**CEE 110 Probability and Statistics for Engineers and Scientists**
**Practice Midterm**

Problem 1: Short answer:

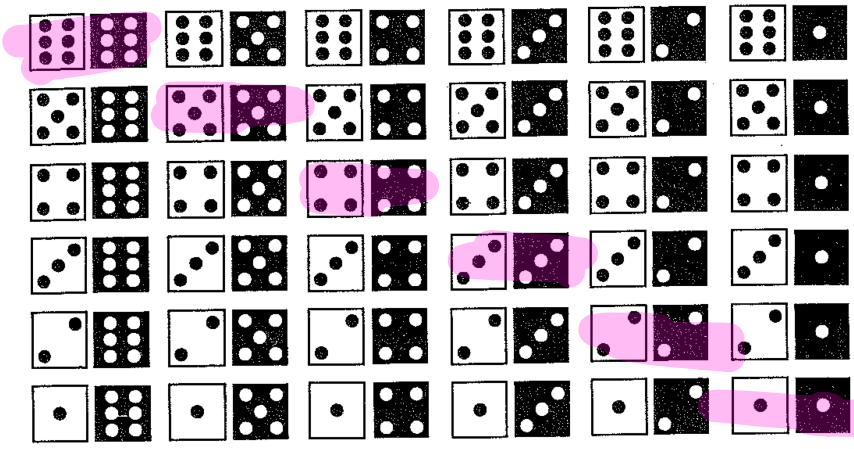
A)
More and more fuses are selected until three defective fuses are found. The number of fuses tested up to and including the third defective fuse is recorded.

$S_X$ = (3, ... ∞)

B)
Four students from this class are selected to each rate a new movie on a scale of 1-5 stars. The sample space contains how many possible outcomes?

$5 \cdot 5 \cdot 5 \cdot 5 = 625$

C)
What are the odds of rolling doubles of any digit (double 1's, 2's, 3's, 4's, 5's, or 6's)?



$P(A) = \frac{N(E)}{N}$

$= \frac{6}{36}$

$= \frac{1}{6}$

D) What is the probability of rolling at least one "1" in three rolls of a die?

$1 - \left(\frac{5}{6}\right)^3$

$\approx .5787$

Problem 2:

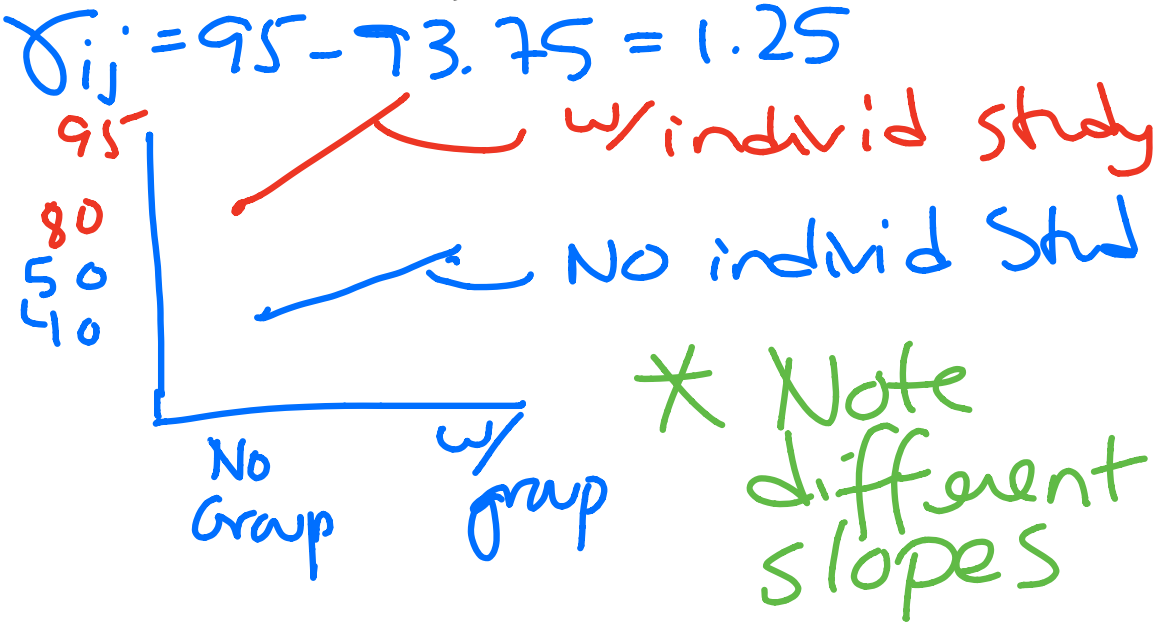
Both study groups and individual studying can be important components in performing successfully in a class. Consider the following data, and please calculate main effects, identify whether the factors are additive or interaction, and please quantify any interaction, if any.

| | No individual studying | With individual studying | Row averages | Main row effects |
|---|---|---|---|---|
| No Study Group | 40 | 80 | 60 | $\alpha_1 = -6.25$ |
| With Study Group | 50 | 95 | 72.5 | $\alpha_2 = +6.25$ |
| Column averages | 45 | 87.5 | 66.25 | |
| Main column effects | $\beta_1 = -21.25$ | $\beta_2 = +21.25$ | | |

$\mu_{2,2} = 66.25 + 6.25 + 21.25 = 93.75$

Please draw the graph used to visualize interaction (one factor along x axis and lines drawn for each case for the other factor).

$\gamma_{ij} = 95 - 73.75 = 1.25$



Problem 3:

A leadership committee of four students is to be formed from twelve students.

a) How many different committees are possible?
b) The twelve students are five biology majors, four chemists, and three physicists. How many committees would have two biology majors, one chem major, and one physics major?
c) If all committees were equally likely to form, what is the likelihood of a committee as outlined in part b.

a) $\binom{12}{4} = \frac{12!}{4!\,8!} = \frac{12\times11\times10\times9}{4\times3\times2} = \frac{11880}{24}$

$= 495$

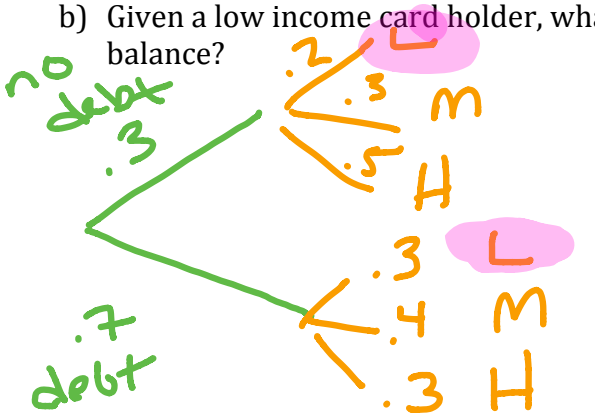
b) $\binom{5}{2}\binom{4}{1}\binom{3}{1} = 120$

c) $\frac{120}{495} = 0.24$

Problem 4:

Thirty percent of all credit card holders carry no monthly balance while 70% do. Of those carrying a balance, 30% could be categorized as "low income", 40% could be categorized as "middle income" and 30% could be categorized as "high income." Of those not holding a balance, 20%, 30%, and 50% have incomes in the low, middle and high categories, respectively.

a) What is the probability that a randomly chosen card holder has low income?
b) Given a low income card holder, what is the probability that s/he carries a balance?



a) $(.3)(.2) + (.7)(.3) = .27$

b) $\frac{.3 \times .7}{.27} = 0.78$

Problem 5:

A simple random sample of three items is selected from a shipment of 20 items, of which four are defective. Let $X$ be the number of defective items in the sample.

a) Find the PMF of $X$.
b) Find the mean value and variance of $X$.

Hint: first find the total number of arrangements that could be made with 3 items taken from 20.

$\binom{20}{3} = \frac{20!}{3!17!} = 1140$

$P(0) = \frac{\binom{16}{3}}{\binom{20}{3}} = \frac{16 \times 15 \times 14}{6} = \frac{560}{1140} = .49$

$P(1) = \frac{\binom{4}{1}\binom{16}{2}}{\binom{20}{3}} = 0.4211$

$P(2) = \frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}} = 0.084$

$P(3) = \frac{\binom{4}{3}}{\binom{20}{3}} = .0035$

$E(X) = \sum x \cdot p(x) = 0 \cdot .49 + 1 \cdot .42 + 2 \cdot .083 + 3 \cdot .0035$

$E(X) = .6$

Problem 6:

A habitat contains 30 owls, of which 12 are fitted with a radio collar.
5 owls are captured at a later time, and let X be the number of collared owls in the group (assuming the collar does not affect the probability of being captured).

a) Is *X*:
   a. Binomial
   b. Hypergeometric
   c. Negative binomial
   d. Poisson
b) What is the probability that $X = 2$?
c) What is the expected value of *X*?

Prob 5 cont'

$\sigma_x^2 = E(X^2) - [E(X)]^2$

$= 0 \cdot .49 + 1 \cdot .421 + 2^2(.084) + 3^2(.0035) - .6^2$

$\sigma_x^2 = .429$

6 a) hypergeometric

b) $P(X=2) = \dfrac{\binom{12}{2}\binom{18}{3}}{\binom{30}{5}} =$

c) $5 * \frac{12}{30}$ $\quad \mu_x = n\frac{M}{N}$

Problem 7:

The resistance for resistors of a certain type is a random variable X having a normal distribution with mean 9 ohms and standard deviation 0.4 ohms. A resistor is acceptable if its resistance is between 8.6 and 9.8 ohms.

a) What is the probability that a randomly chosen resistor will be acceptable?
b) What is the probability that of four randomly and independently chosen resistors, two are acceptable?

$\sigma = .4$ ohms

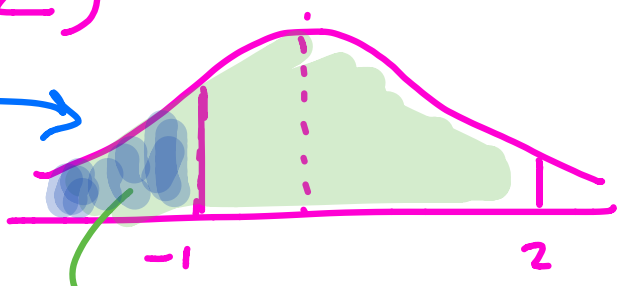
$8.6 \le X \le 9.8$

$\frac{8.6-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{9.8-\mu}{\sigma}$ (Z)

$\frac{8.6-9}{.4} \le Z \le \frac{9.8-9}{.4}$

$P(-1 \le Z \le 2)$

To subtract off → find $Z = 1$




0.977 from table


$\Phi(2) - (1 - \Phi(1))$

.84

.16

$P = 0.82$

b) $p = 0.82 \qquad n = 4$

$$\binom{4}{2}(.82)^2(.18)^2 = 0.13$$