[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name :

Student ID :

Problem	Points	Score
1	10	10
2	15	15 + 1
3	15	15
4	15	7
5	20	16
6	10	10.
7	~~15~~ 9	9
Total	~~100~~ 94	82

15 0000 8 100 000000

16,000,000

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

   $\boxed{32 \text{ bits}}$

   b. Hexadecimal representation

   $16 = 2^4$ 16^6 $16^n - 1$ FF $\cancel{16+15}$ $15 \cdot 16 + 15$ $(16+1)15$ 30 $2^{10} = 1024$ $2^{20} =$

   $\boxed{6 \text{ bits}}$

   1F1 $16^2 + 16 + 15$

   1024 × 1024 = 4096, 20480, 00000, 1024000 — 1,048,576 × 8 = 8388608

   2^{20} 2^3 2^{23} 2^{24}

   Which representation is more efficient? Why?

   Hexadecimal is more efficient b/c it uses significantly fewer bits

   $16^5 = 2^{20}$

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	$(5, 1, 7)_{16}$	1303
8	$(5, 1, 7)_8$	335

$5 \cdot 16^2 + 1 \cdot 16 + 7$

$5 \cdot 256 + 16 + 7$

256 × 5 = 1280, + 16, + 7 = 1303

$5 \cdot 64 + 8 + 7$

320 + 15 = 335

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$E=(ab')(c')(ac+bc(a'cd)')+a(((b+c)(b+d))'c')$

$=(ab'c')(ac+bc(a+c'+d'))+(ac')((b+c)'+(b+d)')$

$=(ab'c')(ac+abc+bcc'+bcd')+(ac'(b'c'+b'd'))$

$=0+0+0+0+ab'c'+ab'c'd'=ab'c'(1+d')=ab'c'$

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$a'+b+c$

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

G1 G2 G3

X	Y	Z	G	
0	0	0	0	→ AND
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	1	→ NOT
1	0	1	0	
1	1	0	1	
1	1	1	1	

NOT gate

X	NOT
0	1
1	0

AND gate

X	Y	AND
0	0	0
0	1	0
1	0	0
1	1	1

$$\boxed{\begin{array}{l} G(1,0,z) = NOT(z) \\ G(0,y,z) = AND(y,z) \end{array}}$$

$$E(a, b, c) = (a + b')(b + c') = (a'b)'(b'c)'$$

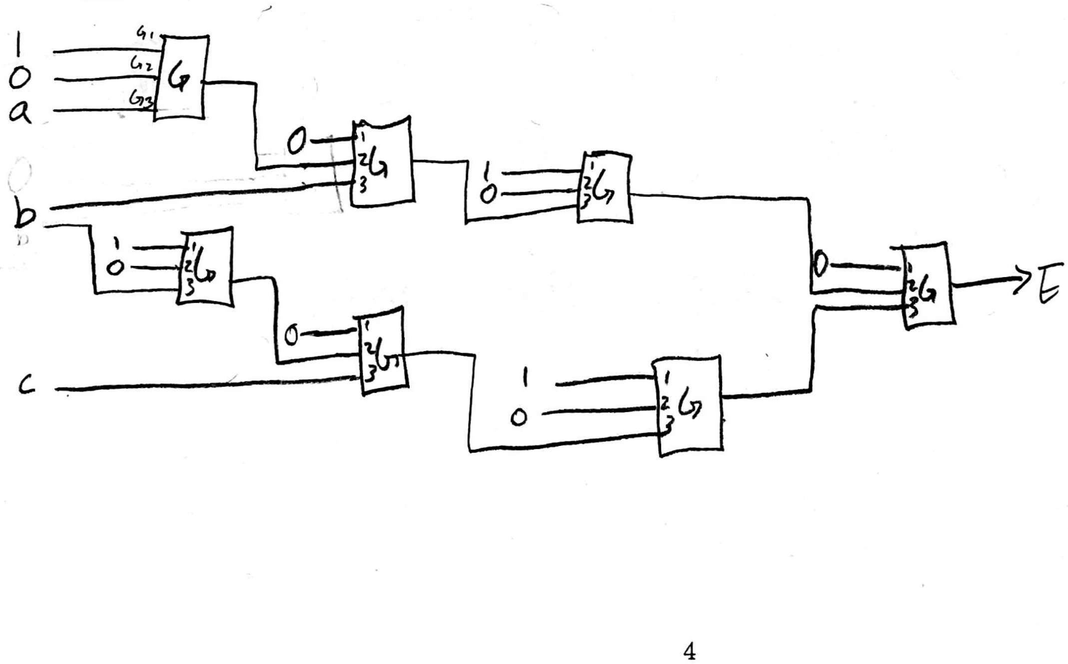
$$= AND(NOT(AND(NOT(a),b)), NOT(AND(NOT(b),c)))$$

$$= AND(NOT(AND(G(1,0,a),b)), NOT(AND(G(1,0,b),c)))$$

$$= AND(NOT(G(0,G(1,0,a),b)), NOT(G(0,G(1,0,b),c)))$$

$$= AND(G(1,0,G(0,G(1,0,a),b)), G(1,0,G(0,G(1,0,b),c)))$$

$$\boxed{E = G(0, G(1,0,G(0,G(1,0,a),b)), G(1,0,G(0,G(1,0,b),C)))}$$



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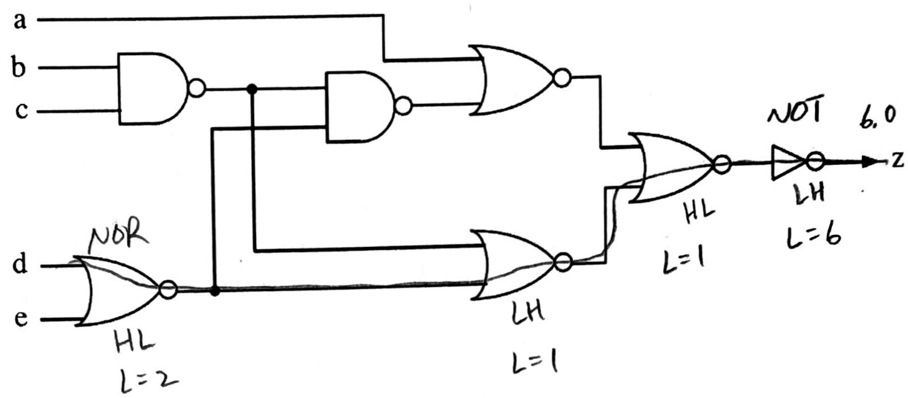
Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



2
+4
0.038
6
0.228

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

$t_{pLH}(d,z) = 0.07 + 0.016(2) + 0.06 + 0.075(1) + 0.07 + 0.016(1) + 0.02 + 0.038(6)$

$= 0.07 + 0.032 + 0.06 + 0.075 + 0.07 + 0.016 + 0.02 + 0.228$

$= 0.102 + 0.135 + 0.086 + 0.248$

$= 0.237 + 0.334$

$\boxed{t_{pLH}(d,z) = 0.571 \text{ ns}}$

0.248
0.086
0.334
0.237
0.571

wrong critical path

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\}$ mod 4

1. **(2 points)** Complete the switching table using binary encoding for all values.

x	y	z (no mod 4)	z
0	0	1	1
0	1	1	1
0	2	1	1
0	3	1	1
1	0	1	1
1	1	4	0
1	2	7	3
1	3	10	2
2	0	1	1
2	1	6	2
2	2	13	1
2	3	19	3
3	0	1	1
3	1	10	2
3	2	19	3
3	3	28	0

x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	1
0	0	0	1	0	1
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	1	1
1	1	1	1	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$Z_1 = x_1' x_0 y_1 y_0' + x_1' x_0 y_1 y_0 + x_1 x_0' y_1' y_0 + x_1 x_0' y_1 y_0 + x_1 x_0 y_1' y_0 + x_1 x_0 y_1 y_0'$

$Z_0 = x_1' x_0' y_1' y_0' + x_1' x_0' y_1' y_0 + x_1' x_0' y_1 y_0' + \cancel{x_1' x_0' y_1 y_0} + x_1' x_0 y_1' y_0' + x_1' x_0 y_1 y_0' + x_1 x_0' y_1' y_0' + x_1 x_0' y_1 y_0'$
$+ x_1 x_0' y_1 y_0 + x_1 x_0 y_1' y_0' + x_1 x_0 y_1 y_0'$

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3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

Z_1 KMap

y_1y_0 \ x_1x_0	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	1	0	1
10	0	1	1	0

prime implicants: $x_1' x_0 y_1,\ x_0 y_1 y_0',\ x_1 y_1' y_0,\ x_1 x_0' y_0$

essential prime implicants: (same as prime implicants)

$Z_1 = NAND(NAND(x_1', x_0, y_1), NAND(x_0, y_1, y_0'), NAND(x_1, y_1', y_0), NAND(x_1, x_0', y_0))$

Z_0 kmap

y_1y_0 \ x_1x_0	00	01	11	10
00	1	1	1	1
01	1	0	0	0
11	1	0	0	1
10	1	1	1	1

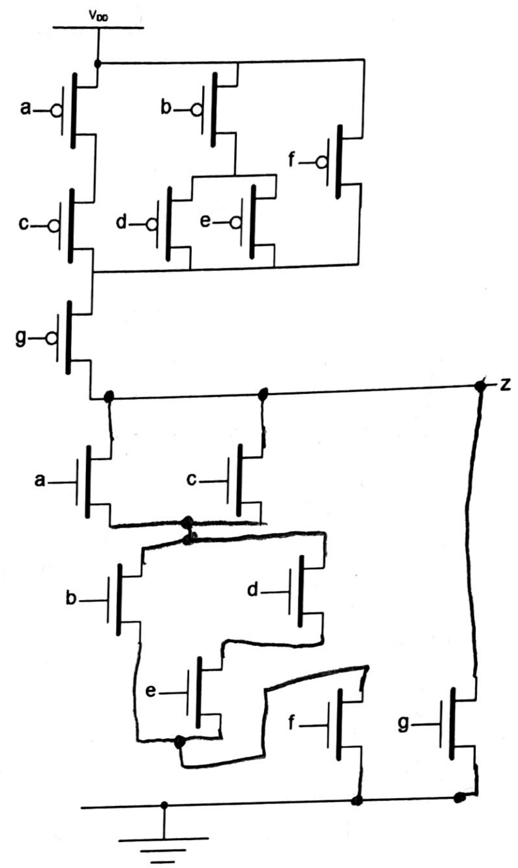
prime implicants: $y_0',\ x_0' y_1,\ x_1' x_0',\ x_0' y_0'$

essential prime implicants: $y_0',\ x_0' y_1,\ x_1' x_0'$

$Z_0 = NAND(NAND(y_0', y_0'), NAND(x_0', y_1), NAND(x_1', x_0'))$

Problem 6 (10 points)

We are given the following partial CMOS network.



$(a+c)(b+de)(f) + g$

1. **(5 points)**
   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra. pull-up : $Z = (a'c' + b'(d'+e') + f')g'$

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

pull-down:

$$z' = ((a'c' + b'(d'+e') + f')g')' = g + (a'c' + b'(d'+e') + f')'$$

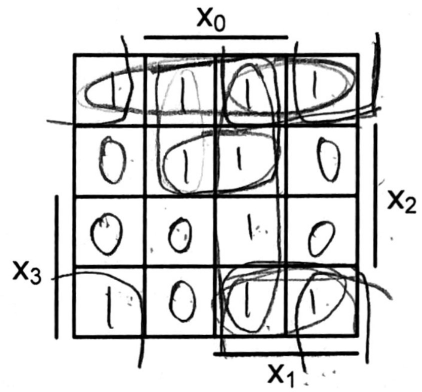
$$= g + ((a'c')'(b'(d'+e'))'(f)) = g + ((a+c)(b + (d'+e')')(f))$$

$$\boxed{z' = g + (a+c)(b+de)(f)}$$

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1. **(2 points)** Fill out the following K-map.



2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

~~(a) x_1~~ ~~(b) x_3x_1~~ (c) $x_3'x_2'$

~~(d) $x_3'x_1$~~ (e) $x_3'x_0$ ~~(f) x_2x_1~~

~~(g) $x_2'x_0$~~ (h) x_1x_0 ~~(i) x_1x_0'~~

~~(j) $x_3'x_2'x_1$~~ ~~(k) $x_2x_1x_0$~~ ~~(l) $x_3x_2x_1x_0$~~

$X_2' X_1$

$X_2' X_0'$

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$X_3'X_0, X_1X_0, X_2'X_0'$

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$f = X_3'X_0 + X_1X_0 + X_2'X_0'$

It's unique