Name...................................

Student ID.........................

MIDTERM I

Physics 1C - Dantchev - Summer Quarter 2017

Good luck!

Indicate your reasoning clearly and include all essential calculations on the pages given. If your work is not entirely on the given problem's page, indicate exactly on what page it continues, or is located. Use the other side of the page if you need that much extra space. It is in your interest to have the answer clearly stated at the end of the solution of a given problem.

Start with the problem that is simplest for you, solve it and go to the next one. First solve those parts of the problems that you find easy. After that concentrate on more difficult parts.

Problem **1**

A group of particles is traveling in a magnetic field of unknown magnitude and direction. You observe that a proton moving at 2.50 km/s in the $+x$ direction experiences a force of 2.50×10^{-16} N in the $+y$ direction and an electron moving at 4.75 km/s in the $-z$ direction experiences a force of 4.75×10^{-16} N in the $+y$-direction.

(a) What are the magnitude and direction of the magnetic field?

(b) What are the magnitude and direction of the magnetic force on an electron moving in the $-y$-direction at 3.20 km/s?

c) If the proton is replaced by a positron moving with the same velocity what will be the magnitude and the direction of the force acting on it? (The positron has an electric charge of $+1$ e, a spin of $1/2$, and the same mass as an electron. The positron is the antiparticle or the antimatter counterpart of the electron.)

Solution of Problem 1

Since $\vec{F} = q\vec{v} \times \vec{B}$ one has that $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$.

a) Since the force on the proton is $\vec{F}_p = \vec{j}F_p$, with $F_p = 2.50 \times 10^{-16}$ N, one shall have no component of $\vec{B}$ along $\vec{j}$, i.e., one has $B_y = 0$. Thus, $\vec{B} = \vec{i}B_x + \vec{k}B_z$, i.e., the magnetic field is in the xz-plane. We know that $\vec{v}_p = \vec{i}v_p$, with $v_p = 2.50$ km/s. Then for the force on the proton we have $\vec{F}_p = e\vec{v}_p \times \vec{B} = e[\vec{i}v_p] \times [\vec{i}B_x + \vec{k}B_z] = -e\vec{j}v_pB_z$. Therefore,

$$B_z = -F_p/(ev_p) = -0.625 \text{ T}.$$

For the electron we have $\vec{F}_e = -e\vec{v}_e \times \vec{B}$. One has $F_e\vec{j} = (-e)(-\vec{k}v_e) \times [\vec{i}B_x + \vec{k}B_z] = ev_eB_x\vec{j}$. Solving for B_x gives

$$B_x = F_e/(ev_e) = 0.625 \text{ T}.$$

The magnitude of the field is $B = \sqrt{B_x^2 + B_z^2} = 0.884$ T.

The magnetic field is in the xz-plane. Calling θ the angle that the magnetic field makes with the $+x$-axis, we have $\tan\theta = B_z/B_x = -1$, so $\theta = -45.0°$.

Therefore the magnetic field is in the xz-plane directed at $45.0°$ from the $+x$-axis toward the $-z$-axis, having a magnitude of 0.884 T.

b) For the electron it is given that $\vec{v}_e = -\vec{j}v_e$, with $v_e = 3.20$ km/s. Then, for the force on the electron we obtain $\vec{F}_e = (-e)\vec{v}_e \times \vec{B} = (-e)[-\vec{j}v_e] \times [\vec{i}B_x + \vec{k}B_z] = ev_e[-\vec{k}B_x + \vec{i}B_z] = [-3.20 \times 10^{-16}\vec{i} - 3.20 \times 10^{-16}\vec{k}]$ N. For magnitude of the force one has $F = \sqrt{F_x^2 + F_z^2} = 4.525 \times 10^{-16}$ N.

Calling θ the angle that the force makes with the $-x$-axis, we have $\tan\theta = F_z/F_x = 1.$, which gives $\theta = 45.0°$. The force is in the xz-plane and is directed at $45.0°$ from the $-x$-axis toward the $-z$-axis.

c) The magnetic force on the positron will be the same as that one of the proton because the force depends only on the charge and the velocity of the particle which are given to be the same. The force does not depend, e.g., on the mass of the particle.

Problem 2

The figure shows two long wires carrying currents I_1 and I_2, respectively. The rectangular loop carries a current I. Calculate:

a) the resultant force (give magnitude and direction) acting on the sides AB, DC, AD, and BC due to the magnetic fields of the two wires;

b) the net force acting on the loop due to the straight wires;

c) the magnetic dipole moment of the loop $ABCD$ - give magnitude and direction;

d) the net torque on the loop due to the straight wires.

Assume that $|AB| = a = 30$ cm, $|BC| = b = 22$ cm, $h_1 = 3$ cm, $h_2 = 1$ cm, $I_1 = 25$ A, $I_2 = 10$ A, and $I = 40$ A.
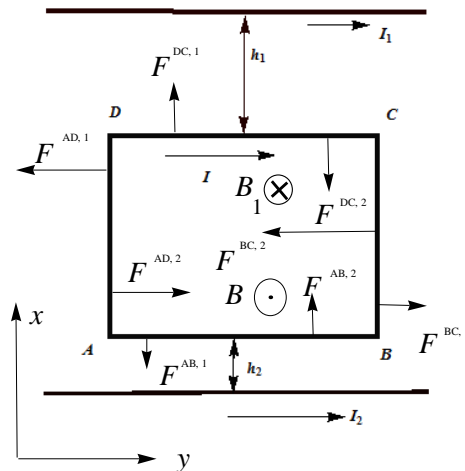
Solution of Problem 2

a) In the region of the loop the field $\vec{\mathbf{B}}_1$, due to the wire 1, is into the page, while $\vec{\mathbf{B}}_2$ is out of the page. The forces on the sides of the loop due to $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ are shown in the figure. One has:

$$F_y^{AB} = 0; \quad F_x^{AB} = F_x^{AB,2} - F_x^{AB,1} = \frac{\mu_0}{2\pi} a \frac{I_2 I}{h_2} - \frac{\mu_0}{2\pi} a \frac{I_1 I}{h_1 + b} = \frac{\mu_0}{2\pi} aI \left(\frac{I_2}{h_2} - \frac{I_1}{h_1 + b} \right) \simeq 2.16 \times 10^{-3} \text{N}.$$

$$F_y^{DC} = 0; \quad F_x^{DC} = F_x^{DC,1} - F_x^{DC,2} = \frac{\mu_0}{2\pi} a \frac{I_1 I}{h_1} - \frac{\mu_0}{2\pi} a \frac{I_2 I}{h_2 + b} = \frac{\mu_0}{2\pi} aI \left(\frac{I_1}{h_1} - \frac{I_2}{h_2 + b} \right) \simeq 1.90 \times 10^{-3} \text{N}.$$

$$F_x^{AD} = 0; \quad F_y^{AD} = F_y^{AD,2} - F_y^{AD,1} = \frac{\mu_0}{2\pi} I_2 I \int_{h_2}^{h_2+b} \frac{dx}{x} - \frac{\mu_0}{2\pi} I_1 I \int_{h_1}^{h_1+b} \frac{dx}{x} = \frac{\mu_0}{2\pi} I \left(I_2 \ln \frac{h_2+b}{h_2} - I_1 \ln \frac{h_1+b}{h_1} \right)$$
$$\simeq -1.7 \times 10^{-4} \text{N}.$$

$\vec{\mathbf{F}}^{BC} = -\vec{\mathbf{F}}^{AD}$. Thus

$$F_x^{BC} = 0, \quad F_y^{BC} = 1.7 \times 10^{-4} \text{N}.$$

b) The net force on the loop is $\vec{\mathbf{F}} = \vec{\mathbf{F}}^{AB} + \vec{\mathbf{F}}^{BC} + \vec{\mathbf{F}}^{DC} + \vec{\mathbf{F}}^{AD} = \vec{\mathbf{F}}^{AB} + \vec{\mathbf{F}}^{DC}$. Thus, it follows that $F_y = 0$, $F_x = F_x^{AB} + F_x^{DC} = 4.06 \times 10^{-3} \text{N}$.

c) $\vec{\mu} = I\vec{\mathbf{A}}, |\vec{\mathbf{A}}| = ab$; $\vec{\mathbf{A}}$ points into the page in accord with the direction of the current shown on the figure. Thus $\mu = Iab = 2.64$ A.m^2.

d) The net torque on the loop is $\vec{\tau} = 0$ because:

- Explanation 1: $\vec{\mu}$ is colinear with both $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$.

- Explanation 2: All the forces lie in the same plane as the loop.

Problem **3**

A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c, see the figure. The central conductor and tube carry equal currents I in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude $B(r)$ of the magnetic field **B**

(a) at points inside the central, solid conductor, i.e., for $r < a$.

(b) at points outside the central, solid conductor but inside the tube, i.e., for $a < r < b$.

(c) at points inside the outer conductor, i.e., for $b < r < c$.

(d) outside the tube, i.e., for $r > c$.

(e) Repeat the above for the case in which the current in the central, solid conductor is I_1, the current in the tube is I_2, with $I_1 = I_2 = I$, and these currents are in the same direction rather than in opposite directions.
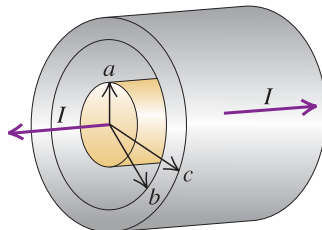
Solution of Problem 3

a) We apply the Ampere's law and choose a circular loop with radius r centered on the axis of the inner conductor. Because of the symmetry $\oint \mathbf{B}(r) \cdot d\mathbf{l} = B(r)(2\pi r)$, independent on the value of r. When $r < a$, for the enclosed current I_{encl} one has that $I/(\pi a^2)$ is the current density in the inner conductor, and, thus, $I_{enlc} = (\pi r^2) \times [I/(\pi a^2)] = Ir^2/a^2$. From $B(r)(2\pi r) = \mu_0 Ir^2/a^2$, solving for $B(r)$, one derives $B(r) = \mu_0 Ir/(2\pi a^2)$.

b) In that case the enclosed current is $I_{encl} = I$. Thus $B(r) = \mu_0 I/(2\pi r)$.

c) The current density in the outer conductor is $I/[\pi(c^2 - b^2)]$. In calculating the enclosed current if the contribution of the inner conductor is to be taken with a positive sign, the contribution of the outer conductor has to be with a negative sign. Thus, the enclosed current is $I_{encl} = I - I[\pi(r^2 - b^2)]/[\pi(c^2 - b^2)] = I\{1 - [(r^2 - b^2)]/(c^2 - b^2)\} = I(c^2 - r^2)/(c^2 - b^2)$. Solving for $B(r)$, one obtains $B(r) = \mu_0 I(c^2 - r^2)/[2\pi r(c^2 - b^2)]$.

d) In this case the enclosed current is $I_{encl} = I - I = 0$. Therefore $B(r) = 0$.

e) When $0 < r < a$ the answer stays the same as in the sub-case a): $B(r) = \mu_0 Ir/(2\pi a^2)$. When $a < r < b$ the answer is again the same as in sub-case b): $B(r) = \mu_0 I/(2\pi r)$. This is so, because the enclosed current in these two sub-cases is the same as in the previous situation with the currents in the two conductors in opposite directions. When case when $b < r < c$ for calculating the enclosed current one has to add the contributions from the inner conductor and the corresponding portion from the outer one. One obtains $I_{encl} = I + I[\pi(r^2 - b^2)]/[\pi(c^2 - b^2)] = I(r^2 + c^2 - 2b^2)/(c^2 - b^2)$ and, thus, $B(r) = \mu_0 I(r^2 + c^2 - 2b^2)/[2\pi r(c^2 - b^2)]$. For $r > c$ the enclosed currents is $I_{encl} = I + I = 2I$ and, therefore, $B(r) = \mu_0 I/(\pi r)$.