b) &= G(0, a, b) \\ NOT(a) &= G(1, 0, a) \end{aligned}}$$

$$E(a,b,c) = (a + b')(b + c')$$
$$= (a'b)'(b'c)' =$$



$(a'b)'(b'c)' =$

$G(1, 0, G(0, G(1, 0, a), b))$ and $G(1, 0, G(0, G(1, 0, b), c))$

$G(0, G(1, 0, G(0, G(1, 0, a), b)), G(1, 0, G(0, G(1, 0, b), c)))$

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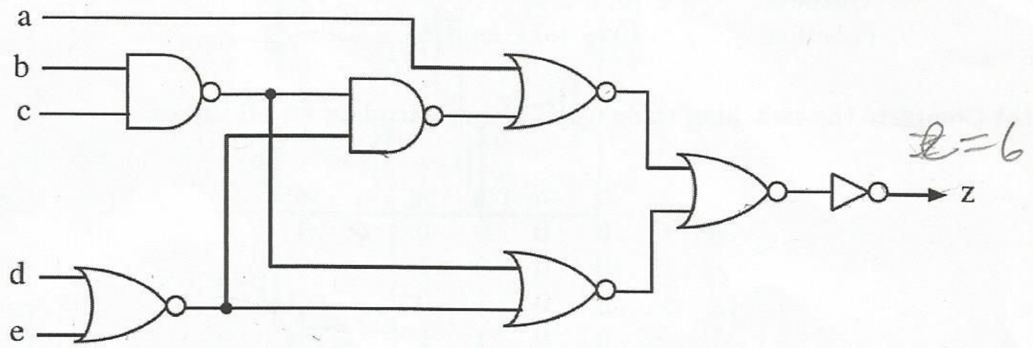
Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

LH HL LH HL LH

$0.06 + 0.075(2) + 0.05 + 0.038(1) +$

$\underline{0.08} + 0.027 \times 1$

$0.06 + 0.075(1) + 0.07 + 0.016(1) +$

$0.02 + 0.038(6) =$

$= 0.767$ ns X

10

Problem 5 (20 points)

Obtain a two-level gate network of the following system.

x y
0 0
0 1
0 2
0 3

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\} \bmod 4$

1. **(2 points)** Complete the switching table using binary encoding for all values.

		x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	0	1
0	1	0	0	0	1	0	1
0	2	0	0	1	0	0	1
0	3	0	0	1	1	0	1
1	0	0	1	0	0	0	1
1	1	0	1	0	1	0	0
1	2	0	1	1	0	1	1
1	3	0	1	1	1	1	0
2	0	1	0	0	0	0	1
2	1	1	0	0	1	1	1
2	2	1	0	1	0	0	1
2	3	1	0	1	1	1	1
3	0	1	1	0	0	0	1
3	1	1	1	0	1	1	0
3	2	1	1	1	0	1	1
3	3	1	1	1	1	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

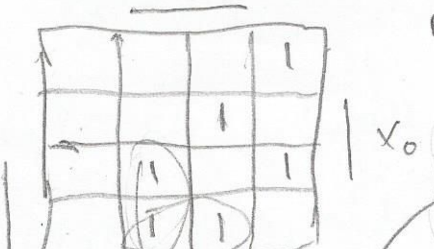
$z_1 = x_1'x_0y_1y_0' + x_1'x_0y_1y_0 + x_1x_0'y_1y_0 + x_1x_0y_1'y_0 + x_1x_0y_1y_0'$

5

$z_0 = x_1'x_0'y_1'y_0' + x_1'x_0'y_1'y_0 + x_1'x_0'y_1y_0' + x_1'x_0'y_1y_0 + x_1'x_0y_1'y_0' + x_0'x_0y_1y_0' + x_1x_0'y_1'y_0' + x_1x_0'y_1y_0' + x_1x_0y_1'y_0' + x_1x_0y_1y_0'$

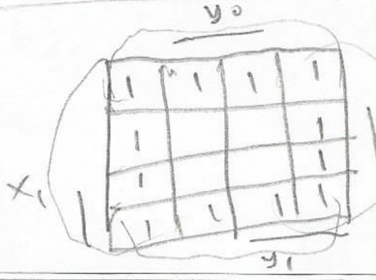
3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

z_1: $x_1y_0y_1' + x_1x_0'y_0 + x_1'x_0'y_1y_0 + x_1'x_0y_1y_0 + x_1x_0y_1y_0$



prime = essential = $\{x_1y_0y_1', x_1x_0'y_0, x_1'x_0'y_1y_0', x_1'x_0y_1y_0, x_1x_0y_1y_0'\}$

z_0:

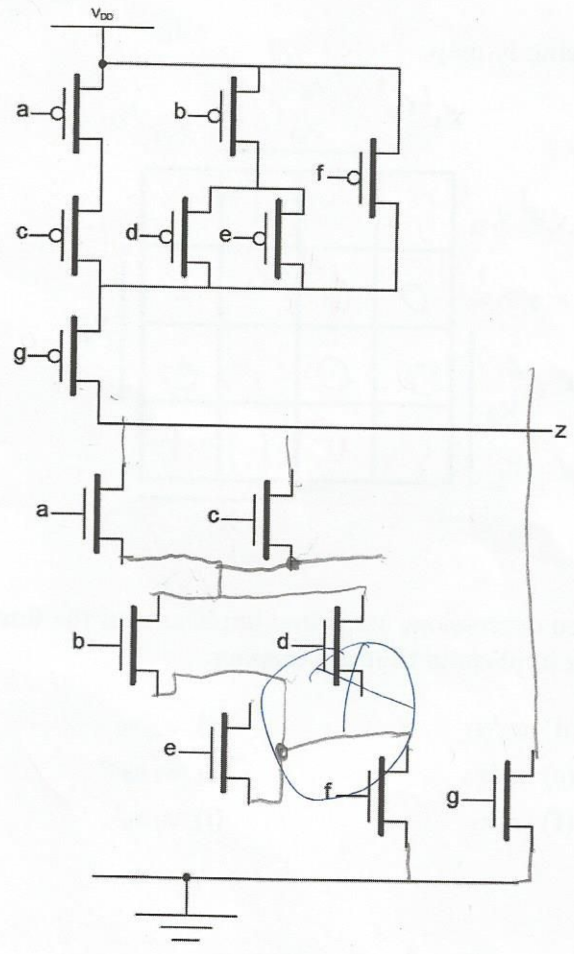


prime = essential = $\{x_0', y_0'\}$

$x_0' + y_0'$

Problem 6 (10 points)

We are given the following partial CMOS network.



1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1) $z = a'c'g' + b'(d'+e') + f'$

2) $z' = (a'c'g' + b'd' + b'e' + f')' =$

$(a+c+g)(b+d)(b+e)(f) =$

$\Pi M(4,5,6,12,13,14)$
$= \Sigma m($

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1. **(2 points)** Fill out the following K-map.

	$x_1'x_0'$	$x_1'x_0$	x_1x_0	x_1x_0'
$x_3'x_2'$	1	1	1	1
$x_3'x_2$	0	1	1	0
x_3x_2	0	0	1	0
x_3x_2'	1	1	1	1

-1

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

(a) x_1
(b) x_3x_1
(c) $x_3'x_2'$
(d) $x_3'x_1$
(e) $x_3'x_0$
(f) x_2x_1
(g) $x_2'x_0$
(h) x_1x_0 ✓
(i) x_1x_0'
(j) $x_3'x_2'x_1$
(k) $x_2x_1x_0$
(l) $x_3x_2x_1x_0$

-3

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$x_1x_0,\ x_2'$

-2

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$x_1x_0 + x_2'$, is unique