MECH&AE 103
Lec 1
Public Affairs Building 1246

MECH&AE 105A
Dis 1A
Boelter Hall 2444

MAT SCI 104
Lec 2
Boelter Hall 5249

MECH&AE 101
Dis 1B
Public Affairs Building 2270

&AE 105A

MECH&AE 101
Lec 1

MECH&AE 105A
Lec 1
Mathematical Sciences

MAT SCI 104

m 1 (10 pts): A typical AM radio station transmits at 1.2 MHz and puts out 40 kW of
er. For simplicity, assume that this power is radiated in all directions uniformly.

a) (2 pts) What is the wavelength of these radio waves?
b) (4 pts) What is the intensity of these radio waves at 20 km distance?
c) (4 pts) What is the RMS amplitude of the electric fields at 20 km distance?

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$P = 40\times10^3 W$

$f = 1.2\times10^6$ Hz

(a) $c = \lambda f$

$\frac{c}{f} = \lambda$

$\lambda = \frac{3\times10^8 m/s}{1.2\times10^6 Hz}$

$\lambda = 250$ m

+2

(b) Intensity = $\frac{\text{Power}}{\text{area}}$

$I = \frac{40\times10^3 W}{4\pi r^2}$

radiating outward as a sphere

$r = 20\times10^3 m$

$I = 7.96\times10^{-6} W/m^2$

+4

(c) E_{rms}

known: $I = \frac{c\varepsilon_0 E_0^2}{2}$ (constants)

$\sqrt{\frac{2I}{c\varepsilon_0}} = E_0 = \sqrt{\frac{2(7.96\times10^{-6} W/m^2)}{(3\times10^8 m/s)(8.85\times10^{-12} F/m)}}$

$E_0 = .0774$ J

$E_{rms} = \frac{E_0}{\sqrt{2}}$

$E_{rms} = \frac{.0774 J}{\sqrt{2}} = 0.0547$ J

+3

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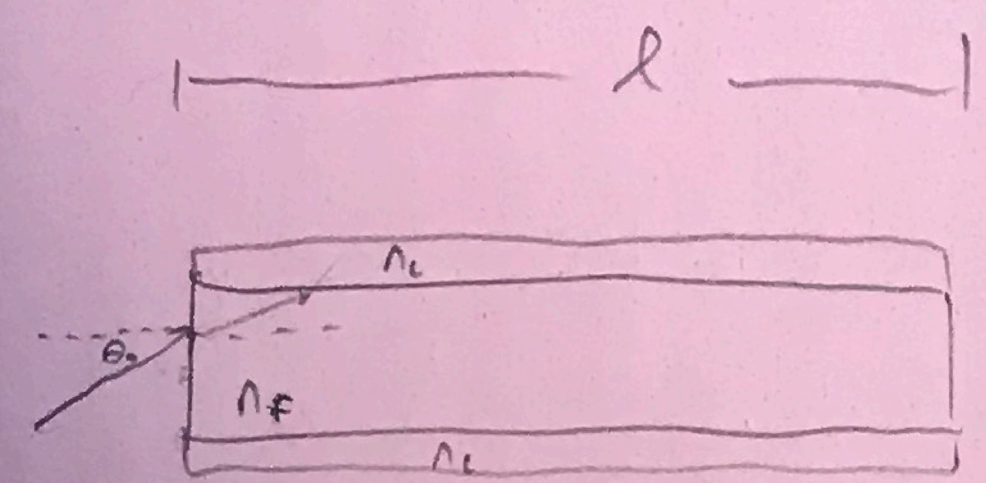
oblem 2 (10 pts): A fiber optic cable has length l and its core fiber has an index of efraction n_f. The thin cladding around the fiber has an index of refraction $n_c < n_f$.

a) (4 pts) Determine the time that it takes a ray passing at an incident angle zero to make it from one end of the fiber to the other end in the fiber in terms of the variables given and the speed of light c. θ_0 $\theta_0 = 0$

b) (6 pts) Another ray reflects off the fiber/cladding interface at the critical angle. Determine the time that it takes for the ray to pass from one end of the fiber to the other end of the fiber in terms of the variables given and the speed of light c.



$v = \frac{c}{n}$

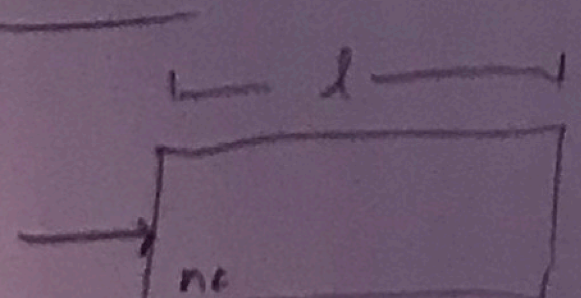
a) $n = \frac{c}{v}$ $\quad v = \frac{d}{t} = \frac{\text{"distance"}}{\text{"time"}}$

$v = \frac{c}{n_f}$ $\quad \frac{d}{t} = \frac{c}{n_f}$

$d = \ell$

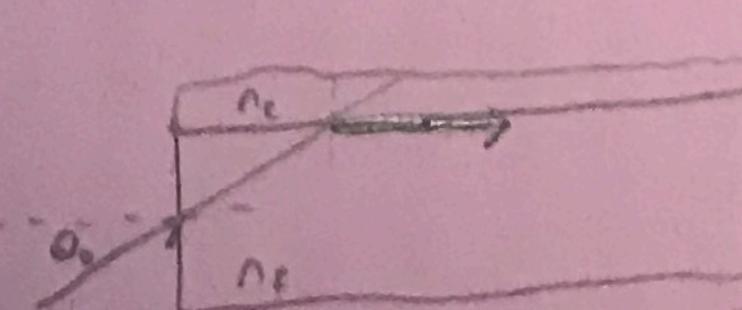
$t = \frac{d n_f}{c} = \boxed{\frac{\ell n_f}{c}}$ ✓

when $\theta_0 = 0$



$v = \frac{c}{n_c}$ $\quad \frac{\ell}{t} = \frac{c}{n_f}$

b) at critical angle $\quad \boxed{\theta_c = \sin^{-1}\left(\frac{n_c}{n_f}\right)}$ +2

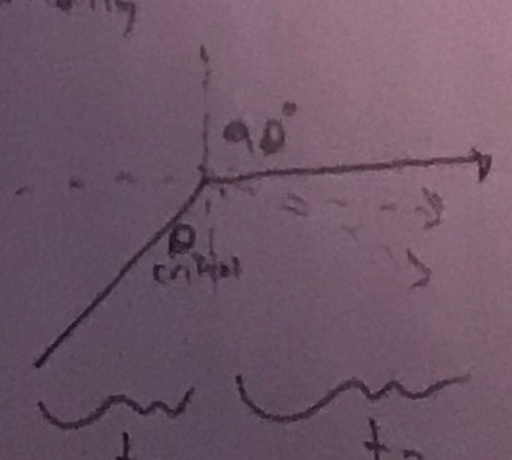


✱ at critical angle, ray is totally internally reflected

break into 2 parts

Snell's

$n_f \sin\theta_2 = n_c \sin\theta_c$



time for distance ray travels + distance all in n_f

(fraction of ℓ)$\left(\frac{n_f}{c}\right)$ + (remainder of ℓ)$\left(\frac{n_c}{c}\right)$

ratio having to do with θ_c relation with ℓ

partial credit

t_1 = time to travel angled distance

t_2 = time to travel after critical angle

3/6

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MAT SCI 104
Dis 2B

Problem 3 (10 pts): Suppose a particle of dust is spherical and has a density ρ=2.5 g/cm^3 and a radius r=4.0x10^{-6} m. The dust particle is light-absorbing (black).

(9)

If the dust particle gets into the beam of a powerful laser that produces 10 W of power, and the laser beam is circular and of diameter 1.5 mm: $r = \frac{1.5}{2}$ mm

a) (4 pts) What is the radiation pressure of the laser beam on something black (in N/m²)?
b) (3 pts) What is the net force on the dust particle due to radiation pressure?
c) (3 pts) What is the acceleration of the dust particle due to radiation pressure?

(a) Intensity = $\frac{\text{power}}{\text{area}} = \frac{P}{A}$

+4

radiation pressure (RP) for perfect absorber = I/c

$RP = \frac{I}{c} = \frac{P/A}{c} = \frac{P}{Ac} = \frac{10\,W}{(3\times10^8\,m/s)(\pi r^2)}$ = .0189 N/m² (radiation pressure of beam)

$r = \left(\frac{1.5}{2}\times10^{-3}\,m\right)$ use radius of laser beam

radiation pressure from part A: .0189 N/m²

(b) * radiation pressure × area = net force

$F_{net} = (.0189\,N/m^2)(\pi r^2) = 9.48\times10^{-13}\,N$

$r = 4\times10^{-6}\,m$ radius of dust

+3

Find mass

volume of a sphere = $\frac{4}{3}\pi r^3$

density · volume = mass

pressure = $\frac{\text{Force}}{\text{area}}$

$\frac{2.5\,g}{cm^3}\cdot\left(\frac{4}{3}\pi\left(.04\times10^{-6}\,cm\right)^3\right) = 6.7\times10^{-22}\,g$ (r of dust particle; mass use in part c)

$= 6.7\times10^{-25}\,kg$

Wrong conversion — change to cm

+2

(c) F=ma

$a = \frac{F}{m}$ (F from part (b); m solved above)

$accel = \frac{9.48\times10^{-13}\,N}{6.7\times10^{-25}\,kg} = 1.41\times10^{12}\,m/s^2$

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$75\times10^{-3}\,H$ $4\times10^{-6}\,F$

…oblem 4 (10 pts): A given RLC circuit has R=3.0 Ω, L=75 mH, and C=4.0 μF. 10

a) (4 pts) Calculate the resonant frequency f of the circuit.
b) (2 pts) What is the impedance Z of the circuit at resonance?
c) (4 pts) What is the impedance Z of the circuit at 110% of the resonant frequency?

(a) resonant frequency

$X_c = X_L$

$\frac{1}{\omega C} = \omega L$

$\omega_0^2 = \frac{1}{LC}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega_0 = \frac{1}{\sqrt{(75\times10^{-3}H)(4\times10^{-6}F)}}$

$\omega_0 = 1825.74$ rad/s +4

$2\pi f = \omega$

$f = \frac{\omega}{2\pi}$

$f = 290.58$ Hz

(b) At resonance $Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

↑ $(X_L - X_c) = 0$

$X_L = X_c$

$Z = \sqrt{R^2 + 0^2}$ +2

$Z = \sqrt{R^2}$

$Z = R = 3.0\,\Omega$

+4

(c) 110% of ω_0

$(\omega_0)(1.1)$

$(1825.74\text{ rad/s})(1.1) = 2008.32$ rad/s

↑ ω_1

$Z = \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = \sqrt{(3\,\Omega)^2 + \left(2008\text{ rad/s}\cdot 75\text{ mH} - \frac{1}{\omega_1 \cdot 4\,\mu F}\right)^2}$

$Z = 26.313\,\Omega$