CS 181 EXAM #1 **NAME** ______________________________

FALL 2015 **UCLA ID** ______________________________

You have 90 minutes to complete this exam. You may assume without proof any statement proved in class.

1 Prove that the following languages are regular:

(2 pts) **a.** binary strings that contain neither 010101 nor 000111 as substrings;

(2 pts) **b.** nonempty binary strings in which the first and last symbols are different;

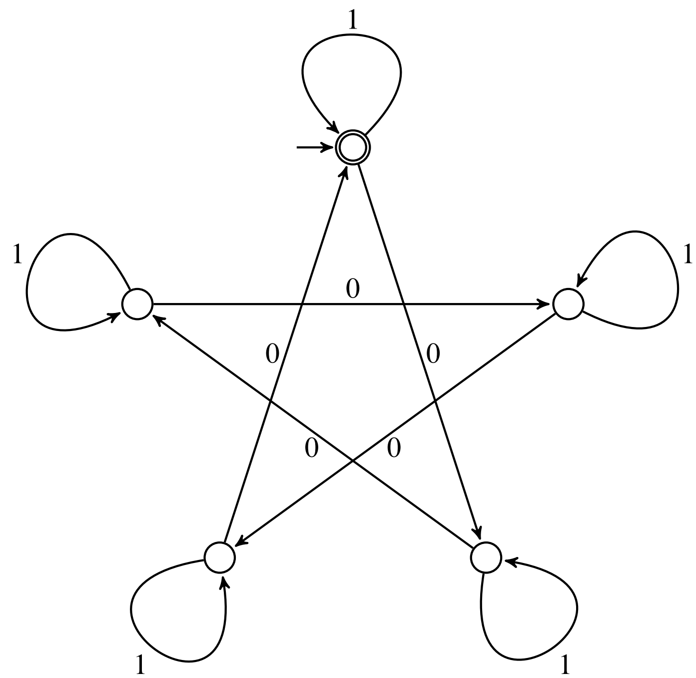
(2 pts) **c.** strings over the decimal alphabet $\{0, 1, 2, \ldots, 9\}$ with symbols in sorted order.

Solution. The languages in (a)–(c) are given by

$$\overline{\{0,1\}^*\{010101\}\{0,1\}^* \cup \{0,1\}^*\{000111\}\{0,1\}^*},$$
$$\{0\}\{0,1\}^*\{1\} \cup \{1\}\{0,1\}^*\{0\},$$
$$\{0\}^*\{1\}^* \cdots \{9\}^*,$$

respectively. Thus, they are obtained by applying the operations of Kleene star, concatenation, union, and complement to the regular languages $\{0\}, \{1\}, \ldots, \{9\}, \{0,1\}, \{010101\}, \{000111\}$. Since regular languages are closed under these operations, the resulting languages are regular as well.

(3 pts) **2** Give a simple verbal description of the language recognized by the following DFA.



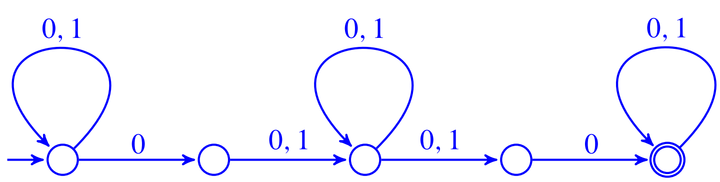
Solution. Binary strings in which the number of 0s is a multiple of 5.

3 Draw NFAs for the following languages, taking full advantage of nondeterminism:

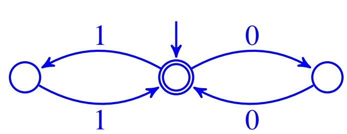
(2 pts) **a.** binary strings that contain a pair of 0s separated by two or more symbols;

(2 pts) **b.** even-length binary strings in which symbols come in pairs, e.g., $\varepsilon, 00, 0011, 1111$, but not $0, 01, 001101$.

a.



b.



(3 pts) **4** Let L be a regular language over a given alphabet Σ. Define L' to be the set of all strings in L whose length is *not* a multiple of 2015. Prove that L' is regular.

Solution. The language, call it A, of strings whose length is not a multiple of 2015 is regular, with DFA $(\{0, 1, 2, \ldots, 2014\}, \Sigma, \delta, 0, \{1, 2, \ldots, 2014\})$ where $\delta(q, \sigma) = (q + 1) \bmod 2015$. Since $L' = L \cap A$ and regular languages are closed under intersection, it follows that L' is regular.

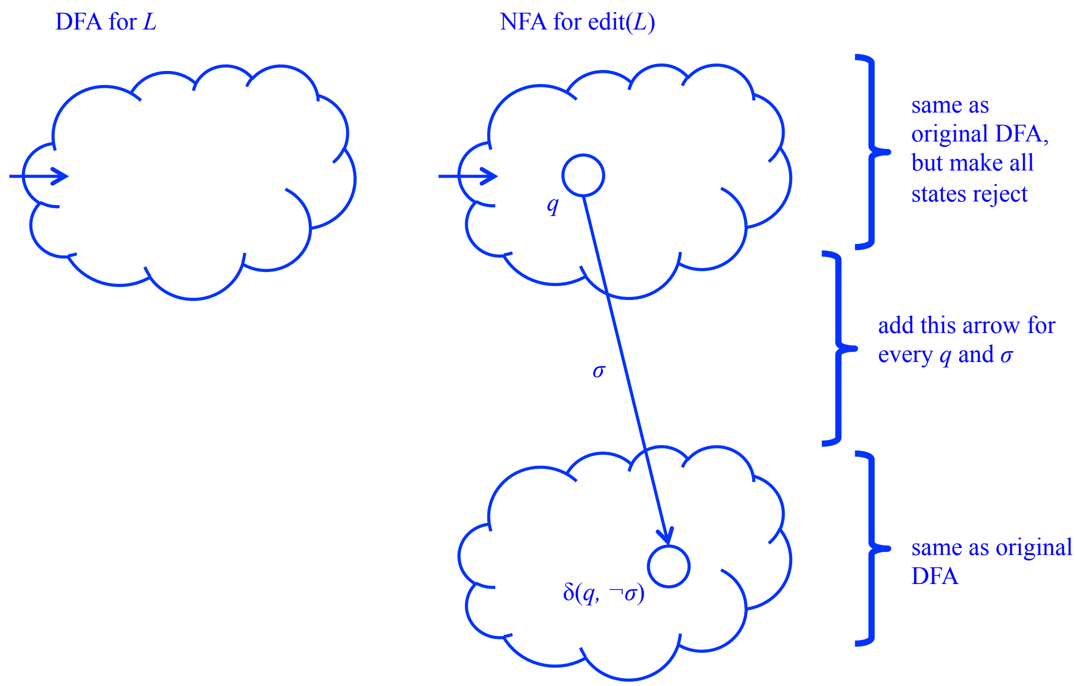
(3 pts) **5** Consider the language L whose strings are binary encodings (with leading zeroes ignored) of multiples of 17. Thus, L contains $\varepsilon, 0, 00, 10001, 000010001, 110011$ but not $10, 11, 0001$. Prove that L is regular.

Solution. We will give a DFA for L. The idea is to convert the input string on the fly to a decimal number and check to see if that number is a multiple of 17. A direct implementation of this idea results in the "infinite" automaton $(\{0, 1, 2, \ldots\}, \{0, 1\}, \delta, 0, \{0, 17, 34, \ldots\})$, where $\delta(q, \sigma) = 2q + \sigma$. The only flaw in this construction is that it uses infinitely many states.

To get an actual DFA for L, recall that we only care about the *remainder* of the decimal number modulo 17. Therefore, we can perform all intermediate computations modulo 17. This corresponds to the automaton $(\{0, 1, 2, \ldots, 16\}, \{0, 1\}, \delta, 0, \{0\})$, where $\delta(q, \sigma) = (2q + \sigma) \bmod 17$.

(3 pts) **6** For a language L over the binary alphabet, let edit(L) denote the set of strings that can be obtained from a string in L by flipping exactly one bit. Prove that edit(L) is regular whenever L is regular.

Solution. A DFA for L can be transformed into an NFA for edit(L) as follows, where δ refers to the DFA's transition function.



(3 pts) **7** Let L be the language of strings over alphabet $\{a, b, c\}$ in which the number of a's, the number of b's, and the number of c's are either all even, or precisely one of them is even. Give a DFA for L with at most four states.

Solution.