LAST Name: SOLUTION

FIRST Name:

University ID:

Discussion Section:

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**PHYSICS 1C: Electrodynamics, Optics, and Special Relativity**

**Fall 2017, Lecture Series 2**

**Practice Midterm 2**

**Time allowed: 50 minutes**

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*Answer* **all** *questions.*

*Calculators* **are** *permitted in this exam*

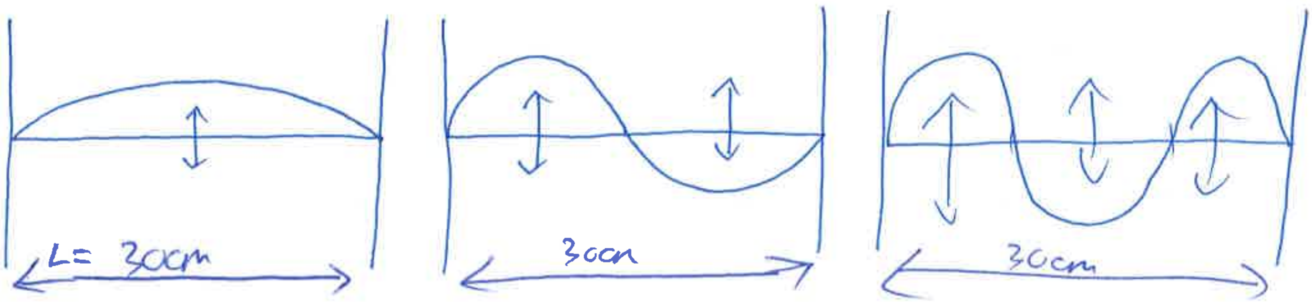
*The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.*

**Do NOT turn over until told that you may do so.**

| Question | Points |
|---|---|
| 1 | /6 |
| 2 | /6 |
| 3 | /12 |
| 4 | /25 |
| 5 | /25 |
| 6 | /26 |

## Section A

**1.** Two perfectly conducting plates are placed 30 cm apart, and an electromagnetic plane standing wave is set up between them. Draw diagrams to show the electric field configuration of the three longest wavelength standing waves that may arise. What is the wavelength in each case? [6]

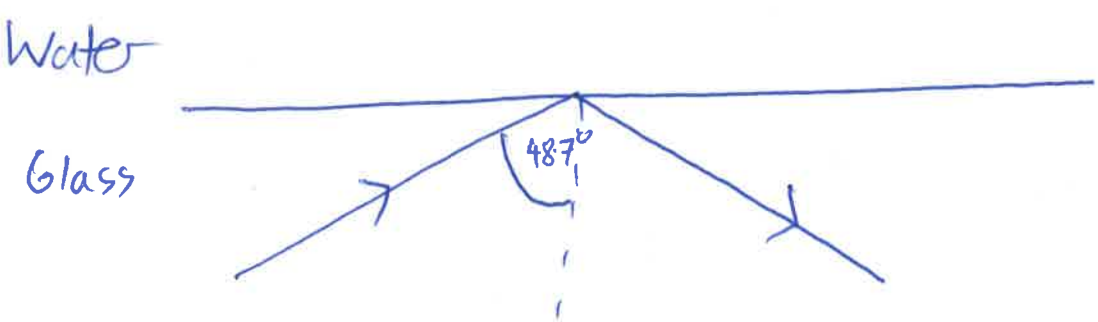



$L = \frac{\lambda}{2}$ $\Rightarrow \lambda = 2L = 60\,\text{cm}$

$L = \lambda$ $\Rightarrow \lambda = 30\,\text{cm}$

$L = \frac{3}{2}\lambda$ $\Rightarrow \lambda = 20\,\text{cm}$

**2.** A ray of light is travelling in a glass cube that is totally immersed in water. You find that if the ray is incident on the glass-water interface at an angle to the normal larger than 48.7° then no light is refracted into the water. The refractive index of water is 1.333. What is the refractive index of the glass? [6]




48.7° is Critical angle: $\sin\theta_{crit} = \frac{n_b}{n_a}$

$\Rightarrow n_a = n_b / \sin\theta_{crit}$

$= (1.333)/\sin(48.7) = 1.7743.$

$= 1.77$ (3s.f.)

**3.** By considering the case of a charging parallel-plate capacitor, explain why Ampère's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 i_C$, is not complete, and explain how the situation can be resolved by including a displacement current. [Hint: you should include a diagram in your answer. The capacitance of a parallel-plate capacitor is given by $C = \epsilon_0 A/d$ and the charge stored on a plate is given by $q = CV$, where the symbols have their usual meanings]. [12]

Ampères law should be independent of the chosen <u>surface</u> (as long as the boundary is the same).
This no longer holds for a charging capacitor!


The charge current through $S_1$ is $i_c$, but is zero through $S_1$.
This can be fixed by adding a 'displacement current' due to changing electric flux between the plates:

$$i_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

It can be shown that this is equal to the ~~...~~ value of the charge current:

$$\text{charge on capacitor plate} = VC = \frac{\epsilon_0 V A}{d} = \epsilon_0 E A = \epsilon_0 \Phi_E$$

$$\Rightarrow \frac{dq}{dt} = i_c = \epsilon_0 \frac{d\Phi_E}{dt}.$$

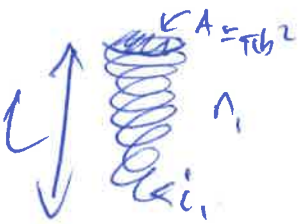
The improved Ampères law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + i_D)$ NOW works for <u>any</u> choice of surface.

## Section B

**4.** A long cylindrical solenoid has radius $b$, length $l$, and $n_1$ turns of wire per unit length, and carries a current $i_1$.

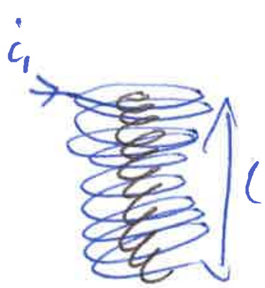
(a) Determine the self-inductance $L_1$ of the solenoid. [The magnetic field inside an ideal solenoid is given by $B = \mu_0 n I$, where the symbols have their usual meanings]. [6]

$$L_1 = \frac{N_1 \Phi_1}{i_1} = \frac{(n_1 l)(BA)}{i_1} = \frac{(n_1 l)(\mu_0 n_1 i_1)(\pi b^2)}{i_1}$$




$$= \underline{\underline{\mu_0 n_1^2 l \pi b^2}}$$

(b) A similar solenoid of radius $a < b$ and $n_2$ turns per unit length is placed inside the first one. Both solenoids have the same length. Calculate the magnetic flux through the smaller solenoid when the current through the larger solenoid is $i_1$. Hence determine the mutual inductance of the system. [8]




$$N_2 \Phi_2 = N_2 B_1 A_2$$

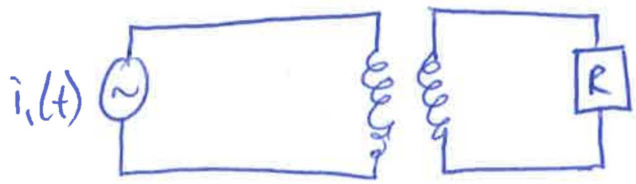
$$= (n_2 l)(\mu_0 n_1 i_1)(\pi a^2)$$

$$= \underline{\underline{\pi \mu_0 n_1 n_2 l i_1 a^2}}$$

Finding $\Phi_2$, the flux through a single loop, is fine too...

$$M_{12} = \frac{N_2 \Phi_2}{i_1} = \underline{\underline{\pi \mu_0 n_1 n_2 l a^2}}$$

(c) The pair of solenoids is used as a transformer in a circuit. A current $i_1(t) = I_1 \cos(\omega t)$ is passed through the first (larger) solenoid. The second solenoid is connected to a circuit containing a resistor with resistance $R$. Calculate the induced emf in the second circuit, the resulting current, and the average power dissipated in the resistor. [Hint: the solenoid in this case is *not* ideal, so you will need to make use of the mutual inductance] [11]




(in terms of M is fine).

$$\mathcal{E}_2(t) = -M\frac{di}{dt} = -M\frac{d}{dt}\left(I_1\cos(\omega t)\right)$$

$$= +MI_1\omega\sin(\omega t) = \pi\mu_0 n_1 n_2 l a^2 I_1\omega\sin(\omega t)$$

$$i_2(t) = \frac{\mathcal{E}_2(t)}{R} = \frac{MI_1\omega}{R}\sin(\omega t)$$

$$P_2(t) = i_2^2 R = \left[\frac{MI_1\omega}{R}\sin(\omega t)\right]^2 R$$

$$= M^2 I_1^2 \omega^2 R \sin^2(\omega t)$$

R should be in denominator here

Average power:

$$P_{av} = \langle M^2 I_1^2 \omega^2 R \sin^2(\omega t)\rangle$$

$$= \frac{1}{2} M^2 I_1^2 \omega^2 R$$

**5.** Solar panels convert sunlight into electrical power. On a sunny day, sunlight at the surface of the earth is 1kilowatt per square meter, which determines the maximum output of a solar panel. You may treat air as a vacuum for this problem.

(a) What is the RMS magnetic field of the sunlight just before hitting the panels? [7]

$$\left(E_{max} = c B_{max}\right) \quad I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{c B_{max}^2}{2\mu_0}$$

$$\Rightarrow B_{max}^2 = \frac{2\mu_0 I}{c} \Rightarrow B_{max} = \sqrt{\frac{2\mu_0 I}{c}}$$

$$B_{rms} = \frac{B_{max}}{\sqrt{2}} = \sqrt{\frac{\mu_0 I}{c}} = \sqrt{\frac{(4\pi \times 10^{-7})(1 \times 10^3)}{3 \times 10^8}}$$

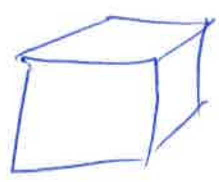
$$= 2.0466\ldots \times 10^{-6} = \underline{2.05 \times 10^{-6}} \ (3\text{sf})$$

(b) How much energy is present in one cubic metre of sunlight, just before hitting the panels? [6]

Energy density: $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

$$= \frac{1}{\mu_0} B^2$$

$$\Rightarrow u_{av} = \frac{B_{max}^2}{2\mu_0}$$



$$U = u_{av} \times \text{vol.} = \left(\frac{B_{max}^2}{2\mu_0}\right)(1\,m^3) = \frac{1}{\cancel{2\mu_0}}\left(\frac{\cancel{2\mu_0} I}{c}\right)(1\,m^3)$$

$$= I/c \times 1\,m^3 = \frac{(1 \times 10^3)(1\,m^3)}{3 \times 10^8} = 3.33\ldots \times 10^{-6}\,J$$

$$= \underline{3.33 \times 10^{-6}\,J} \ (3\text{sf})$$

(c) Assume that the sunlight is absorbed 100% by the black solar panel which has dimensions of 0.5 m×0.5 m. What is the force of the sunlight on the panel in Newtons? [Hint: you do not need any answers from parts (a) or (b) to solve this part.] [6]

$$P_{rad}^{abs} = I/c \qquad F = p \times A = \frac{IA}{c}$$

$$F = \frac{(1\times10^3)(0.5)^2}{(3\times10^8)}$$

$$= 8.333\ldots \times 10^{-7}\ N$$

$$= \underline{8.33 \times 10^{-7}\ N}$$

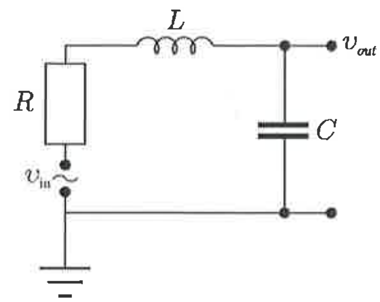
(d) If the solar panels only respond to one linear polarisation of light, what is the maximum power output of the panel? [6]

$$P_{max} = \text{Intensity} \times \text{Area} \times \frac{1}{2}$$ ← unpolarised sunlight has ≈ 50% component along one axis.

$$= (1\times10^3)\times(0.5)^2\times(0.5)$$

$$= \underline{125\ W}$$

**6.** An alternating input voltage $v_{\text{in}}(t)$ with amplitude $V_{\text{in}}$ and angular frequency $\omega$ is applied from an ideal source to the circuit shown below.



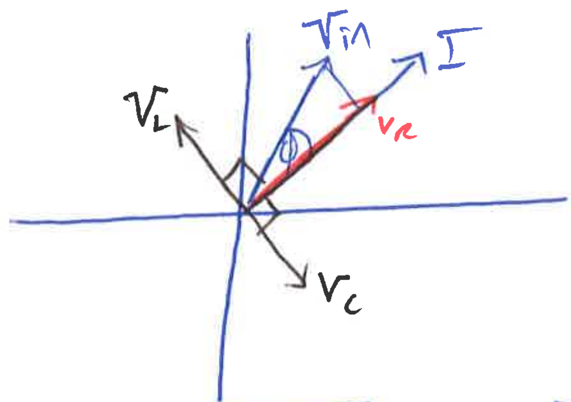

(a) Using phasors, or otherwise, obtain (equilibrium) expressions for $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$, assuming the current through the circuit is given by $i(t) = I\cos(\omega t)$. Determine both the relative amplitude and the relative phase as a function of frequency in each case. [16]

$$i(t) = I\cos(\omega t)$$

$$V_R(t) = IR\cos(\omega t)$$

$$V_L(t) = I\omega L\cos(\omega t + 90^\circ)$$

$$V_C(t) = \frac{I}{\omega C}\cos(\omega t - 90^\circ)$$




$$V_{in} = \sqrt{V_R^2 + (V_L - V_c)^2} = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\phi_{in} = \tan^{-1}\left(\frac{V_L - V_c}{V_R}\right) = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$v_{in}(t) = V_{in}\cos(\omega t + \phi_{in})$$

$V_{out}(t) = \cancel{\ } v_C(t)$

$= \frac{I}{\omega C}\cos(\omega t - 90^\circ)$

$V_{out}$ $\phi_{out}$

(b) Obtain the leading expressions for the amplitude ratio $V_{out}/V_{in}$ in the limit that $\omega$ is very small and in the limit that $\omega$ is very large. Based on these expressions, what would be a possible use for this circuit? [10]

$$\frac{V_{out}}{V_{in}} = \frac{I/\omega C}{I\sqrt{R^2+(\omega L - 1/\omega C)^2}}$$

$$\omega \to 0: \quad \frac{V_{out}}{V_{in}} \to \frac{1/\omega C}{\sqrt{(1/\omega C)^2}} = \underline{\underline{1}}$$

$$\omega \to \infty: \quad \frac{V_{out}}{V_{in}} \to \frac{1/\omega C}{\sqrt{(\omega L)^2}} = \frac{1/\omega C}{\omega L} = \underline{\underline{\frac{1}{\omega^2 LC}}} \to \underline{\underline{0}}$$

Low pass filter $\Rightarrow$ high frequency component of signal are removed.