LAST Name: SOLUTION

FIRST Name:

University ID:

PHYSICS 1C: Electrodynamics, Optics, and Special Relativity

Fall 2017, Lecture Series 2

Practice Final Exam

Time allowed: 3 hours

Answer **all** *questions.*

Calculators are *permitted in this exam*

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Question	Points
1	/10
2	/8
3	/8
4	/12
5	/12
6	/16
7	/18
8	/18
9	/18
Total	/120

Section A

1.

(a) In class we performed a demonstration where an aluminium disk at the end of a pendulum was dropped into the field of a strong magnet and the moving disk immediately stopped. The reason it stopped was primarily because:

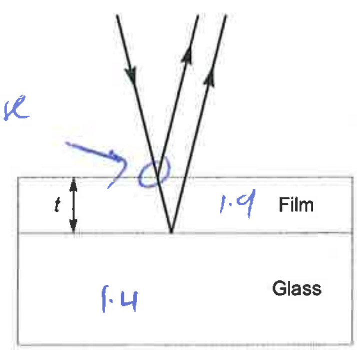
(i) Aluminium is paramagnetic and like-poles (e.g. N and N) repel. ☐
(ii) Eddy currents could not flow easily because of slits cut in the disk. ☐
(iii) Eddy currents could flow easily and experienced a Lorentz force. ☒ ✓
(iv) Aluminium is non-magnetic and shields magnetic field lines. ☐
(v) Aluminium is diamagnetic and like-poles (e.g. N and N) repel. ☐

[2]

(b) Light is normally incident on a thin film of thickness t which is coating a piece of glass. The film has refractive index $n_{\text{film}} = 1.9$ and the glass has refractive index $n_{\text{glass}} = 1.4$.

$$\Delta\phi = \frac{4\pi t}{\lambda} + \delta \overset{!}{=} 2\pi\left(m+\tfrac{1}{2}\right)$$

π phase shift



$$\Rightarrow \frac{4\pi t}{\lambda} = 2\pi m$$

$$\Rightarrow t = \frac{\lambda m}{2}$$

If λ is the wavelength of the incident light in the film, which of following film thicknesses will minimise the reflected intensity? [Hint: there may be more than one correct answer].

$m = 1, 2$

(i) $t = \lambda/4$. ☐
(ii) $t = \lambda/2$. ☒
(iii) $t = 3\lambda/4$. ☐
(iv) $t = \lambda$. ☒
(v) $t = 5\lambda/4$. ☐

[2]

(c) Without correcting lenses, a person with myopia (nearsightedness):

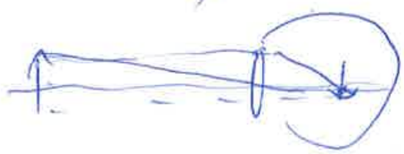
(i) Cannot see near objects because the image forms in front of the retina. ☐

(i) Cannot see near objects because the image forms behind the retina. ☐

(i) Cannot see far objects because the image forms in front of the retina. ☒

(i) Cannot see far objects because the image forms behind the retina. ☐

[2]



(d) Two protons (each with rest mass $m_p = 1.67 \times 10^{-27}$ kg) are initially moving with equal speeds in opposite directions. The protons continue to exist after a collision that also produces an η^0 particle (which has rest mass 9.75×10^{-28} kg). If the two protons and the η^0 particle are all at rest after the collision, find the initial speed of the protons, expressed as a fraction of the speed of light. [4]

Before: $(p) \xrightarrow{v} \qquad \xleftarrow{v} (p)$

After: $(p)\ (\eta^0)\ (p)$

$$E_{before} = 2\gamma m_p c^2$$

$$E_{after} = 2m_p c^2 + m_\eta c^2$$

Conserve: $2\gamma m_p \cancel{c^2} = 2m_p \cancel{c^2} + m_\eta \cancel{c^2}$

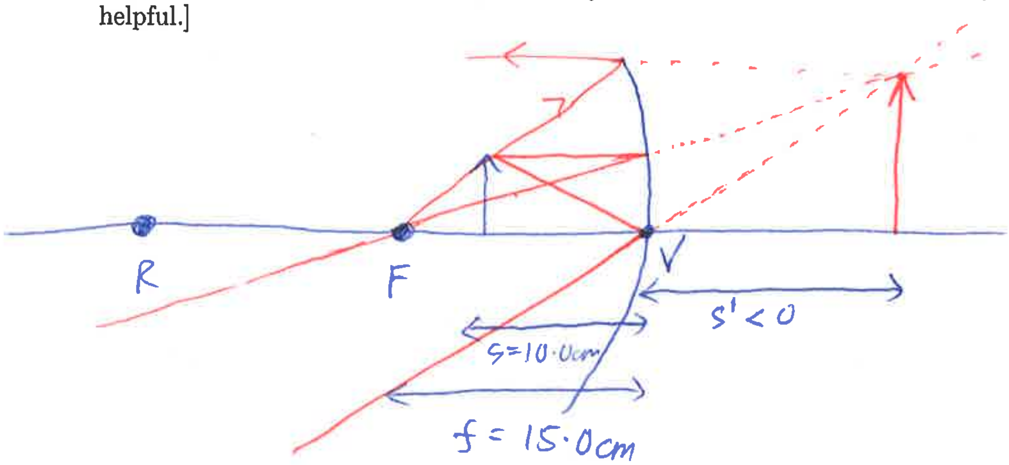
$$\Rightarrow \gamma = 1 + \frac{m_\eta}{2m_p} = 1.2919\ldots$$

$$\gamma^2 = \frac{1}{1-\frac{v^2}{c^2}} \Rightarrow 1 - \frac{1}{\gamma^2} = \frac{v^2}{c^2} \Rightarrow v = c\sqrt{1-\frac{1}{\gamma^2}}$$

$$= (0.63313\ldots)c$$

$$= (0.633)c \text{ (3s.f.)}$$

2. An object 2.5 cm tall is placed 10 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 30.0 cm. Determine the position, size, orientation, and nature (real or virtual) of the image. [Hint: you may find a principal ray diagram helpful.] [8]



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{1}{(15\times10^{-2})^{-1} - (10\times10^{-2})^{-1}}$$

$$= -30 \text{ cm}$$

1) $s' = \underline{-30\text{cm}} \Rightarrow$ 30cm behind mirror

2) size: $m = -\frac{s'}{s} = \frac{30}{10} = 3 \Rightarrow y' = 3\times2.5\text{cm} = \underline{7.5\text{cm}}$

3) <u>virtual</u>

4) <u>upright</u>

3. Write an equation describing the magnetic field as a function of distance and time for a plane wave of microwave radiation. Assume it has a wavelength 6.28 cm propagating along the x-axis in the positive direction with a linear polarisation and the electric field points along the y-axis. The electric field has its maximum value of 3.0 V/m at $t = 0$ at the origin of the coordinate system. Put SI units on all numbers. [8]

$\lambda = 6.28 \times 10^{-2}\,m$ $\qquad \vec{E} \propto \hat{j}$ $\qquad \vec{E} \times \vec{B} \propto \hat{i}$

$\Rightarrow \vec{B} \propto \hat{k}$

$E_{max} = 3.0\,V/m$

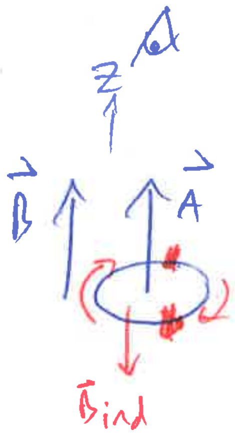
$E_{max} = c B_{max} \Rightarrow B_{max} = \frac{3}{3\times10^{8}} = (1\times10^{-8}\,T)$

$k = \frac{2\pi}{\lambda} = 100$ (3sf.)

$\omega = ck = (3\times10^{8}) \times (100) = 3\times10^{10}$ rad/s

$\therefore \boxed{\vec{B} = (1\times10^{-8}\,T)\,\hat{k}\,\cos\left((100\,\text{rad/m})\,x - (3\times10^{10}\,\text{rad/s})\,t\right)}$

4. A circular coil of 500 turns of wire has a radius of 0.04 m and a total resistance of 600 Ω and is fixed to the origin of a coordinate system so that it cannot move or rotate. It is placed in a uniform magnetic field that varies according to time as $B(t) = at + bt^4$, starting from $t = 0$, where $a = 2.0$, $b = 2.0 \times 10^{-3}$, and each constant has units to make $B(t)$ have units of Tesla. Both $\vec{\mathbf{B}}(t)$ and the vector area $\vec{\mathbf{A}}$ of the coil point along the z-axis.



(a) Looking from above the coil (i.e., looking in the negative z-direction from a positive z-position) does the current induced in the loop flow clockwise or anticlockwise? [2]

$\vec{B}$ increases in z-direction → induced current opposes this

⇒ clockwise

(b) What is the magnitude of current through the loop at $t = 10.0$ s? [6]

$$\Phi(t) = \vec{B}(t) \cdot \vec{A}$$
$$= (at + bt^4) A$$

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -NA\left[a + 4bt^3\right]$$

current: $$|I| = \left|\frac{\mathcal{E}}{R}\right| = +\frac{NA}{R}\left[a + 4bt^3\right]$$

$$= \frac{(500)(\pi \times 0.04^2)}{600}\left[2 + (8\times10^{-3})(10)^3\right]$$

$$= \frac{\pi}{75} A = 0.0419 \, A \text{ (3s.f.)}$$

(c) What is the magnetic torque on the coil at $t = 10.0$ s? [4]

[Hint: this does not take the whole page]

$$\cancel{\vec{\mu} = I N \vec{A} = \frac{\pi}{75}(500)(\pi \times 0.04^2)}$$ ← not needed

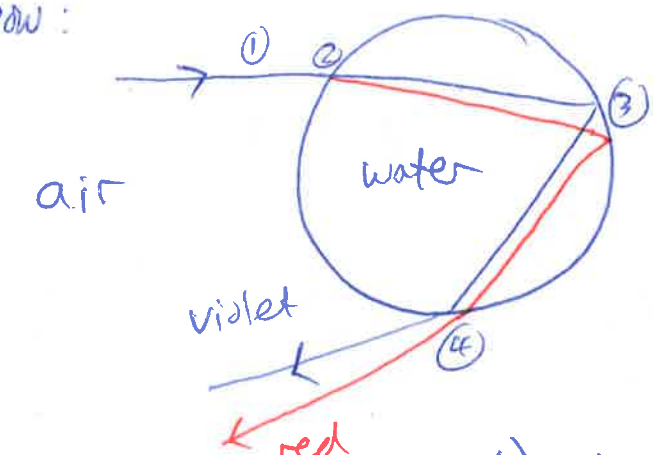
$$\cancel{= 0.10527\ldots \text{ Am}^2}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \underline{0}! \quad \vec{\mu} \parallel \vec{B}$$

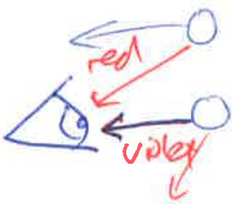
5. Explain, with the aid of diagrams, how single and double rainbows form. [Hint: be sure to describe the ordering of the colours observed in each case]. [12]

Rainbows are formed by the refraction and reflection of light from water droplets.

Single Rainbow:



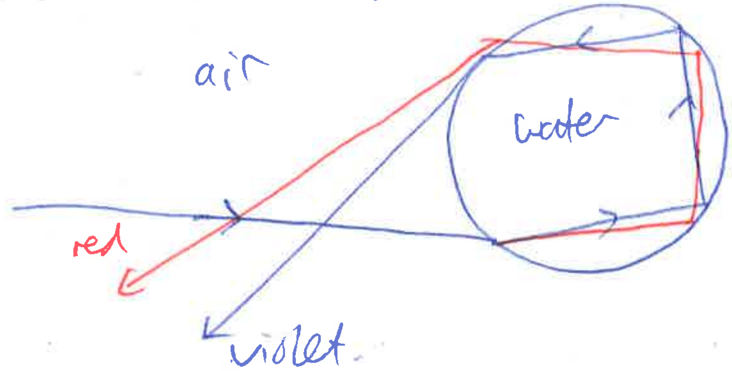
1) white light from sun enters droplet
2) light is refracted
3) light is (totally internally) reflected
4) light is refracted again leaving the droplet.



Water is dispersive — refractive index varies with wavelength. Shorter wavelength colours are refracted more.

For a single rainbow, we observe red at the top.

For a double rainbow, there is an additional (total internal) reflection, so the order is reversed:



Section B

6. A particular cable consists of two concentric conductors. The inner conductor is a thin cylindrical sheet with radius a which carries current I. The outer conductor is a thin cylindrical sheet which has radius $b > a$ and carries current I in the opposite direction.

(a) Determine the magnitude and direction of the magnetic field $\vec{\mathbf{B}}$ at radius r:

(i) when $r < a$

(ii) when $a < r < b$

(iii) when $r > b$. [8]

[Hint: be sure to state any symmetry assumptions you make].

Ampères Law: $\oint \vec{B} \cdot d\vec{l} = \mu I_{encl}$

Cylindrical symmetry $\Rightarrow \vec{B}$ must be azimuthal around cylinder.

i) $I_{encl} = 0 \Rightarrow \vec{B} = 0$ for $r < a$

iii) $I_{encl} = 0 \Rightarrow \vec{B} = 0$ for $r > b$

ii) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow 2\pi r B = \mu_0 I$

$\Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

$\Rightarrow \vec{B} = \dfrac{\mu_0 I}{2\pi r} \hat{\theta}$

(b) If the wire has length l, calculate the energy stored in the magnetic field in the cable. [Hint: you may require the integral $\int_a^b \frac{1}{x}\mathrm{d}x = \ln(b/a)$.] [8]

Energy density: $u = \frac{1}{2\mu_0} B^2$

$$\Rightarrow u(r) = \frac{1}{2\mu_0}\left[\frac{\mu_0 I}{2\pi r}\right]^2$$

$$= \frac{\mu_0 I^2}{8\pi^2 r^2}$$

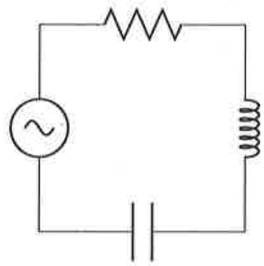
Total energy: $U = \int u\,dV = l \int u\,dA$

$$= \int 2\pi r l\,dr$$

$$= \frac{\mu_0 I^2 l}{4\pi} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right)$$

7. An L-R-C seris circuit is connected to an $f = 60$ Hz source that has $V_{\text{rms}} = 120V$. The resistor has a resistance of 400 Ω and someone has measured the impedance of the combined inductor, resistor and capacitor at this frequency to be 500 Ω.



(a) What is the average power delivered to the circuit by the source? [5]

impedance: $Z = 500\Omega$ $\quad V = IZ$

$$\Rightarrow \quad I_{rms} = \frac{120V}{500\Omega} = 0.24A$$

Power is only dissipated by resistor.

$$P_{AV} = P_{diss} = \frac{1}{2} VI\cos\phi \overset{\text{resistor}}{=} V_{rms} I_{rms} = I_{rms}^2 R$$

$$= (0.24)^2 (400)$$

$$= 23.04 \text{ W}$$

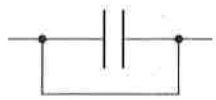
$$= 23.0\text{W } (3s.f.)$$

Alternatively: $P_{AV} = V_{rms} I_{rms} \cos\phi$

$$= V_{rms} I_{rms} \frac{R}{Z} \text{ for LRC}$$

$$= (120)(0.24)\left(\frac{400}{500}\right) = 23.04\text{W} = 23.0\text{W } (3s.f.)$$

(b) You discover that the capacitor has been 'shorted' the whole time by a little bridge of conducting solder, which effectively modifies the circuit as shown below.



This was true the entire time, even when the impedance was measured. What is the value of the inductor? [Hint: include units]. [5]

$$Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \frac{Z^2 - R^2}{\omega^2} = L^2$$

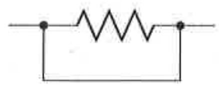
$$\Rightarrow L = \frac{\sqrt{Z^2 - R^2}}{\omega} \qquad [\omega = 2\pi f]$$

$$= \frac{\sqrt{500^2 - 400^2}}{(2\pi \times 60)}$$

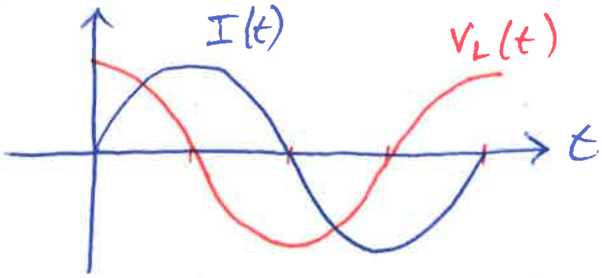
$$= 0.79577\ldots$$

$$= \underline{0.796\text{ H}} \quad (3\text{s.f.})$$

(c) Now you deliberately 'short' the resistor as well by connecting a wire to both sides.



Sketch the voltage and current versus time across the source for one cycle. Assume that $I(t) = 0$ at $t = 0$ and is increasing. You do not need to put numerical values for current, voltage, time, etc. on your plot. [5]



$V(t) = V_L(t)$

shift

$V(t) = I\omega L \cos(\omega t + 90°)$

$+90°$ shift

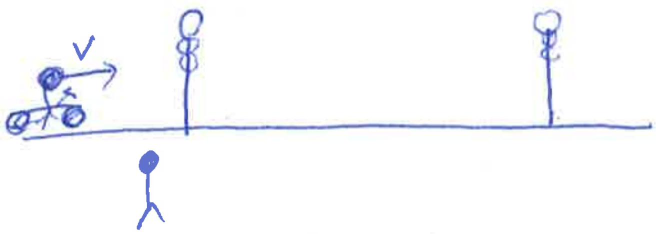
(d) What is the average power dissipated in part (c)? [For simplicity, assume the wires have no resistance.] [3]

$P_{AV} = 0$

[~~capacitor~~ inductor purely reactive]

8. Two traffic lights are separated by 300 m. They turn green at the same time, as measured by Stanley, in a car stopped at a light (meaning he is at rest in the rest frame of the lights). Mavis is driving a relativistic motorcycle at $v = (4/5)c$ along the road, which is straight and contains the two lights. Mavis is at rest in the frame of the motorcycle. [Hint: Don't worry about the Doppler shifts to the colour of the traffic lights in this question, because everyone can tell which light is which by whether its on the top or bottom of the traffic light].

(a) In Mavis's frame of reference, what is the distance between the lights? [5]



Mavis sees length-contracted distance:

$$l = \frac{l_0}{\gamma} = 300 \times \sqrt{1-\left(\frac{4}{5}\right)^2}$$

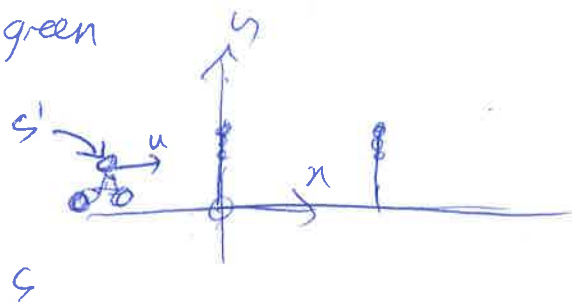
$$= 300 \times \sqrt{\frac{9}{25}}$$

$$= \underline{180\text{m}}$$

(b) Mavis goes through the second light just as it turns green in her frame. How much earlier or later than the first light did the second light turn on in her frame? Does she deserve a traffic ticket for 'running a red light', meaning she went past the first light when it was red in her frame? [5]

Event 1: 1st light turns green

Event 2: 2nd light turns green



E1: S (Stanley) S' (Mavis)

$x_1 = 0$ $x_1' = 0$

$t_1 = 0$ $t_1' = 0$

E2: S (~~Stanley~~ Stanley) S' (Mavis)

$x_2 = 300\text{m}$ $x_2' = ?$ (not needed)

$t_2 = 0$ $t_2' = ?$

Lorentz transformation: $t_2' = \gamma\left(t_2 - \frac{u x_2}{c^2}\right)$

$$= \frac{1}{\sqrt{1-(\frac{4}{5})^2}}\left(0 - \frac{(\frac{4}{5}c)(300)}{c^2}\right)$$

$$= -1.333\ldots \times 10^{-6}\,\text{s}$$

$$= -1.33\times10^{-6}\,\text{s} \quad \text{(earlier)}$$

2nd light turns earlier than first → 1st light was red when Mavis passed it → yes ticket!

(c) We often saw that 'simultaneity is relative', meaning events that were simultaneous in one inertial reference frame might not be simultaneous in another. Why then in this case can 'everyone' (meaning all observers in all intertial reference frames) agree whether the light was green or not when Mavis passed through it? [Hint: What are the relevant events?] [4]

Two events that occur at different points in space may be simultaneous in some frames but not in others. The traffic light turning green and Mavis passing the traffic light occur at the same point in space, and so must be simultaneous in all frames. [In other words, they are one event!]

$(\Delta t' = \gamma(\Delta t - 0) \Rightarrow \Delta t' = \gamma \Delta t)$.

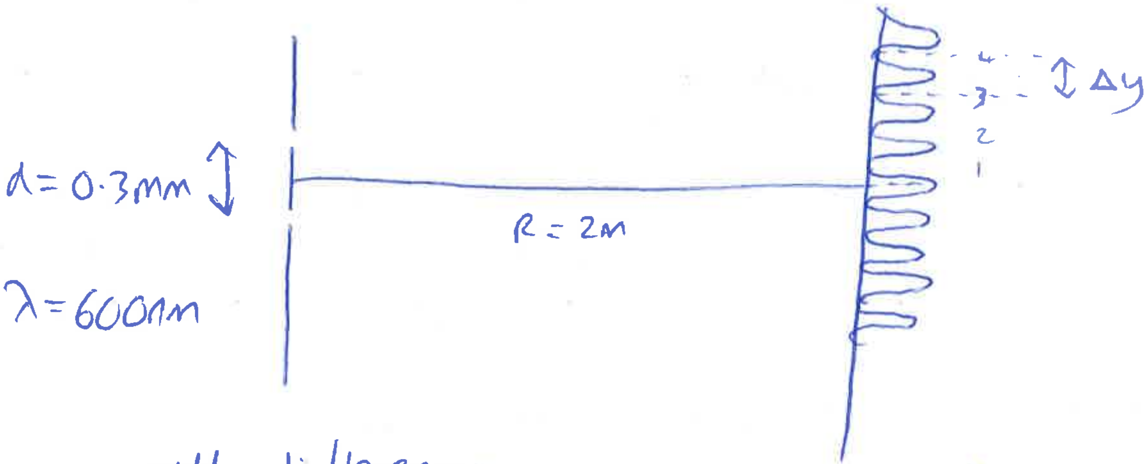
(d) In Mavis's frame, her kinetic energy is zero. But if Mavis's mass (sometimes called 'rest mass') is $m = 50$ kg, then her rest energy is $E_0 = mc^2$, which is 4.5×10^{18} Joules, which is her total energy in her frame. What is her kinetic energy in Stanley's frame (in Joules)? [4]

$$K = (\gamma - 1)mc^2 = \left(\frac{5}{3} - 1\right) \times 4.5 \times 10^{18}$$

$$= \underline{3 \times 10^{18}\ \text{J}}$$

9. Two thin slits spaced 0.300 mm apart are placed a distance of 2.0 m from a screen. They are illuminated with a laser of wavelength 600 nm.

(a) What is the distance between the third and fourth dark interference fringe? [6]



path difference: $\Delta r = d\sin\theta \approx \frac{dy}{R}$

dark fringe ⇒ destructive interference.

$$\therefore \Delta r = \frac{dy}{R} = \left(m+\frac{1}{2}\right)\lambda$$

$$\Rightarrow y = \frac{R\lambda}{d}\left(m+\frac{1}{2}\right)$$

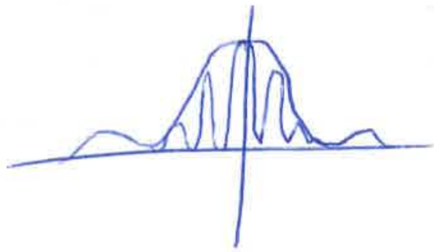
$$y_3 - y_2 = \frac{R\lambda}{d}\left[\left(3+\frac{1}{2}\right) - \left(2+\frac{1}{2}\right)\right]$$

$$= \frac{R\lambda}{d} = \frac{(2)(600\times10^{-9})}{0.3\times10^{-3}}$$

$$= 4\times10^{-3}\,m$$

(b) If the slits are now 0.1 mm wide, how many interference *minima* fit inside the first diffraction peak? [6]

New pattern:



minima of single-slit pattern: $\sin\theta = \frac{m\lambda}{a}$

central maximum: $-\frac{\lambda}{a} < \sin\theta < \frac{\lambda}{a}$

$\Rightarrow -6\times10^{-3} < \sin\theta < 6\times10^{-3}$

minima of ~~single~~ two-slit pattern: $\sin\theta = \left(m+\frac{1}{2}\right)\frac{\lambda}{d}$

$= \left(m+\frac{1}{2}\right)(2\times10^{-3})$

$m=0$: $\sin\theta = 1\times10^{-3}$ in range ✓

$m=1$: $= 3\times10^{-3}$ ✓

$m=2$: $= 5\times10^{-3}$ ✓

$m=3$: $= 7\times10^{-3}$ ✗

$m=-1$: $= -1\times10^{-3}$ ✓

$m=-2$: $= -3\times10^{-3}$ ✓

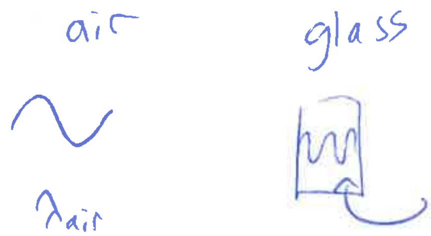
$m=-3$: $= -5\times10^{-3}$ ✓

$m=-4$: $= -7\times10^{-3}$ ✗

6 minima

(c) Normally the interference pattern has a bright line at its centre. If you cover one of the slits with glass ($n = 1.4$), what is the minimum thickness to change the bright line to a dark line? [6]

[Hint: for simplicity, the angles are small enough that you can assume the light enters and leaves the glass perpendicularly to its surface.]



$$\lambda_{glass} = \frac{\lambda_{air}}{n}$$

There is an extra phase difference:

$$\Delta\phi = \phi_{glass} - \phi_{air}$$

$$= \frac{2\pi}{\lambda_{glass}} t - \frac{2\pi}{\lambda_{air}} t$$

$$= \frac{2\pi t}{\lambda_{air}} \left[n - 1 \right]$$

Extra phase difference should be π (to change constructive to destructive, and vice versa).

$$\pi \stackrel{!}{=} \Delta\phi = \frac{2\pi t}{\lambda_{air}} (n-1)$$

$$\Rightarrow \quad 1 = \frac{2t}{\lambda}(n-1) \quad \Rightarrow \quad t = \frac{\lambda}{2(n-1)}$$

$$= \frac{(600 \times 10^{-9})}{2(0.4)} = \underline{7.5 \times 10^{-7}\ m}$$