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CEE101: Statics and Dynamics - Exam 1: Statics – November 8, 2011
Instructor: Gaurav Sant, Teaching Assistant: Shih-Po Lin

Exam Policies: The exam consists of four equally weighed questions (25 points each) and is designed to be a 1 hour exam (100 points total). However, 2 hours are made available. The exam is closed notes. As as such; no reference materials (textbooks, note-cards etc.) are permitted. Calculators are permitted. Units are critical, wrong units imply a wrong answer or incomplete understanding. Errors of this nature will be penalized accordingly. Math errors will be penalized once for multi-part problems. Please show work and free-body diagrams. This will ensure partial credit even in case of incorrect solutions. Please indicate if forces are tensile or compressive. Please maintain standard principles of academic integrity. Failure to do so will result in rigorous administrative action and a failing grade in the exam and course. In case of any questions please consult with the proctoring staff immediately.

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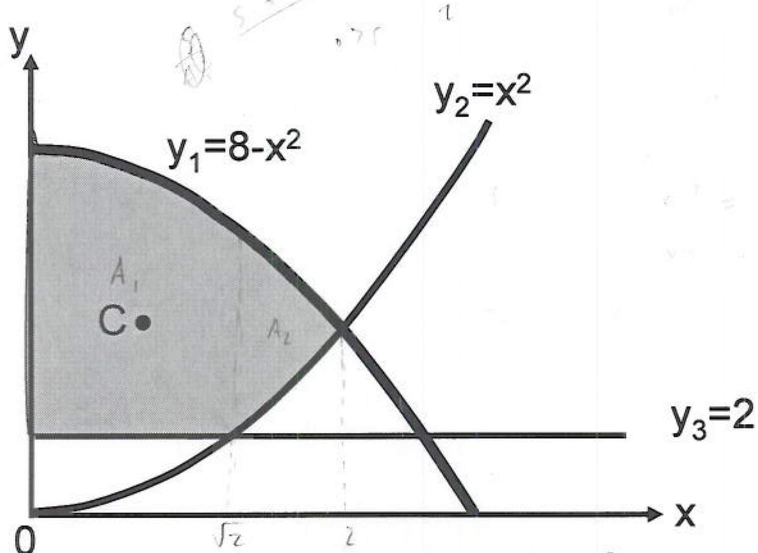
Problem 1: Calculate the area and centroid of the shaded region shown in the figure below.



$A_1 = \int_0^{\sqrt{2}} 8 - x^2 - 2 \, dx$

$= \int_0^{\sqrt{2}} 6 - x^2 \, dx = 6x - \frac{x^3}{3} \Big]_0^{\sqrt{2}}$

$= 6\sqrt{2} - \frac{2\sqrt{2}}{3}$

$= \frac{16\sqrt{2}}{3} \approx 7.54$

$A_2 = \int_{\sqrt{2}}^{2} 8 - x^2 - x^2 \, dx$

$= \int_{\sqrt{2}}^{2} 8 - 2x^2 \, dx$

$= 8x - \frac{2x^3}{3} \Big]_{\sqrt{2}}^{2}$

$= 8(2) - \frac{2}{3}(2)^3 - 8(\sqrt{2}) + \frac{2}{3}(\sqrt{2})^3$

$= 16 - \frac{16}{3} - 8\sqrt{2} + \frac{4\sqrt{2}}{3} = \frac{32}{3} - \frac{24\sqrt{2}}{3} + \frac{4\sqrt{2}}{3}$

$= \frac{32 - 20\sqrt{2}}{3}$

$A_{tot} = A_1 + A_2 = \frac{16\sqrt{2}}{3} + \frac{32 - 20\sqrt{2}}{3}$

$A_{tot} = \frac{32 - 4\sqrt{2}}{3} \approx 8.78 \text{ units}^2$

$(8-x^2)(8-x^2)$
$= 64 - 16x^2 + x^4$

$X_{c(A_1)} = \frac{\int_0^{\sqrt{2}}\int_2^{8-x^2} x \, dy\, dx}{A_{tot}}$ $\quad \int_2^{8-x^2} dy = y\Big]_2^{8-x^2} = 6 - x^2$

$\int_0^{\sqrt{2}} 6x - x^3 \, dx = 3x^2 - \frac{x^4}{4}\Big]_0^{\sqrt{2}} = 6 - 1 = 5$

$\frac{5}{A_{tot}} = .663$ units

$Y_c(A_1) = \frac{\int_0^{\sqrt{2}}\int_2^{8-x^2} y \, dy\, dx}{A_{tot}}$ $\quad \int_2^{8-x^2} y\, dy = \frac{y^2}{2}\Big]_2^{8-x^2}$

$= \frac{(8-x^2)^2}{2} - 2 = 30 - 8x^2 + \frac{x^4}{2}$

$\int_0^{\sqrt{2}} 30 - 8x^2 + \frac{x^4}{2} = 30x - \frac{8}{3}x^3 + \frac{x^5}{10}\Big]_0^{\sqrt{2}}$

$= \frac{376\sqrt{2}}{15}$ $\quad \frac{\frac{376\sqrt{2}}{15}}{A_{tot}} = 4.7$ units

$$X_c(A_2) = \frac{\int_{\sqrt{2}}^{2}\int_{x^2}^{8-x^2} x\,dy\,dx}{A_{2tot}} \qquad \int_{\sqrt{2}}^{2} 8x - 2x^3 = 4x^2 - \frac{x^4}{2}\Big]_{\sqrt{2}}^{2}$$

$$= 8 - 6 = 2$$

$$\frac{2}{A_{2tot}} = 1.61 \text{ units}$$

$$Y_c(A_2) = \frac{\int_{\sqrt{2}}^{2}\int_{x^2}^{8-x^2} y\,dy\,dx}{A_{2tot}}$$

$$\int_{x^2}^{8-x^2} y\,dy = \frac{y^2}{2}\Big]_{x^2}^{8-x^2} = \frac{(8-x^2)^2}{2} - \frac{(x^2)^2}{2}$$

$$= 32 - 8x^2 + \cancel{\frac{x^4}{2} - \frac{x^4}{2}}$$

$$\int_{\sqrt{2}}^{2} 32 - 8x^2 = 32x - \frac{8x^3}{3}\Big]_{\sqrt{2}}^{2} = 64 - \frac{8}{3}(2^3) - 32\sqrt{2} + \frac{8}{3}(\sqrt{2})^3$$

$$= 4.95 \text{ units}$$

$$\frac{4.95}{A_{2tot}} = 4$$

$$X_c(A_{tot}) = \frac{(.663)(7.54) + (1.61)\left(\frac{32-20\sqrt{2}}{3}\right)}{8.78} = .797 \text{ units}$$

$$Y_c(A_{tot}) = \frac{(4.7)(7.54) + (4)\left(\frac{32-20\sqrt{2}}{3}\right)}{8.78} = 4.60 \text{ units}$$

$$\boxed{(X_c, Y_c) = (.797 \text{ units}, 4.60 \text{ units})}$$

$$\boxed{A_{tot} = 8.78 \text{ units}^2}$$

Problem 2: A beam is clamped (fixed) to a wall at end A. Determine the resistance forces and moment on the clamped end. If the allowable stress on the beam is 3×10^8 N/m^2 is the beam safe under the loading conditions shown in the following figure? Note: the maximum stress on the section can be expressed as: $\sigma_{max} = \dfrac{Mc}{I}$, where M is the bending moment acting on the section, c = 0.25 m is the half depth of the beam, and I is the second moment of area. For the uniform beam shown below, I = 0.008 m^4

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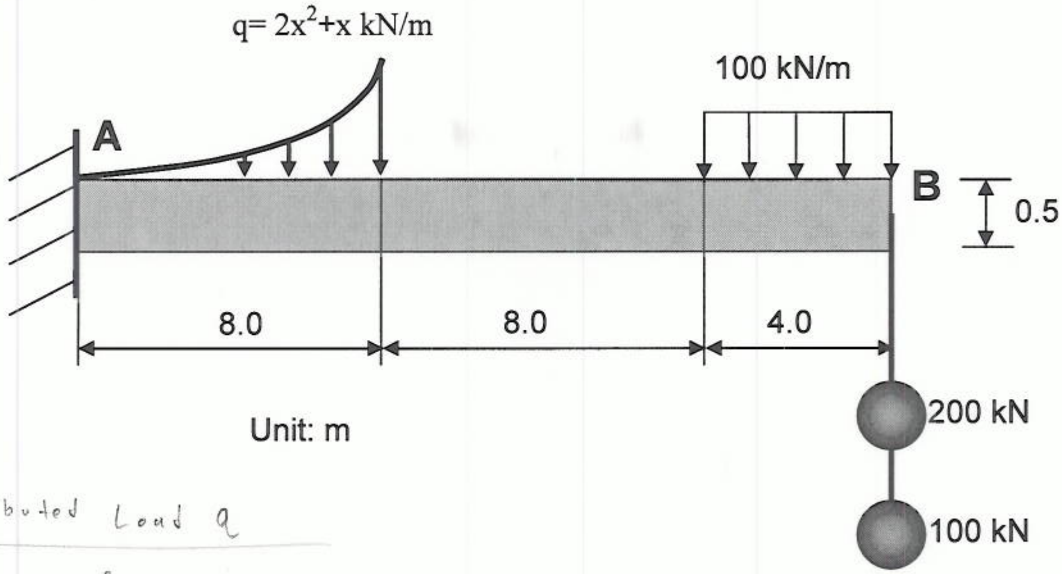

Distributed Load q

Total load $= \int_0^8 2x^2 + x \, dx$

$= \left[\frac{2}{3}x^3 + \frac{x^2}{2}\right]_0^8 = \frac{1120}{3}$ kN

Equiv. Load spot:

$\dfrac{\int_0^8 2x^3 + x^2}{1120/3} = \dfrac{\left[\frac{2x^4}{4} + \frac{x^3}{3}\right]_0^8}{1120/3}$

$= \frac{208}{35}$ m

FBD

$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y - \frac{1120}{3} - 400 - 300 = 0$

$A_y = \frac{3220}{3}$ kN

$\Sigma M_A = M_{Az} - \frac{1120}{3}\left(\frac{208}{35}\right) - 400(18) - 300(20)$

$M_{Az} = 15420$ kN·m

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$$\boxed{\begin{aligned} A_x &= 0 \\ A_y &= \frac{3220}{3}\ kN = 1073\ kN \\ M_{A_z} &= 15420\ kN\cdot m \end{aligned}}$$

Rounding?

Max stress will be @ base

$$\sigma_{max} = \frac{(15420 \times 10^3\ N\cdot m)(0.25\ m)}{(.008\ m^4)} = 4.82 \times 10^8\ N/m^2$$

So the beam is not safe because
$4.82 \times 10^8\ N/m^2 > 3 \times 10^8\ N/m^2$

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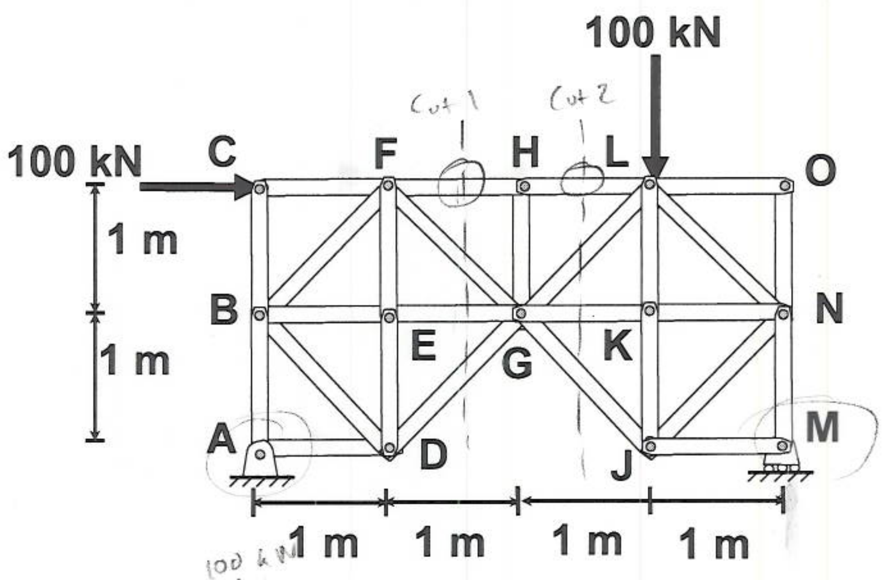
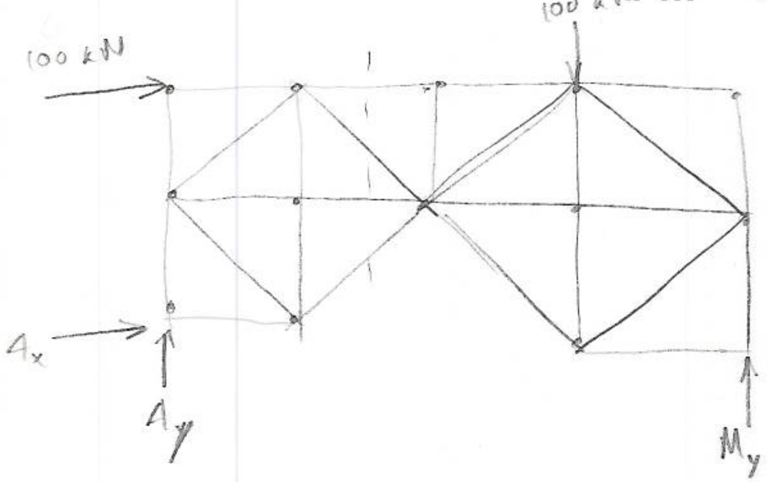
Problem 3: Determine the reaction forces at joints A and M and the internal forces acting on the members FH and HL of the complex truss system shown below.





Global

$\Sigma F_x = A_x + 100 = 0$

$A_x = -100\ kN$

$\Sigma F_y = A_y + M_y - 100\ kN = 0$

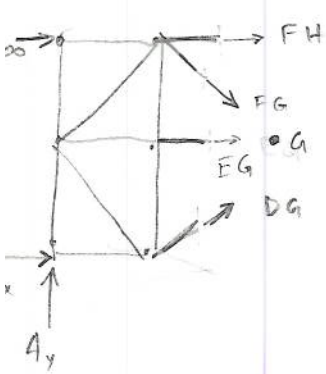
$A_y + M_y = 100$

$\Sigma M_A = M_y(4) - 100(3) - 100(2) = 0$

$M_y = 125\ kN$

$A_y = -25\ kN$

Cut 1



$\Sigma M_{G_z} = -A_y(2) + A_x(1) - 100(1) - FH(1) = 0$

$FH = A_x - 2A_y - 100 = -150\ kN$

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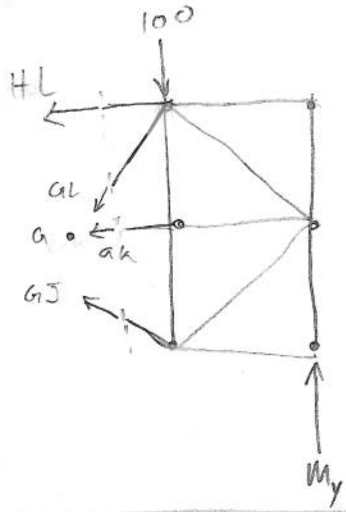
Cut 2



$$\Sigma M_{Gz} = M_y(2) - 100(1) + HL(1) = 0$$

$$HL = 100 - 2M_y$$

$$= 100 - 2(125)$$

$$= -150 \text{ kN}$$

Forces @A

$A_x = -100$ kN

$A_y = -25$ kN

Forces @ M

$M_y = 125$ kN

FH

$F_{FH} = -150$ kN (compression)

HL

$F_{HL} = -150$ kN (compression)

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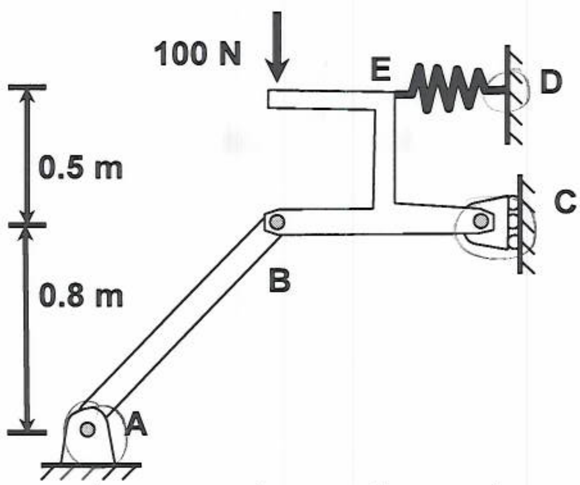
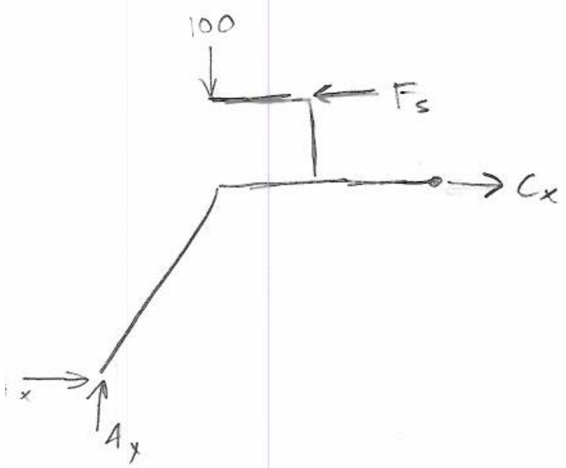
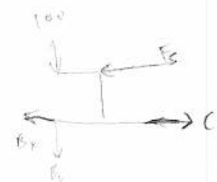
Problem 4: Consider the frame shown in the following figure. Determine the reaction forces acting at A, C and D and the deformation of spring DE with a spring constant k_S = 50000 N/m ($F=k_S*d$, where F is the spring force and d is the deformation).







Global

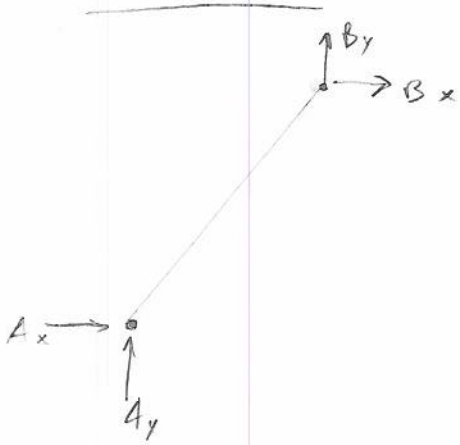
$\Sigma F_x = A_x + C_x - F_s = 0$

$\Sigma F_y = A_y - 100 = 0$

$A_y = 100$ N

$\Sigma M_A = F_s(1.3) - 100(.6) - C_x(.8) = 0$

Member AB



$\Sigma F_y = A_y + B_y = 0$

$B_y = -100$ N

$B_x = -75$ N

$\Sigma M_B = A_x(.8) - A_y(.6) = 0$

$A_x = \frac{A_y(.6)}{(.8)} = 75$ N

$A_x = 75$ N

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$$C_x - F_s + 75 = 0$$

$$C_x = F_s - 75$$

$$1.3 F_s - .8 C_x = 60$$

$$1.3 F_s - .8(F_s - 75) = 60$$

$$.5 F_s + \cancel{60} = \cancel{60}$$

$$F_s = 0 \quad \text{so} \quad D_x = 0$$

$$C_x = -75\ N$$

$F_s \rightarrow$ MMM $\leftarrow D_x$

$$D_x = F_s = 0$$

$$F_s = k_s d$$

$$0 = (50\,000\ \tfrac{N}{m})\, d$$

$$d = 0m$$

Forces @ A

$A_x = 75N$
$A_y = 100N$

Forces @ C

$C_x = -75\ kN$

Forces @ D

$D_x = 0$

Spring Deformation

$d = 0m$