CS M146 Final Exam

TOTAL POINTS

92 / 100

QUESTION 1

1 True or False 14 / 14

✓ - 0 pts **all correct**

- **2 pts** a. incorrect - **2 pts** b. incorrect
- **2 pts** c. incorrect
- **2 pts** d. incorrect - **2 pts** e. incorrect
- **2 pts** f. incorrect
- **2 pts** g. incorrect

QUESTION 2

Hidden Markov Models 9 pts

2.1 a 3 / 3

✓ - 0 pts **Correct**

- **3 pts** incorrect

2.2 b 3 / 3

✓ - 0 pts **Correct**

- **1 pts** partially incorrect computation
- **2 pts** incorrect computation
- **3 pts** incorrect

2.3 c 3 / 3

✓ - 0 pts **Correct**

- **2 pts** incorrect justification
- **3 pts** incorrect

QUESTION 3

3 Naive Bayes 10 / 12

- **0 pts** Correct
- **1.5 pts** a) incomplete
- **3 pts** a) incorrect

✓ - 1 pts **b) minor mistake**

- **4 pts** b) incorrect
- **2 pts** c) incorrect

✓ - 1 pts **c) minor mistake**

- **3 pts** d) incorrect

QUESTION 4

Kernels and SVM 25 pts

4.1 a.i 3 / 3

✓ - 0 pts **Correct**

- **1 pts** minor error
- **2 pts** unclear prove, but partially correct
- **3 pts** Incorrect

4.2 a.ii 5 / 5

✓ - 0 pts **Correct**

- **1 pts** minor error
- **2 pts** Partially correct
- **3 pts** You haven't reach the key point yet, but you are on the way
- **4 pts** Wrong way!! 1 point for proving v^Av >=0 with a special v (or B)
- **5 pts** Incorrect

4.3 b.i 4 / 4

✓ - 0 pts **Correct**

- **2 pts** first blank incorrect
- **2 pts** second blank incorrect

4.4 b.ii 4 / 4

✓ - 0 pts **Correct**

- **2 pts** minor incorrect
- **4 pts** incorrect. (A typical wrong statement is saying that it turns out to be hard SVM.)

4.5 b.iii 3 / 3

✓ - 0 pts **False statement, with reasonable explanation**

- **2 pts** False statement, without/ with wrong explanation
- **3 pts** True statement

4.6 b.iv 3 / 3

✓ - 0 pts **True statement, mentioned dual form, support vectors or similar**

- **1 pts** True statment, mentioned a perceptron-style update, but fail to discuss the difference with SVM (i.e. stating that alpha is # of mistakes) - **2 pts** True statement without/ with wrong explanation - **3 pts** False statement

4.7 b.v 3 / 3

- **1 pts** W is wrong: either w1/w2 does not equal to +1 or w1,w2 are negative
- **0.5 pts** b is wrong: either b is positive; or b does not suitable for W
- **1.5 pts** svs are wrong (0.5 for each)

✓ - 0 pts **Correct**

QUESTION 5

Short Answer Questions 38 pts

5.1 Adaboost 3 / 3

✓ - **0 pts** **Correct**

- **2 pts** Correct point, Incorrect justification
- **3 pts** Incorrect answer
- **1 pts** Wrong decision stump
- **2 pts** Circled positive points on one side of the decision stump but reasoning is correct

5.2 Clustering 4 / 4

✓ - **0 pts** **Correct**

- **2 pts** Incorrect explanation
- **4 pts** Incorrect
- **1 pts** Insufficient explanation

5.3 LOOCV 3 / 3

✓ - **0 pts** **Correct**

- **3 pts** Incorrect

5.4 Probability 4 / 4

✓ - **0 pts** **Correct**

- **1 pts** Wrong denominator
- **1 pts** Wrong numerator
- **4 pts** Incorrect

5.5 Multiclass 2 / 6

- **0 pts** Correct
- **2 pts** Minor mistake / Didn't sum over all examples
- **1 pts** minor mistake

5.7 PAC_ii 3 / 3

✓ - **0 pts** **Correct (PAC theorem only shows the upper bound)**

- **1 pts** Incorrect explanation
- **3 pts** Incorrect

5.8 Generative vs Discriminative 4 / 4

✓ - **0 pts** **Both correct**

- **2 pts** One incorrect answer
- **4 pts** Both incorrect

5.9 VC Dimension 6 / 8

- **0 pts** Correct
- **8 pts** Incorrect
- **4 pts** VC(DT3)=8 w/ explanation or examples
- **2 pts** incorrect Prove VC(DT3) >= 8
- **2 pts** Prove VC(DT_k) < 9 (=2^3 + 1)

✓ - **2 pts** **minor mistake**

Your answer is right but the proof is too handwaving.

QUESTION 6

6 Name and Id 2 / 2

✓ - **0 pts** **Correct**

/ Didn't sum over all classes

✓ - **4 pts** **Only procedure / Attempt to derive(taken log somewhere in the derivation)**

- **6 pts** Incorrect
- **3 pts** Mostly correct formulation
- **1 pts** Tiny mistake

5.6 PAC_i 3 / 3

✓ - **0 pts** **Correct (200 examples)**

- **3 pts** Incorrect
- **2 pts** Correct approach but no answer

CM146: Introduction to Machine Learning	Winter 2018
Final Exam	
Mar. 22^{nd}, 2018	

- This is a closed book exam. Everything you need in order to solve the problems is supplied in the body of this exam.
- This exam booklet contains **five** problems.
- You have 150 minutes to earn a total of 100 points.
- **Besides giving the correct answer, being concise and clear is very important. To get the full credit, you must show your work and explain your answers.**

Good Luck!

Name and ID: (2 Point)	

Name		/2
True/False Questions		/14
Hidden Markov Models		/9
Naive Bayes		/12
Kernels and SVM		/25
Short Answer Questions		/38
Total		/100

1 True or False [14 pts]

Choose either True or False for each of the following statements. For the statement you believe it is *False*, please give your brief explanation of it. Two points for each question. *Note: the credit can **only** be granted if your explanation for the false statement is correct. Also note, a negated statement is not counted as a correct explanation.*

(a) Training a k-class classification model using one-against-all is always faster than using one-vs-one because one-vs-one requires to train more binary classifiers.

False, depends on computational complexity of classifier

(b) We would expect the support vectors to remain the same in general as we move from a linear kernel to higher order polynomial kernels.

False, the decision boundary changes so vectors far from margin in linear case can become support vectors in high order.

(c) In a mistake-driven algorithm such as the Perception algorithm, if we make a mistake on example x_i with label y_i, we update the weights w so that we can guarantee that we now predict y_i correctly.

False, it is close to being correct but not necesarily, it may take multiple mistakes on (x_i, y_i)

(d) Consider a classification problem with n features. The VC dimension of the corresponding (linear) SVM hypothesis space is larger than that of the corresponding logistic regression hypothesis space.

False, both have linear decision boundaries

(e) A 3-layer neural network with non-linear activation functions can learn non-linear decision boundaries.

True

(f) In AdaBoost, the weight associated with each weak learner can be negative (less than 0).

False, each weight is between 0 and 1

(g) Using MAP to estimate model parameters always give us better performance.

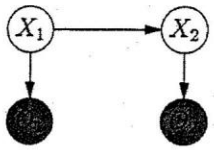
False, the learned $P(h)$ could overfit

0 -> 1

A -> B

2 Hidden Markov Models [9 pts]

Consider the following Hidden Markov Model.



X_1	$\Pr(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$\Pr(X_{t+1}\|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$\Pr(O_t\|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that $O_1 = A$ and $O_2 = B$ is observed.

check math

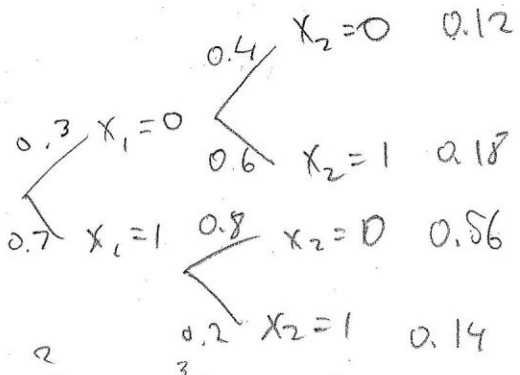
(a) **(3 pts)** What is the probability of $P(O_1 = A, O_2 = B, X_1 = 0, X_2 = 1)$?

$$\Pr(X_1=0)\,\Pr(X_2=1 \mid X_1=0)\,\Pr(O_1=A \mid X_1=0)\,\Pr(O_2=B \mid X_2=1)$$
$$= 0.3 \cdot 0.6 \cdot 0.9 \cdot 0.5$$
$$= 0.09 \cdot 0.9 = 0.081$$

12
9
18
9
108

(b) **(3 pts)** What is the most likely assignment for X_1 and X_2?

$$\Pr(X_1=1)\,\Pr(X_2=0 \mid X_1=1)$$
$$= 0.7 \cdot 0.8 = 0.56$$



00	$0.12 \cdot 0.9 \cdot 0.1 = 0.0108$
01	$0.18 \cdot 0.9 \cdot 0.5 = 0.081$
10	$0.56 \cdot 0.5 \cdot 0.1 = 0.028$
11	$0.14 \cdot 0.5 \cdot 0.5 = 0.0350$

2
14
25
70
28
350

3
56
5
280

$X_1=0,\ X_2=1$

(c) **(3 pts)** [True/False] Based on the independent assumptions in HMM, the random variable O_1 is independent of the random variable X_2. Justify your answer.

$$\Pr(O_1 \mid X_1) = \frac{P(X_1 \mid O_1)\,P(O_1)}{P(X_1)} \qquad \Pr(X_2 \mid X_1) = \frac{P(X_1 \mid X_2)\,P(X_2)}{P(X_1)}$$

False, they both depend on X_1, so they are not necessarily independent

$p^1(1-p)^5$

3 Naive Bayes [12 pts]

Data the android is about to play in a concert on the Enterprise and he wants to use a Naive Bayes classifier to predict whether he will impress Captain Picard. He believes that the outcome depends on whether Picard has been reading Shakespeare or not for the three days before the concert. For the previous five concerts, Data has observed Picard and noted on which days he read Shakespeare. His observations look like this:

D1 (Day 1)	D2 (Day 2)	D3 (Day 3)	LC (LikedConcert)
1	1	0	yes
0	0	1	no
1	1	1	yes
1	0	1	no
0	0	0	no

(a) **(3 pts)** What does the modeling assumption make in the Naive Bayes model?

conditional independence

$$P(X,Y|Z) = P(X|Z)P(Y|Z)$$

(b) **(4 pts)** Show the Naive Bayes model that Data obtains using maximum likelihood from these instances. (Write down the numerical values of the model parameters.)

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

$$P(D|LC=yes) = 1^{D1}\,1^{D2}\left(\frac{1}{2}\right)^{D3}$$

$$P(D1=1|LC=yes) = \frac{2}{2} = 1$$

$$P(LC=yes) = \frac{2}{5}$$

$$P(D2=1|LC=yes) = \frac{2}{2} = 1$$

$$P(LC=yes|D) =$$

$$P(D3=1|LC=yes) = \frac{1}{2}$$

(c) **(2 pts)** If Picard reads Shakespeare only on day 1 and day 2, how likely is he to enjoy Data's concert?

$$P(LC=yes|\,D1=1, D2=1, D3=0) = 1$$

(d) **(3 pts)** Estimate $P(LC = yes|D_2 = 1)$.

$$= \frac{\text{count}(LC=yes, D_2=1)}{\text{count}(D_2=1)} = \frac{2}{2} = 1$$

4 Kernels and SVM [25 pts]

(a) **(8 pts)** Properties of Kernels

i. **(3 pts)** Given n training examples $\{x_i\}_{i=1}^n$, the kernel matrix $\mathbf{A}$ is an $n \times n$ square matrix, where $\mathbf{A}(i,j) = K(x_i, x_j) = \Phi(x_i)^T \Phi(x_j)$. Prove that the kernel matrix is symmetric (i.e, $A_{i,j} = A_{j,i}$).
hints: Your proof will not be longer than 2 or 3 lines.

$$A(i,j) = \Phi(x_i)^T \Phi(x_j) = \Phi(x_i)_1 \Phi(x_j)_1 + \cdots + \Phi(x_i)_m \Phi(x_j)_m$$
$$= \Phi(x_j)_1 \Phi(x_i)_1 + \cdots + \Phi(x_j)_m \Phi(x_i)_m$$
$$= \Phi(x_j)^T \Phi(x_i) = A(j,i)$$

ii. **(5 pts)** Prove that the kernel matrix $\mathbf{A}$ is positive semi-definite.
hints: (1) Remember that an $n \times n$ matrix $\mathbf{A}$ is positive semi-definite if and only if for any n dimensional vector $\mathbf{v} \neq \mathbf{0}$, we have $\mathbf{v}^T\mathbf{A}\mathbf{v} \geq 0$. (2) Consider a matrix $\mathbf{B} = [\Phi(x_1), \cdots, \Phi(x_n)]$ and use it to prove A is positive semi-definite.

$$A = BB^T$$
$$v^T A v = v^T B B^T v$$
$$= \left[\sum_i v_i \Phi(x_i)_1 \quad \cdots \quad \sum_i v_i \Phi(x_i)_m\right] \begin{bmatrix} \sum v_i \Phi(x_i)_1 \\ \vdots \\ \sum v_i \Phi(x_i)_m \end{bmatrix}$$
$$= \sum_i \left(v_i^2 \Phi(x_i)_1^2 + \cdots + v_i^2 \Phi(x_i)_m^2\right)$$
$$\geq 0$$

v: $n \times 1$, B: $n \times m$, B^T: $m \times n$, $B^T v$: $m \times 1$, $v^T B$: $1 \times m$

(b) **(17 pts)** Given a dataset $D = \{x_i, y_i\}, x_i \in \mathbb{R}^k, y_i = \{-1, +1\}, 1 \leq i \leq N$.
A hard SVM solves the following formulation

$$\min_{w,b} \quad \frac{1}{2} w^T w \qquad \text{s.t} \quad \forall i, y_i(w^T x_i + b) \geq 1, \tag{1}$$

and soft SVM solves

$$\min_{w,\xi_i,b} \quad \frac{1}{2} w^T w + C \sum_i \xi_i \qquad \text{s.t} \quad \forall i, y_i(w^T x_i + b) \geq 1 - \xi_i, \quad \forall i, \xi_i \geq 0 \tag{2}$$

i. **(4 pts)** Complete:
If $C =$ ∞, soft SVM will behave exactly as hard SVM.
In order to reduce over-fitting, one should decrease (decrease or increase) the value of C.

ii. **(4 pts)** Show that when $C = 0$, the soft SVM returns a trivial solution and cannot be a good classification model.

$\min_{w, \xi_i, b} \frac{1}{2} w^T w$ s.t. $\forall i, y_i(w^T x_i + b) \geq 1 - \xi_i$, $\forall i, \xi_i \geq 0$

$w = \vec{0}$, $b = 0$, $\forall i, \xi_i = 1$ minimizes $\frac{1}{2} w^T w$

iii. **(3 pts)** [True/False] The slack variable ξ_i in soft SVM for a data point x_i always takes the value 0 if the data point is correctly classified by the hyper-plane. Explain your answer. False, it can be classified correctly but within the margin s.t. $0 < \xi_i < 1$

iv. **(3 pts)** [True/False] The optimal weight vector w can be calculated as a linear combination of the training data points. Explain your answer. [You do not to prove this.]

True, that is the dual representation of SVM where $w = \sum \alpha_i x_i$

v. **(3 pts)** We are given the dataset in Figure 1 below, where the positive examples are represented as black circles and negative points as white squares. (The same data is also provided in Table 1 for your convenience). Recall that the equation of the separating hyperplane is $\hat{y} = \mathbf{w}^T \mathbf{x} + b$.

i. Write down the parameters for the learned linear decision function.

$W = (w_1, w_2) =$ (1, 1). $b =$ −1

ii. Circle all support vectors in Figure 1.

$x_2 = -x_1 + 1$

$0 = -x_1 - x_2 + 1$

$0 = x_1 + x_2 - 1$

index	x_1	x_2	y
1	0	0	−
2	0	-4	−
3	-1	-1	−
4	-2	-2	−
5	3	0	+
6	0	3	+
7	1	1	+
8	3	-1	+

Table 1: The dataset S

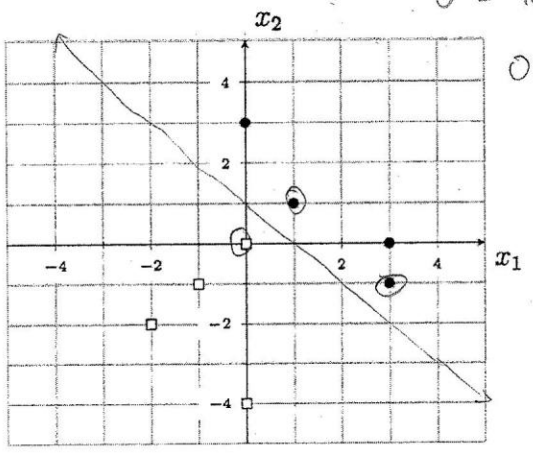
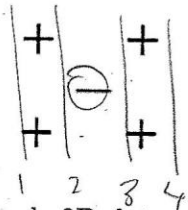


Figure 1: Linear SVM

5 Short Answer Questions [38 pts]

Most of the following questions can be answered in one or two sentences. Please make your answer concise and to the point.

(a) **(3 pts)** Consider training a classifier using AdaBoost with decision stumps (pick a horizontal or a vertical line, and one side of the half-space is positive and the other one is negative) on the following dataset:



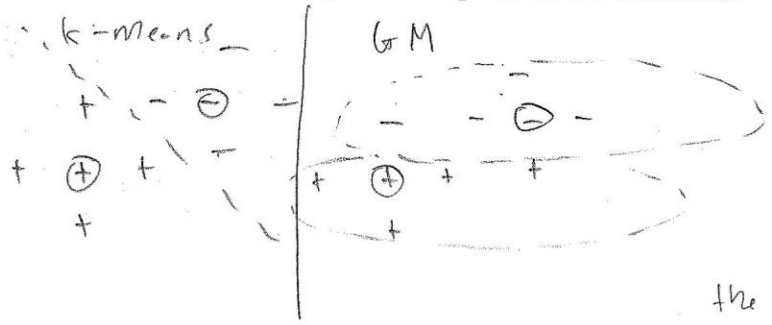
1 - 1
2 - 3
3 - 2
4 - 4

Figure 2: Example 2D dataset for Boosting

Which example(s) will have their weights increased at the end of the first iteration? Circle them and justify.

If the right side of the line is classified (+) then line 1 has the lowest error and in that case the middle (-) is the training error. (symmetrical for horizontal)

(b) **(4 pts)** Suppose we clustered a set of N data points using two different clustering algorithms: k-means and Gaussian mixtures. In both cases we obtained 2 clusters and both algorithms return the same set of cluster centers. Can 2 points that are assigned to different clusters in the kmeans solution be assigned to the same cluster in the Gaussian mixture solution? If no, explain. If so, sketch an example and explain in 1-2 sentences.



Σ can be set in a way that points with further Euclidian distance are included since the Gaussian distribution is skewed

(c) **(3 pts)** Suppose you are running a learning experiment on a new algorithm for binary classification. You have a data set consisting of 100 positive and 100 negative examples. You plan to use leave-one-out cross-validation (i.e., 200-fold cross-validation) to evaluate a baseline method: a simple majority function (i.e., returns the most frequent label on the training set as the prediction). What is the average cross-validation accuracy of the baseline? (Only need to write down the number).

0%

(d) **(4 pts)** P(Good Movie | Includes Tom Cruise) = 0.01
P(Good Movie | Tom Cruise absent) = 0.1
P(Tom Cruise in a randomly chosen movie) = 0.01
What is P(Tom Cruise is in the movie | Not a Good Movie)?

$$P(G|TC) = \frac{P(TC|G)P(G)}{P(TC)} \qquad P(G|!TC) = \frac{P(!TC|G)P(G)}{P(!TC)}$$

$$0.01 = \frac{P(TC|G)P(G)}{0.01} \qquad = \frac{(1-P(TC|G))P(G)}{1-P(TC)}$$

$$\frac{0.0001}{P(G)} = P(TC|G) \qquad 0.1 = \frac{(1-P(TC|G))P(G)}{1-0.01}$$

$$0.099 = \left(1-\frac{0.0001}{P(G)}\right)P(G) \qquad 0.099 = (1-P(TC|G))P(G)$$

$$0.099 = P(G) - 0.0001 \rightarrow P(G) = 0.0991$$

$$P(TC|!G) = \frac{P(!G|TC)P(TC)}{P(!G)} = \frac{(1-P(G|TC))P(TC)}{1-P(G)} = \frac{(1-0.01)\,0.01}{1-0.0991} = \frac{0.99 \cdot 0.01}{1-0.0991}$$

(e) **(6 pts)** We can easily extend the binary Logistic Regression model to handle multi-class classification. Lets assume we have K different classes, and posterior probability for class k is given as

$$P(y=k|X=x) = \frac{\exp(w_k^T x)}{\sum_{k'=1}^{K} \exp(w_{k'}^T x)} \qquad (3)$$

where x is a d dimensional vector and w_k is the weight matrix for the k^{th} class.
Assuming dataset D consists of n examples, derive the log likelihood condition for this classifier.

hints: Let I_{ik} be an indicator function, where $i = 1, \ldots, n$ and $I_{ik} = \begin{cases} 1 & \text{if } y_i = k \\ 0 & \text{if } y_i \neq k \end{cases}$

(Full points if the derivation is mathematically correct. 2 points if you can describe the procedure for deriving.)

$$\log P(y=k|X=x) = \log \exp(w_k^T x) - \log \sum_{k'=1}^{K} \exp(w_{k'}^T x)$$

$$= w_k^T x - \sum_{k'=1}^{K} w_{k'}^T x$$

(f) **(6 pts)** In class we learned the following PAC learning bound for consistent learners:
Theorem 1. Let H be a finite concept class. Let D be an arbitrary, fixed unknown distribution over X. For any $\epsilon, \delta > 0$, if we draw a sample S from D of size

$$m > \frac{1}{\epsilon}\left(ln(|H|) + ln\frac{1}{\delta}\right) \tag{4}$$

then with probability at least $1 - \delta$, all hypothesis $h \in H$ have $err_D(h) \leq \epsilon$. Our friend Kai is trying to solve a learning problem that fits in the assumptions above.

i. Kai tried a training set of 100 examples and observed some test error, so he wanted to reduce the test error to half. How many examples should Kai use, according to the above PAC bound?

$m_0 = 100 \rightarrow err_D(h) = \epsilon_0$

He should use 200 examples

ii. Kai took your suggestion and ran his algorithm again, however the error on the test set did not halve. Do you think it is possible? explain briefly.

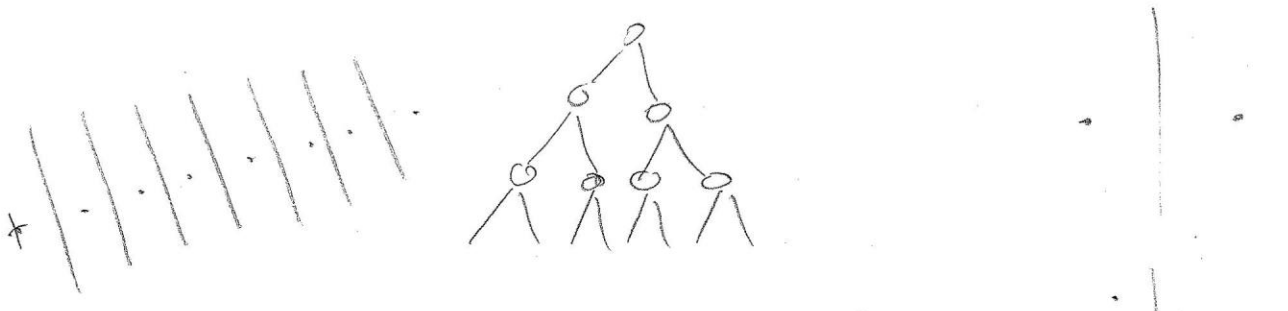
Yes, it is not guaranteed since PAC learning has a $1-\delta$ probability of success. In order to increase chance of error bound, more examples are needed

(g) **(4 pts)** List two differences between generative and discriminative learning models.

- discriminative learns $P(Y|X)$ while generative learns $P(X|Y)$ and $P(Y)$
- discriminative tends to underfit while generative tends to overfit

(h) **(8 pts)** We define a set of functions $T = f(x) = I[x > a] : a \in \mathbb{R}^1$, where $I[x > a]$ is the indicator function returning 1 if $x > a$ and returning 0 otherwise. For input domain $X = \mathbb{R}^1$, and a fixed positive number k, consider a concept class DT_k consisting of all decision trees of depth at most k where the function at each non-leaf node is an element of T. Note that if the tree has only one decision node (the root) and two leaves, then $k = 1$.

Determine the VC dimension of DT_3, and prove that your answer is correct.



$VC(DT_3) = 7$

shatters 7 points

Since
$VC(DT_3) \geq 7$

result shatter 8 points

and
$VC(DT_3) < 8$