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MIDTERM II

Physics 1C - Dantchev - Summer Quarter 2017

Good luck!

Indicate your reasoning clearly and include all essential calculations on the pages given. If your work is not entirely on the given problem's page, indicate exactly on what page it continues, or is located. Use the other side of the page if you need that much extra space. It is in your interest to have the answer clearly stated at the end of the solution of a given problem.

Start with the problem that is simplest for you, solve it and go to the next one. First solve those parts of the problems that you find easy. After that concentrate on more difficult parts.

Problem **1**

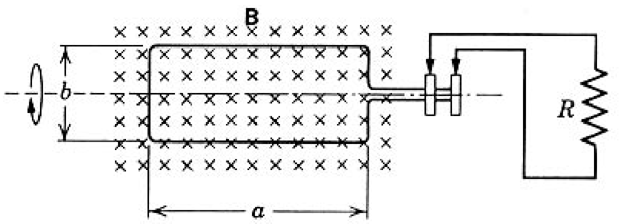
As a new electrical engineer for a given power company, you are assigned the project of designing a commercial alternating current generator based on a rectangular loop of N turns, each of length $a = 2\ m$ and width $b = 0.5\ m$, that is rotating in a uniform magnetic field $B = 1\ mT$ perpendicular to the axis of the rotation of the loop (see the figure).

A) What should be the number of turns in such a generator if it is to be used in the US, where it has to supply $\mathcal{E}_{max} = 170\ V$ at $f = 60\ Hz$. Here $\mathcal{E}_{max}$ is the maximal magnitude of the produced emf.(The well-known value of $\mathcal{E}_{rms} = 120\ V$ in a commercial power supply (in the US) is the so-called root-mean-square value of $\mathcal{E}$. It is connected to $\mathcal{E}_{max}$ by $\mathcal{E}_{max} = \sqrt{2}\mathcal{E}_{rms}$.)

B) Find N if you have to design such a generator for Europe, where it has to supply $\mathcal{E}_{max} = 340\ V$ at $f = 50\ Hz$.

C) How will the supplied emf $\mathcal{E}$ vary with time in the both cases? Make a sketch.

D) How does the flux Φ_B through the loop depend on the time? Make a sketch.



Solution of Problem 1

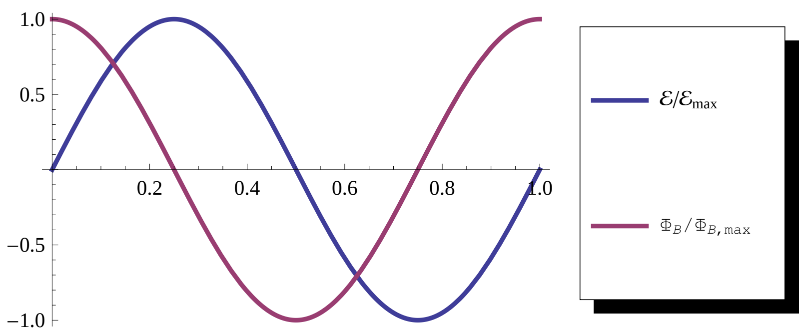
A) $\mathcal{E}_{\max} = \omega(NBA), A = ab, \omega = 2\pi f \Rightarrow \mathcal{E}_{\max} = 2\pi f NBab$. Thus

$$N_{\text{USA}} = \frac{\mathcal{E}_{\max}^{\text{USA}}}{2\pi f NBab} = 451 \quad \text{turns.}$$

B)

$$N_{\text{Europe}} = \frac{\mathcal{E}_{\max}^{\text{Europe}}}{2\pi f NBab} = 1082 \quad \text{turns.}$$

C) and D) In both cases $\mathcal{E}$ and Φ_B will vary sinusoidally with time $\mathcal{E} = \mathcal{E}_{\max}\sin(2\pi ft)$, $\Phi_B = \Phi_{B,\max}\cos(2\pi ft)$, where $f = 60$ Hz in USA and $f = 50$ Hz in Europe. Here $\Phi_{B,\max} = Bab$.

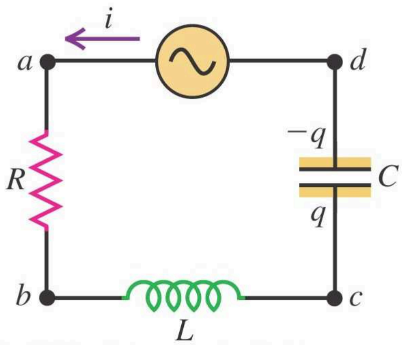


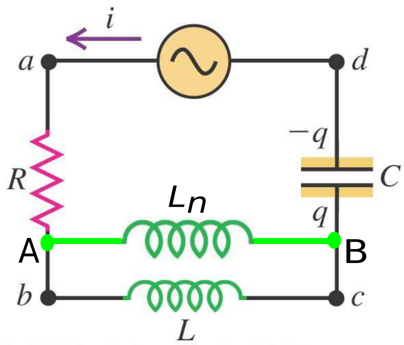
Problem **2**

In the $L-R-C$ series circuit shown in the left figure it is known that $v = V\cos(\omega t)$, where $f = \omega/(2\pi) = 60$ Hz, the voltage amplitude of the source is $V = 120\ V$, $R = 80.0\ \Omega$, and the reactance of the capacitor is $X_C = 480\ \Omega$. The voltage amplitude across the capacitor is $V_C = 360\ V$.

For the circuit in the left figure: 1. What is the current amplitude I in the circuit? 2. What is the impedance Z of the circuit? 3. What two values can the reactance of the inductor have? For which of these two values is the angular frequency less than the resonance angular frequency? Explain your answer. 4. Determine the average power P dissipated in the circuit. 5. Determine the possible values of the phase angle Φ of the current $i = I\cos(\omega t - \Phi)$ in the circuit.

A student makes a connection between the points A nd B (right figure) through an inductor $L = 1\ H$ and measures the current i. He establishes that the current i differs from the one on the left figure, i.e., he measures different I and Φ. Can you find 6) what are the possible values of I and 7) of Φ he measures?





Solution of Problem 2

1. One has $V_C = IX_C$, thus $I = V_C/X_C = 0.750\ A$.

2. One has $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $Z = VI$. One knows V from the problem and I from 1). Thus, $Z = V/I = 160\ \Omega$.

3. Since $Z = \sqrt{R^2 + (X_L - X_C)^2}$ one has $Z^2 - R^2 = (X_L - X_C)^2$. Thus, $X_L - X_C = \pm\sqrt{Z^2 - R^2}$, i.e. $X_L = X_C \pm \sqrt{Z^2 - R^2}$. Thus, it is possible that $X_L = 619\ \Omega$, or $X_L = 341\ \Omega$. At the resonant frequency $X_L = X_C$, with X_C fixed at its given value of $X_C = 480\ \Omega$. Obviously, since $X_L = \omega L$ and $341 < 480$, $X_L = 341\ \Omega$ corresponds to the frequency smaller than the resonant ones.

4. One has $P = \frac{1}{2}IV cos\phi = \frac{1}{2}IVR/Z = \frac{1}{2}I^2R = 22.5W$

5. One has $\tan\phi = \frac{X_L - X_C}{R} = \pm\sqrt{Z^2 - R^2}/R = \pm 1.74$. Thus, $\phi = \pm 60°$.

6. and 7.

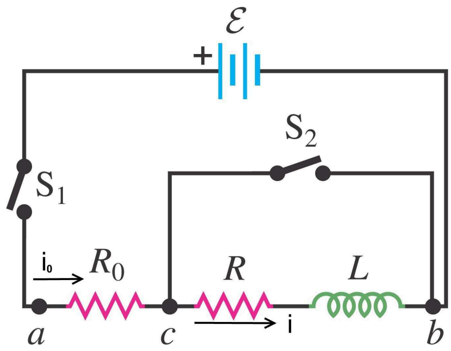
The two inductors are connected in parallel and are equivalent to a single inductor with $L_{\text{eff}}^{-1} = L^{-1} + L_n^{-1}$ (see problem Alternative Exercise 30.91 from the homework #2), i.e., $X_{L_{\text{eff}}}^{-1} = X_L^{-1} + X_{L_n}^{-1}$. For the corresponding $R - L_{\text{eff}} - C$ circuit one has $\tan\Phi = (X_{L_{\text{eff}}} - X_C)/R$ and $I = V/\sqrt{R^2 + (X_{L_{\text{eff}}} - X_C)^2}$. With $X_L = 619\ \Omega$ one obtains $X_{L_{\text{eff}}} = 234.297\ \Omega$ and, thus $\tan\Phi = -3.071$, i.e., $\Phi = -71.965°$, $I = 0.464$ A. With $X_L = 341\ \Omega$ one obtains $X_{L_{\text{eff}}} = 179.047\ \Omega$ and, thus $\tan\Phi = -3.7626$, i.e., $\Phi = -75.114°$, $I = 0.385$ A.

Problem **3**

In the circuit shown in the figure $\mathcal{E} = 36.0\ V$, $R_0 = 50.0\ \Omega$, $R = 150\ \Omega$ and $L = 4.00\ H$.

Initially, both the switch S_1 and the switch S_2 are left open. a) Find the values of the currents i_0 through R_0 and i through R, as well as the potential differences v_{ab} and v_{cb}, immediately after the switch S_1 is closed. b) Find the expressions for $i_0(t)$, $i(t)$, $v_{ac}(t)$ and $v_{cb}(t)$ as a function of time t. c) With the switch S_2 still open what will be the values of i_0, i, v_{ab} and v_{cb} after S_1 has been closed for a very long time, so that the currents have reached their steady final values?

After the currents have reached their steady values with S_1 closed and S_2 open, the switch S_2 has been closed too. d) What are i_0, i, v_{ab} and v_{cb} just after S_2 is closed. e) What are i_0, i, v_{ab} and v_{cb} a long time after S_2 is closed.



Solution of Problem 3

a) Once the switch S_1 is closed one obtains a complete circuit with $\mathcal{E}$, R_0, R and L connected in series. Thus, $i = i_0$. At the moment $t = 0$ the inductor blocks the circuit and, therefore, $i(t = 0) = i_0(t = 0) = 0$. In this case $v_{ac}(0) = i_0(0)R_0 = 0$. From the loop rule $\mathcal{E} = v_{ab} = v_{ac} + v_{cb}$. Thus, $v_{cb}(0) = \mathcal{E} - v_{ac}(0) = \mathcal{E}$.

b) At any instant $i(t) = i_0(t)$. The two resistors are equivalent to one with resistance $R_{\rm tot} = R_0 + R = 200\ \Omega$. The circuit is then equivalent to the $L - R$ one studied in lecture with R replaced by $R_{\rm tot}$. Thus, one has

$$i(t) = i_0(t) = \frac{\mathcal{E}}{R_{\rm tot}}\left[1 - \exp\left(-t/\tau_L\right)\right], \quad \text{where} \quad \tau_L = \frac{L}{R_{\rm tot}} = 0.020\ \text{s}.$$

In addition $v_{ab} = \mathcal{E}$, while

$$v_{ac}(t) = R_0 i_0(t) = \frac{\mathcal{E}R_0}{R_{\rm tot}}\left[1 - \exp\left(-t/\tau_L\right)\right], v_{cb}(t) = \mathcal{E} - v_{ac}(t) = \mathcal{E}\left\{1 - \frac{R_0}{R_{\rm tot}}\left[1 - \exp\left(-t/\tau_L\right)\right]\right\}$$

c) When $t \to \infty$, from *b*) one obtains $i(\infty) = i_0(\infty) = \mathcal{E}/R_{\rm tot} = 0.180\ A$; $v_{ac}(\infty) = R_0\ i_0(\infty) = 9.0\ V$; $v_{cb}(\infty) = \mathcal{E} - v_{ac}(\infty) = 36 - 9 = 27\ V$; $v_{ab} = 36$ V.

d) Immediately after the switch S_2 is closed, the inductor maintains the current trough R, thus $i = 0.180\ A$ (see c)). Since c and b are now connected with conductor of zero resistance $v_{cb} = 0$. From the internal loop containing only R_0 and $\mathcal{E}$, one has $\mathcal{E} - i_0 R_0 = 0$, thus $i_0 = \mathcal{E}/R_0 = 36/50 = 0.720\ A$ and $v_{ac} = i_0 R_0 = 36\ V$.

e) After a long time $i = 0$, $v_{cb} = 0$, $i_0 = \mathcal{E}/R_0 = 36/50 = 0.720\ A$, $v_{ac} = i_0 R_0 = 36\ V$.

Problem 4

An airplane flying at a distance of 11.3 km from a radio transmitter receives a signal of intensity 7.83 $\mu W/m^2$. Calculate

a) the amplitude of the electric field E_m at the airplane due to this signal.
b) the maximum value of the magnetic field B_m at the airplane.
c) the total power radiated by the transmitter, assuming the transmitter to radiate uniformly in all directions.
d) what is the average value of the Poynting vector S and its direction at the airplane? Make a sketch.

Solution of Problem 4

a)

$$I = 7.83 \times 10^{-6}\ \mathrm{W/m^2} = \frac{1}{2\mu_0 c} E_m^2 \Rightarrow E_m = \sqrt{2\mu_0 c I} = 76.8\ \mathrm{mV/m}.$$

b) $B_m = E_m/c = 256\ \mathrm{pT} = 2.56 \times 10^{-10}\mathrm{T}$.
c) $P = 4\pi r^2 I$, where $r = 11.3$ km $\Rightarrow P = 12.56\ kW$.
d) $\overline{S} = I = 7.83\ \mu\mathrm{W/m^2}$.