C&EE 108 Introduction to Mechanics of Deformable Solids

Midterm Exam (May 6, 2016)

Name: solution **UID:**

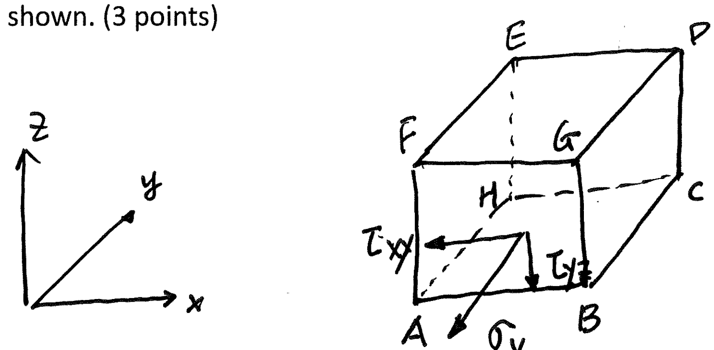
Problem 1 (10 Points).

(a) In order to use "Principal of Superposition", what conditions must be satisfied? (2 points)

① linear material property

② small deformations

(b) Please draw and mark the positive stress components on plane ABGF, given the coordinate system as shown. (3 points)



ABGF is a negative plane

(c) What is the normal stress in an axially loaded bar if P=10 kN and A=500mm²? (1 point)

A) 0.02 kPa B) 20 Pa C) 20 kPa D) 200 N/mm² E) 20 MPa

(d) The Poisson's ratio, v of common engineering materials lies in the range (1 point):

A) $0 \le v \le 1$ B) $0 \le v \le 0.5$ C) $-1 \le v \le 1$ D) $-0.5 \le v \le 0.5$

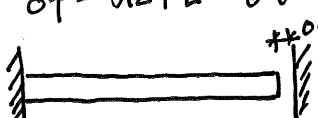
(e) The stress distributions at different cross sections are different. However, at locations far enough away from the support and the applied load, the stress distribution becomes uniform. This is due to (1 point):

A) Principle of superposition B) Inelastic property C) Poisson's effect D) Saint Venant's Principle

(f) A 40 ft long steel beam is fixed at one end and the other end is free yet located 0.3in. away from the wall at the room temperature of $30F^o$. The steel has a thermal coefficient of $\alpha = 6.6\times10^{-6}/F^o$. What kind stress will the beam experience if the surrounding temperature is increased to $120F^o$? (2 points)

A) Tension stress B) Compression stress C) No Stress D) Can not tell

$\delta_T = \alpha \Delta T L = 6.6\times10^{-6} \times (120-30) \times 40 \times 12 = 0.285 \text{ in.} < 0.3 \text{ in.}$

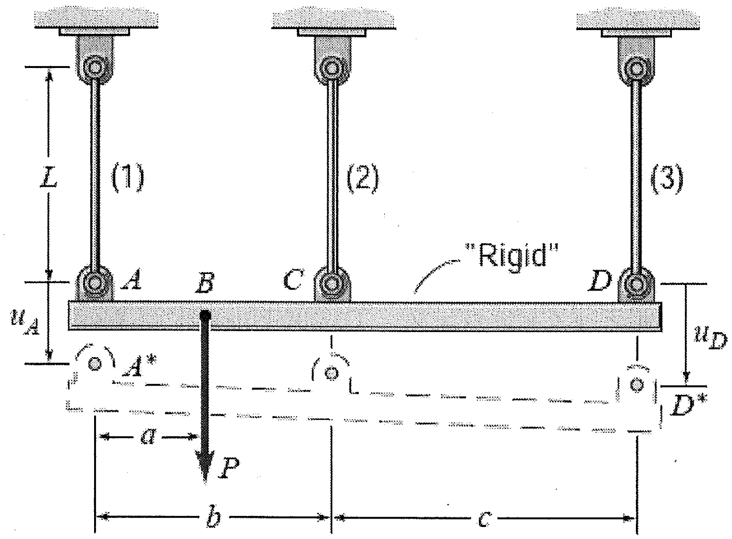


Problem 2 (30 Points).

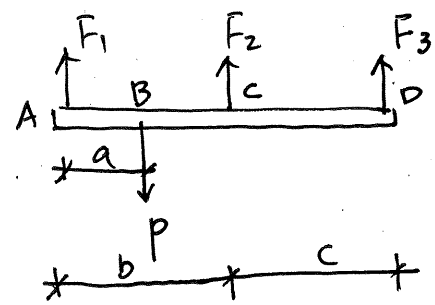
A rigid beam AD is supported by three vertical rods that are attached to the beam at points A, C and D. When originally assembled and P=0, the rods are all stress free and beam AD is horizontal. Under the load and parameters give as: $A = 1.0in^2$, $L = 60in.$, $E = 30\times10^3 ksi$, $a = 20in.$, $b = 40in.$, $c = 60in.$ and $P = 10kips$.

(a) Solve for the axial forces F_1, F_2 and F_3 in the support rods (Hint: let F_2 be the redundant force, use the moment equilibrium of point A and D as well as the displacement compatibility condition to solve the problem) (20 points)

(b) Determine the vertical displacement u_A and u_D (10 points)



(a)



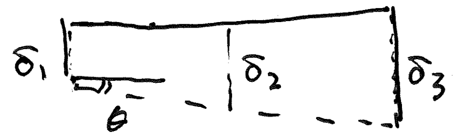
Equilibrium :

$$\Sigma M_A = 0 \quad bF_2 + F_3(b+c) = Pa \quad \text{---} ①$$

$$\Sigma M_D = 0 \quad F_1(b+c) + F_2 \cdot c = P(b+c-a) \text{-} ②$$

Load-displacement :

$$\delta_1 = \frac{F_1 L}{EA} \qquad \delta_2 = \frac{F_2 L}{EA} \qquad \delta_3 = \frac{F_3 L}{EA}$$



Compatibility :

$$\delta_2 = \delta_1 + \frac{\delta_3 - \delta_1}{b+c} \cdot b$$

$$\Rightarrow \delta_1 c + \delta_3 b - \delta_2 (b+c) = 0$$

$$\Rightarrow \frac{Lc}{EA} F_1 + \frac{Lb}{EA} F_3 - \frac{L(b+c)}{EA} F_2 = 0$$

$$\Rightarrow c F_1 - (b+c) F_2 + b F_3 = 0 \quad \cdots \quad (3)$$

Solve (1) to (3) simultaneously

From (1) $\Rightarrow$ $F_3 = \frac{a}{b+c} P - \frac{b}{b+c} F_2 \quad \cdots \quad (1a)$

From (2) $\Rightarrow$ $F_1 = \frac{b+c-a}{b+c} P - \frac{c}{b+c} F_2 \quad \cdots \quad (2a)$

Substitute (1a) and (2a) into (3) and solve for F_2

$$F_2 = \frac{ab + bc - ac + c^2}{2(b^2 + c^2 + bc)} P = \frac{20 \times 40 + 40 \times 60 - 20 \times 60 + 60^2}{2(40^2 + 60^2 + 40 \times 60)} \times 10$$

$$\boxed{F_2 = 3.6842 \text{ kips}} \Rightarrow \boxed{F_1 = 5.7895 \text{ kips}} \quad \boxed{F_3 = 0.5263 \text{ kips}}$$

(b) $$u_D = \frac{F_3 L}{EA} = \frac{0.5263 \text{ kips} \times 60 \text{ in.}}{30 \times 10^3 \text{ kips/in}^2 \times 1 \text{ in}^2} = \underline{\underline{1.053 \times 10^{-3} \text{ in}}}.$$

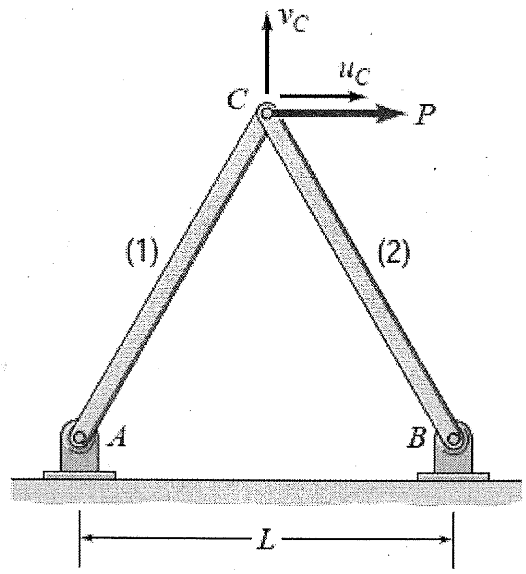
$$u_A = \frac{F_1 L}{EA} = \frac{5.7895 \text{ kips} \times 60 \text{ in.}}{30 \times 10^3 \text{ kips/in}^2 \times 1 \text{ in}^2} = \underline{\underline{1.158 \times 10^{-2} \text{ in}}}.$$

Problem 3 (30 Points).

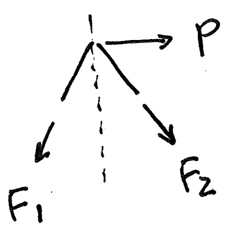
Each of the two bars of the planar truss in the figure below has length L, and they are made of a material with modulus of elasticity E. If $A_1 = A$ and $A_2 = 2A$, and the horizontal load P is applied at joint C.

(a) Determine the stresses σ_1 and σ_2 in the two bars. (**15** points).

(b) Determine the horizontal displacement, u_c and vertical displacement, v_c of the pin joint at C (**15** points).



(a) Equilibrium at C



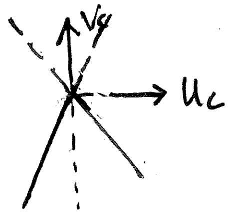
$\Sigma F_x = 0 \quad -F_1 \sin 30^\circ + F_2 \sin 30^\circ + P = 0$

$\Sigma F_y = 0 \quad F_1 \cos 30^\circ + F_2 \cos 30^\circ = 0 \Rightarrow F_2 = -F_1$

$\Rightarrow F_1 = P \quad \& \quad F_2 = -P$

$$\sigma_1 = \frac{F_1}{A_1} = \frac{P}{A} \quad \text{(tension)}$$

$$\sigma_2 = \frac{F_2}{A_2} = -\frac{P}{2A} \quad \text{(compression)}$$

(b) 

$$\delta_1 = u_c \sin 30^\circ + v_c \cos 30^\circ \quad \text{(elongation)}$$

$$\delta_2 = u_c \sin 30^\circ - v_c \cos 30^\circ \quad \text{(shortening)}$$

and $\delta_1 = \dfrac{F_1 L}{EA_1} = \dfrac{PL}{EA}$, $\delta_2 = \dfrac{F_2 L}{EA_2} = \dfrac{PL}{2EA}$

Therefore,

$$u_c (\overbrace{2\sin 30^\circ}^{2\times 0.5=1}) = \delta_1 + \delta_2 = \frac{3PL}{2EA}$$

$$\Rightarrow \boxed{u_c = \frac{3PL}{2EA}}$$

$$v_c (2\cos 30^\circ) = \delta_1 - \delta_2 = \frac{PL}{2EA}$$

$$\Rightarrow v_c = \frac{PL}{2\sqrt{3}EA} = \boxed{\frac{\sqrt{3}PL}{6EA} = v_c}$$

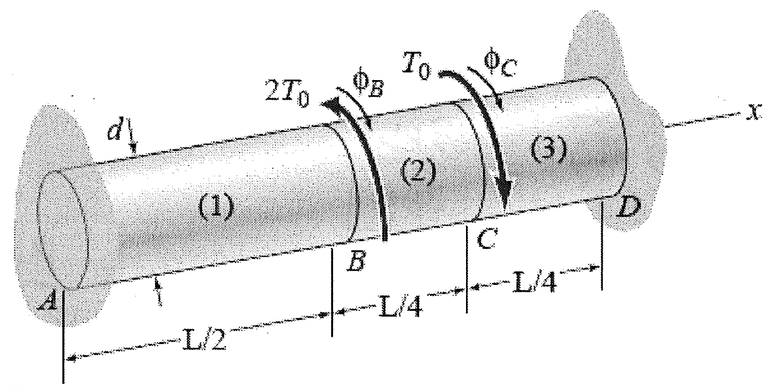
Problem 4 (30 Points).

A uniform shaft with fixed ends at A and D is subjected to external torques of magnitude T_0 and $2T_0$, as shown below. The diameter of the shaft is d, and its shear modulus is G.

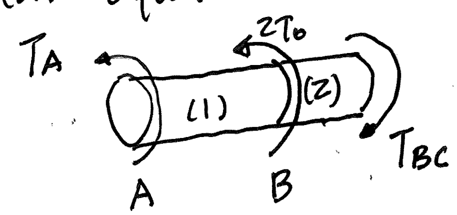
(a) Determine the internal torques T_1, T_2 and T_3 in each segment respectively. (**10** points)

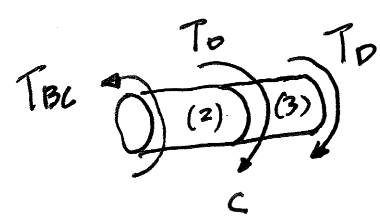
(b) Determine the maximum shear stress in each segment. (**10** points)

(c) Determine the angle of rotation of the shaft at joint B, ϕ_B. (**10** points)



(a) Equilibrium





$T_A + 2T_0 = T_{BC}$

$\Rightarrow T_A = T_{BC} - 2T_0$

$T_{BC} - T_0 - T_D = 0 \Rightarrow T_D = T_{BC} - T_0$

Load-displacement:

$$\phi_{AB} = \frac{T_A L_1}{GJ} \qquad \phi_{BC} = \frac{T_{BC} L_2}{GJ} \qquad \phi_{CD} = \frac{T_D L_3}{GJ}$$

Compatibility $\phi_{AB} + \phi_{BC} + \phi_{CD} = 0$

$$\frac{(T_{BC}-2T_0)(\frac{L}{2})}{GJ} + \frac{T_{BC}(L/4)}{GJ} + \frac{T_{BC}-T_0(L/4)}{GJ} = 0$$

$$\Rightarrow \quad 2(T_{BC}-2T_0) + T_{BC} + T_{BC}-T_0 = 0 \quad \Rightarrow T_{BC} = \frac{5T_0}{4} = 1.25\,T_0$$

Therefore, $T_A = T_{BC} - 2T_0 = -0.75\,T_0$

$T_D = T_{BC} - T_0 = 0.25\,T_0$

$$\Rightarrow \left\{ \boxed{\begin{aligned} T_1 &= -0.75\,T_0 \\ T_2 &= 1.25\,T_0 \\ T_3 &= 0.25\,T_0 \end{aligned}} \right.$$

(b) $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$

$$(\tau_{max})_1 = \frac{T_1(\frac{d}{2})}{J} = \frac{0.75\,T_0(\frac{d}{2})}{\frac{\pi d^4}{32}} = \boxed{\frac{12T_0}{\pi d^3} = (\tau_{max})_1}$$

$$(\tau_{max})_2 = \frac{T_2(\frac{d}{2})}{J} = \frac{1.25\,T_0(\frac{d}{2})}{\frac{\pi d^4}{32}} = \boxed{\frac{20T_0}{\pi d^3} = (\tau_{max})_2}$$

$$(\tau_{max})_3 = \frac{T_3(\frac{d}{2})}{J} = \frac{0.25\,T_0(\frac{d}{2})}{\frac{\pi d^4}{32}} = \boxed{\frac{4T_0}{\pi d^3} = (\tau_{max})_3}$$

(c) $$\phi_B = \int_0^{L/2} \frac{T_1}{GJ}\,dx = \frac{T_1(\frac{L}{2})}{G\,\frac{\pi d^4}{32}} = \frac{-0.75\,T_0 L \times 16}{G\pi d^4} = -\frac{12\,T_0 L}{\pi G d^4}$$

$$\Rightarrow \boxed{\phi_B = -\frac{12T_0L}{\pi G d^4}}$$