MAY 1, 2013
MIDTERM EXAM

(4 pts) **1** Draw a deterministic finite automaton for the language of binary strings in which the number of zeroes and the number of ones are both even.

(4 pts) **2** Draw a nondeterministic finite automaton for $(0 \cup 1)^*01(011 \cup 0^*)^*11^*$.

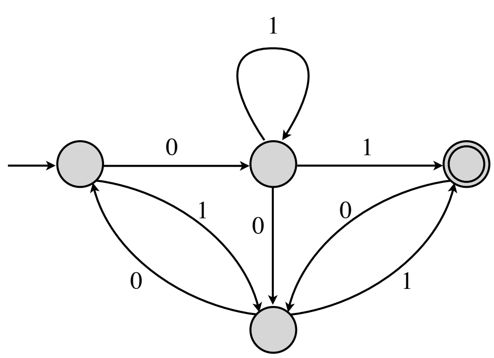
(12 pts) **3** Construct regular expressions for the following languages over the binary alphabet:

a. binary strings that do not begin with a 0;
b. binary strings that contain both 00 and 11 as substrings;
c. binary strings that have at least seven 1's.

(8 pts) **4** Rigorously establish the regularity or nonregularity of the following languages:

a. binary strings of odd length with middle symbol 0;
b. binary strings that contain a three-letter palindrome.

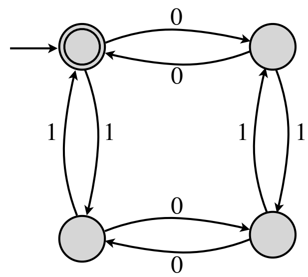
(4 pts) **5** Convert the following automaton to a regular expression, showing the intermediate steps:

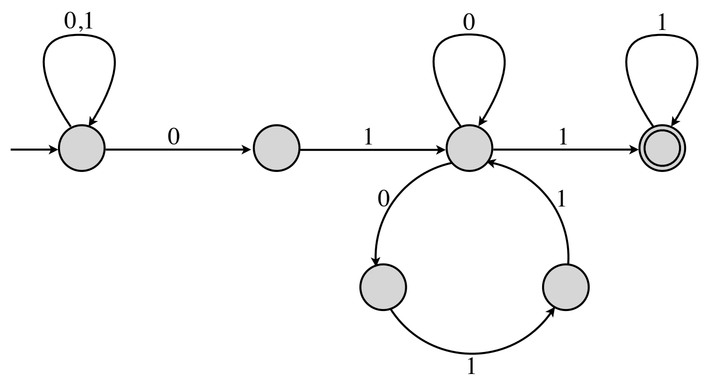


(4 pts) **6** For any integer $k \geqslant 2$, construct a deterministic finite automaton with 2^k states that recognizes the language ~~$(0 \cup 1)^*(0 \cup 1)^{k-1}0$~~ $(0 \cup 1)^*0(0 \cup 1)^{k-1}$.

(4 pts) **7** Give an example of a nonregular language L such that L^* is regular. Prove that your example works.

MIDTERM SOLUTIONS

1 

2 

3 **a.** $\varepsilon \cup 1\Sigma^*$;

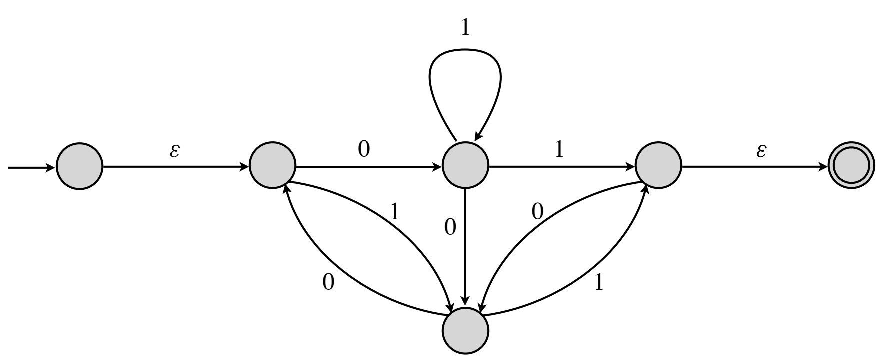
b. $\Sigma^*(00\Sigma^*11 \cup 11\Sigma^*00)\Sigma^*$;

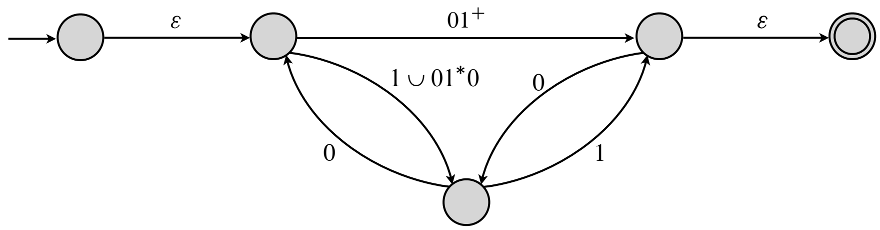
c. $(\Sigma^*1\Sigma^*)^7$.

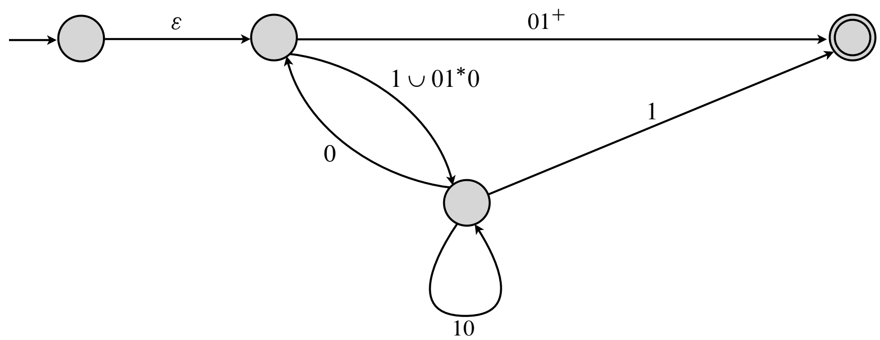
4 **a.** Let L be the language in question. For the sake of contradiction, assume that L is regular. Consider the string $w = 1^p01^p \in L$, where p is a pumping length of L. By the pumping lemma, $w = xyz$ for some x, y, z such that y is nonempty, $|xy| \leqslant p$, and $xy^iz \in L$ for all $i = 0, 1, 2, 3, \ldots$. This contradicts $xz \notin L$. Hence, L is nonregular.

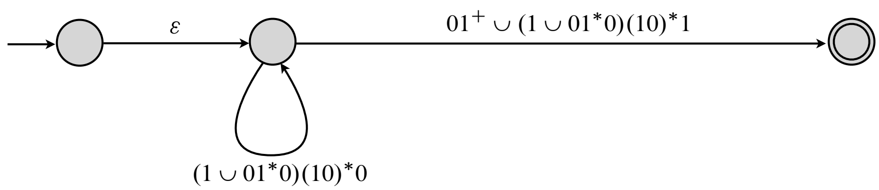
b. The language is regular because it is given by the regular expression $\Sigma^*(000 \cup 010 \cup 101 \cup 111)\Sigma^*$.

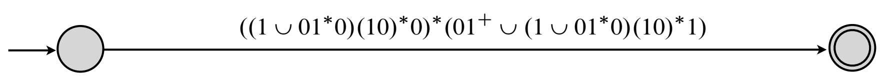
5 One possible sequence of steps is as follows:











The final regular expression is $((1 \cup 01^*0)(10)^*0)^*(01^+ \cup (1 \cup 01^*0)(10)^*1)$.

6 We simply need to keep track of the last k symbols seen, and accept if the kth last symbol is a 0. This is accomplished by the automaton $(\{0,1\}^k, \{0,1\}, \delta, 1^k, 0\{0,1\}^{k-1})$, where $\delta(u_1u_2\ldots u_k, \sigma) = u_2\ldots u_k\sigma$.

7 Consider the language $L = \{w : |w| = n^2 \text{ for some } n \in \mathbb{N}\}$, which was shown in class to be nonregular. Since $0, 1 \in L$, we conclude that $L^* = (0 \cup 1)^*$, which is regular.