LAST Name: SOLUTION

FIRST Name:

University ID:

Discussion Section:

PHYSICS 1C: Electrodynamics, Optics, and Special Relativity

Fall 2017, Lecture Series 2

Practice Midterm 1

Time allowed: 50 minutes

Answer **all** *questions.*

Calculators **are** *permitted in this exam*

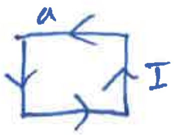
The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Question	Points
1	/6
2	/6
3	/12
4	/25
5	/25
6	/26

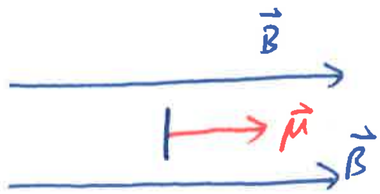
Section A

1. A square loop of wire with side length a has a current I passing through it. What is the magnitude of its magnetic dipole moment μ? [2]



$\mu = Ia^2$

This current loop is placed in a uniform magnetic field $\vec{\mathbf{B}}$. Draw the configuration which is the lowest energy state, showing the directions of $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{B}}$. [4]



$\vec{B} \parallel \vec{\mu}$

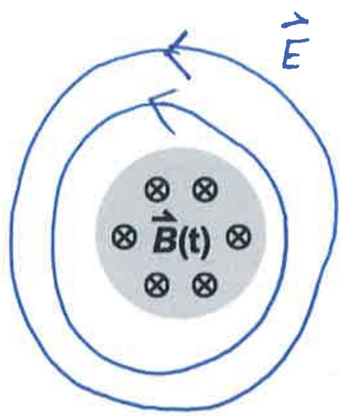
$(U = -\vec{\mu}\cdot\vec{B})$

What is the magnetic potential energy of this configuration? [2]

$U = -\vec{\mu}\cdot\vec{B} = -BIa^2$

2. The magnetic field in a circular region of space is increasing into the page, as shown in the figure below. Add two representative field lines to the diagram to show the direction of the induced electric field outside of this region. [4]

$\mathcal{E} = -\dfrac{d\Phi}{dt} = \oint \vec{E}_n \cdot d\vec{l}$



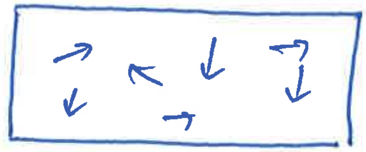
* circular by symmetry
* orientation opposite to $d\Phi/dt$.

Is the induced electric field conservative or nonconservative? [2]

nonconservative (caused by changing $\vec{B}$ field)

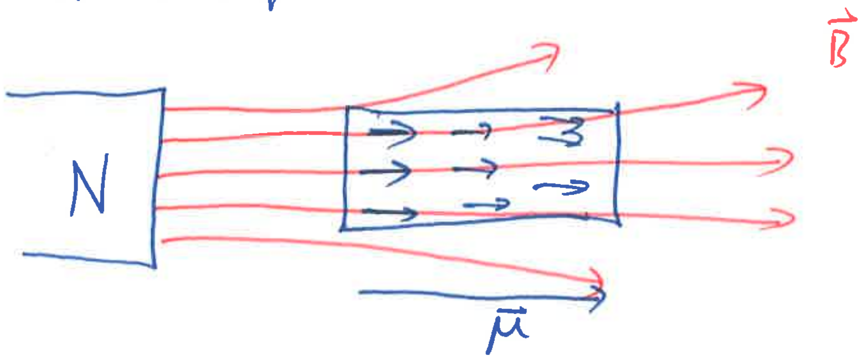
3. Briefly explain, with the aid of diagrams, what happens when a paramagnetic material is brought near to the north pole of a permanent magnet. [12]

Paramagnetic materials have existing dipoles; caused by the motion of electrons:

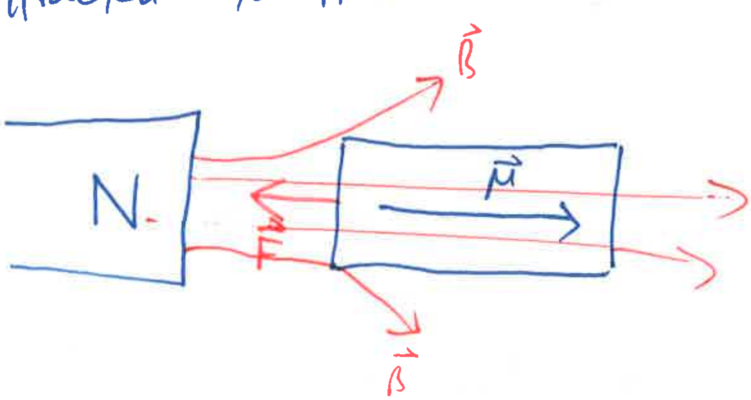


These are randomly aligned if there is no external field due to their thermal motion.

Near the north pole of a permanent magnet, the magnetic field aligns these dipoles so that the material has an overall dipole moment:



The paramagnet now acts like a dipole in the magnetic field of the permanent magnet. The non-uniform field of the permanent magnet causes the paramagnet to be attracted to it:



Section B

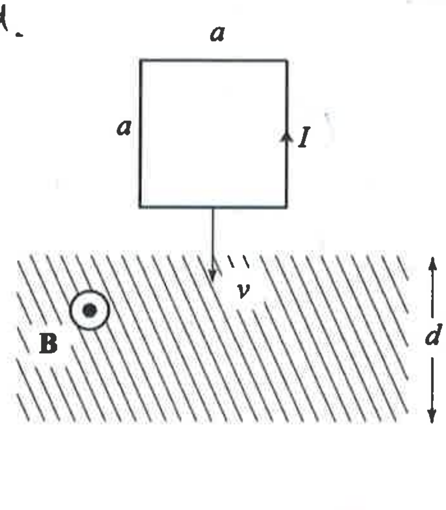
4.

(a) State Faraday and Lenz's laws of electromagnetic induction. [6]

Faraday: $\mathcal{E} = -\frac{d\Phi}{dt}$

Lenz: The effects of electromagnetic induction act to oppose the change that caused them.

(b) A square conducting loop with sides of length a and total electrical resistance R is moved at a constant speed v toward a region with uniform magnetic flux density **B** out of the page as shown in the following figure. The field region is of thickness $d > a$. At $t=0$, the leading edge enters the region of magnetic field.



Determine the magnetic flux Φ_B through the loop as a function of time t for $0 < t < 2d/v$ and sketch $\Phi_B(t)$ versus time for the same interval. Take positive circulation to correspond to the direction shown in the figure. [10]

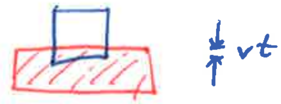
① The square is totally inside the region where $\vec{B} \neq 0$ when $vt = a \Rightarrow t = a/v$.

② The square remains within this region until $vt = d$, when the leading edge leaves this region $\Rightarrow t = d/v$

③ The square then leaves this region, and is completely outside of it when $vt = d + a \Rightarrow t = \frac{d+a}{v}$

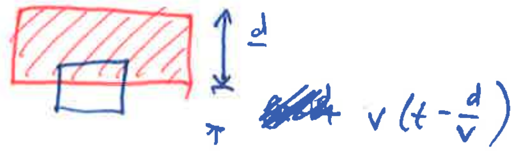
④ The square remains outside of this region until $t = \frac{2d}{v}$

① $\Phi_1 = BA_1 = Bvta$

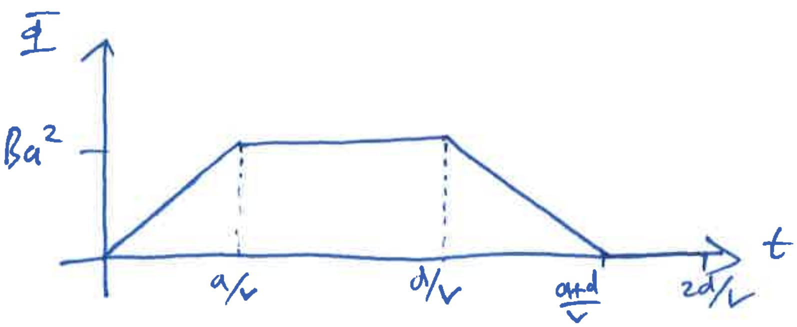


② $\Phi_2 = BA_2 = Ba^2$

③ $\Phi_3 = BA_3 = Ba(a - v(t - d/v))$



④ $\Phi_4 = BA_4 = 0$

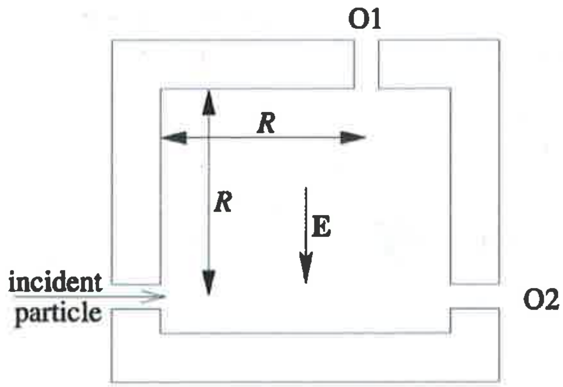


(c) Determine the current $I(t)$ flowing through the loop for $0 < t < 2d/v$. [9]

$$\mathcal{E} = -\frac{d\Phi}{dt} \qquad I = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$$

$$I(t) = \begin{cases} -\dfrac{Bva}{R} & 0 < t < a/v \\ 0 & a/v < t < d/v \\ +\dfrac{Bva}{R} & d/v < t < (d+a)/v \\ 0 & (d+a)/v < t < 2d/v \end{cases}$$

5. A beam of particles, all of the same positive charge q but of differing speeds v and differing masses m, is incident on the device shown:

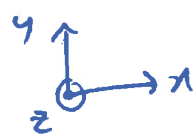


In the device, there is a uniform magnetic field **B** perpendicular to the plane of the paper, and a uniform electric field **E** as indicated; particles may only leave through the openings $O1$ and $O2$ which may be assumed to have negligible width and to be field-free.

(a) Determine the direction and magnitude of the magnetic field that will allow particles of a particular velocity v to continue undeflected and to leave the device through the opening $O2$. [10]

Require $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = 0$

Define axes:



$\vec{E} = -E\hat{j}$

$\vec{v} = \hat{i}$

$q > 0$

$\Rightarrow \quad -E\hat{j} + v\hat{i}\times\vec{B} = 0$

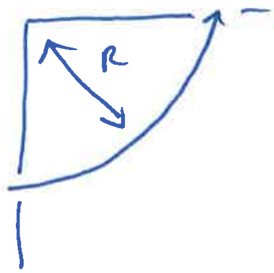
$\Rightarrow \quad v\hat{i}\times\vec{B} = E\hat{j}$

$\Rightarrow \quad \vec{B} = \frac{E}{v}(-\hat{k})$

$\therefore \quad |\vec{B}| = \frac{E}{v}$, into the page

(b) If $\mathbf{E} = 0$, show that only particles of a particular momentum may escape through $O1$ and find an expression for this momentum. Explain why the energy of the particles is unchanged in this situation. [15]

$\vec{E} = 0$:



Follows circular path

Radius R

$$\vec{F}_B = q\vec{v} \times \vec{B} = q v B \hat{j}$$

Magnetic force is always $\perp$ to $\vec{v}$, so provides centripetal force necessary for circular motion.

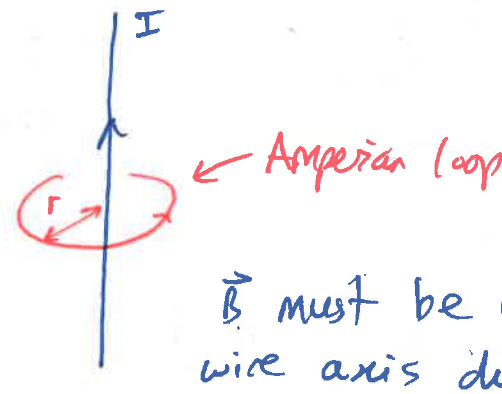
$$\Rightarrow \quad qvB = \frac{mv^2}{R} \qquad [F_B = F_{centripetal}]$$

$$\Rightarrow \quad mv = qBR$$

Magnetic force can do no work on the particle, as it is always perpendicular to its velocity.
Therefore, the energy of the particle is unchanged.

6.

(a) Use Ampère's law to show that the magnetic field a perpendicular distance r from a long straight wire carrying current I has magnitude $B = \mu_0 I/(2\pi r)$. [8]



$\vec{B}$ must be circular around wire axis due to symmetry.

Ampère: $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

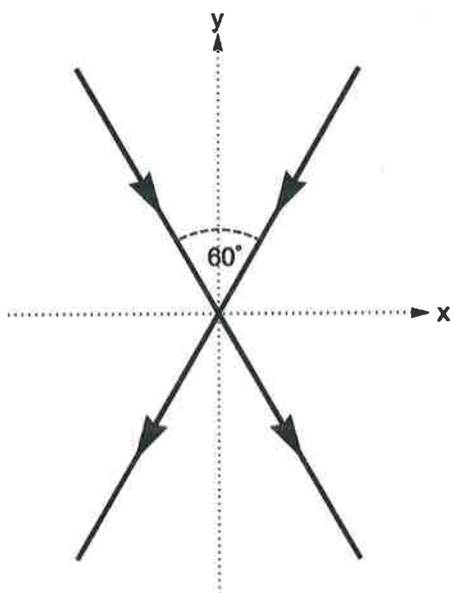
$\vec{B} \parallel d\vec{l}$ so $$2\pi r B = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R}$$

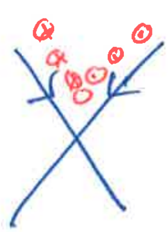
(b) How may we use the principle of superposition to help us calculate the magnetic field at a point in space near a collection of sources? [4]

Total field = vector sum of field from each source.

(c) The diagram below shows two long straight wires lying in the xy-plane, each carrying a current I. The wires cross at an angle of 60° as shown, symmetric about the y-axis and with the intersection at the origin. Find the magnitude of the magnetic field $B(x, 0, 0)$ along the x-axis, and the magnitude of the magnetic field $B(0, y, 0)$ along the y-axis. [14]

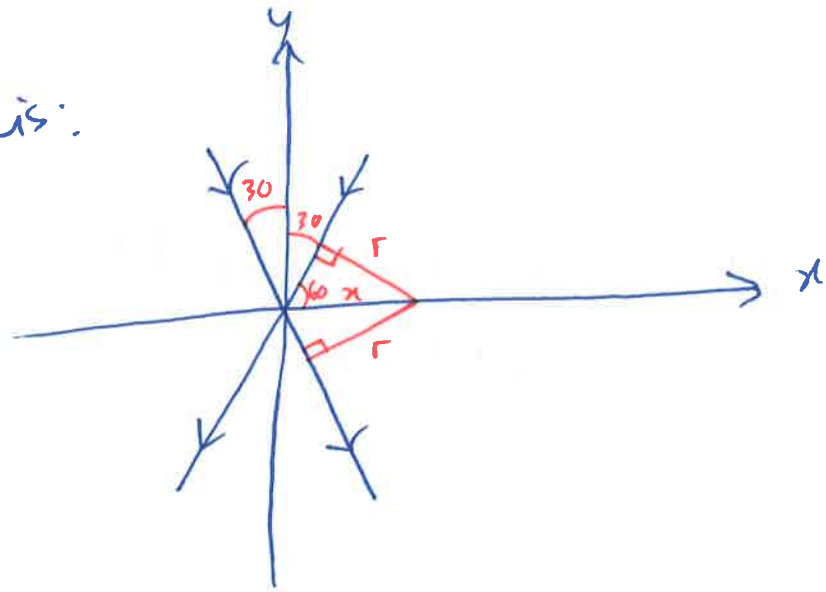


Magnetic field along the y-axis vanishes due to symmetry:

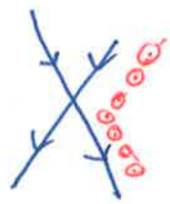


$B(0, y, 0) = 0$

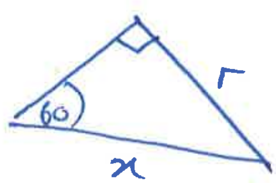
x-axis:



Along x-axis, field from each wire adds constructively:



Need perpendicular distance r:



$$r = x \sin 60$$

$$= \frac{\sqrt{3}\,x}{2}$$

$$B(x,0,0) = B_1(r) + B_2(r)$$

$$= \frac{\mu_0 I}{2\pi\left(\sqrt{3}\,x/2\right)} + \frac{\mu_0 I}{2\pi\left(\sqrt{3}\,x/2\right)}$$

$$= \frac{2\mu_0 I}{\sqrt{3}\,\pi x}$$