River Robles

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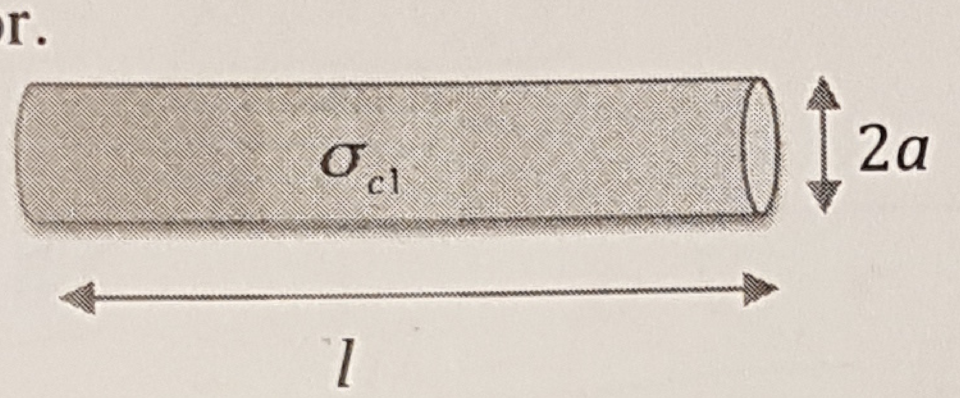
Physics 1BH – Prof. J. Rosenzweig – Wi...
Midterm 2
March 2, 2017

Use only the paper provided for you. Show all of your work for full credit. Write your name on each sheet of paper in your answers, then staple all together in order. You have 1:50 to complete this exam. You are permitted one sheet of paper as notes, with writing on both sides.

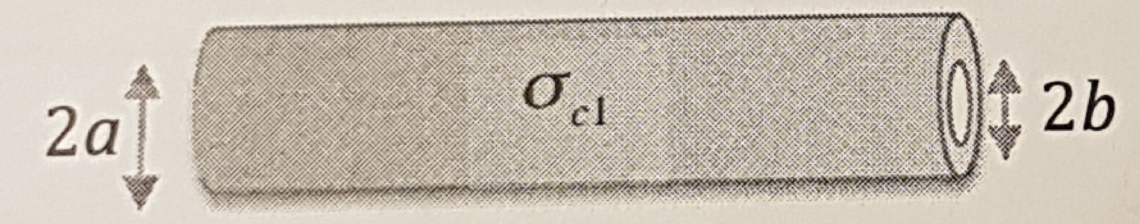
1. Consider a long cylindrical resistor, made of uniform material having conductivity denoted as σ_{c1}. The cylinder has radius a and length l. A voltage difference V is applied over the length of the resistor.



(a) (5 pts) Assuming it is uniform, what is the current density flowing in the resistor? (5 pts) What is the resistance R?

29 (b) (5 pts) In the limit that $a<<l$, the current flow in the resistor generates an azimuthal magnetic field B_ϕ interior to the cylinder ($r<a$) that approximates an infinitely long flow. Use Ampere's law to determine this magnetic field.

(c) Now consider the same geometry cylinder, same material, but cut a concentric cylindrical hole of radius b into it, as shown below.



(5 pts) What is the resistance of this object now? (5 pts) What is the magnetic field in the material? (5 pts) How does this compare to the answer in (b)

2. A charge is placed on the z-axis a distance d above a conducting plane, defined by z=0.
   (a) (5 pts) Where should the image charge be placed in order to guarantee that the z=0 plane is an equipotential (ϕ=0, assuming the potential vanishes at infinity); (5 pts) What is the magnitude of the image charge?

30 (b) (10 pts) Find the (purely normal) electric field at the z=0 surface.
   (c) (5 pts) From this field, find the axially symmetry surface charge density $\sigma(r)$. (5 pts) What is the total charge associated with $\sigma(r)$?

3. An infinite line of charge of linear density λ' is at rest on the z-axis in a certain frame.
   (a) (5 pts) What is the potential ϕ' associated with this line charge?
   (b) Now look at this line charge in a frame moving with respect to the z-axis, so that it appears to be moving forward in z at a speed v_f. (5 pts) What is the line charge density

34 λ in this frame? (5 pts) What is the potential ϕ associated with λ?
   (c) (5 pts) What is the current associated with the moving charge? (5 pts) What is the vector potential component A_z associated with the moving charge?
   (d) (10 pts) What is the Lorentz force experienced by a co-moving charge q at a radius r?

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$$\boxed{R = \frac{L}{\sigma A} = \frac{\ell}{\sigma_{c\ell} \pi a^2}} \qquad V = IR \rightarrow I = \frac{V}{R} = \frac{V\sigma_{c\ell}\pi a^2}{\ell}$$

a.

+9 $J = \frac{I}{A} = \frac{I}{\pi a^2} = \boxed{\frac{V\sigma_{c\ell}}{\ell}}$ direction

b. $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} = \frac{\mu_0 V \sigma_{c\ell} \pi r^2}{\ell}$

$I_{enc} = J A_0 \qquad I_{tot} = J A_{tot}$

$\frac{I_{enc}}{A_0} = \frac{I_{tot}}{A_{tot}}$

$\Rightarrow I_{enc} = \frac{V_0 \sigma_{c\ell} \cancel{\pi a^2}}{\ell} \cdot \frac{\pi r^2}{\cancel{\pi a^2}} = \frac{V\sigma_{c\ell}\pi r^2}{\ell}$

+5 $B \cdot 2\pi r = \frac{\mu_0 V \sigma_{c\ell} \pi r^2}{\ell} \Rightarrow \boxed{B = \frac{\mu_0 V \sigma_{c\ell} r}{2\ell} \hat{\varphi}}$

c. $d\ell$

$A = \pi a^2 - \pi b^2 \Rightarrow dR = \frac{d\ell}{\sigma_{c\ell} A} = \frac{d\ell}{\sigma_{c\ell}\pi(a^2-b^2)}$

$\Rightarrow \boxed{R = \frac{\ell}{\sigma_{c\ell}\pi(a^2-b^2)}}$

$V = IR \rightarrow I = \frac{V\sigma_{c\ell}\pi(a^2-b^2)}{\ell}$

$I_{enc} = J\cdot A_{enc} \qquad I_{tot} = J\cdot A_{tot}$

for $b < r < a$,

$\frac{I_{enc}}{A_{enc}} = \frac{I_{tot}}{A_{tot}}$

$\oint_c \vec{B}\cdot d\ell = \mu_0 I_{enc}$

$I_{enc} = \frac{V\sigma_{c\ell}\pi(a^2-b^2)}{\ell} \cdot \frac{\pi(r^2-b^2)}{\pi(a^2-b^2)}$

for $r < b$, $B \cdot 2\pi r = \mu_0 (0)$

$= \frac{V\sigma_{c\ell}\pi(r^2-b^2)}{\ell}$

+15 $\Rightarrow \boxed{B = 0 \text{ for } r < b}$

for $b < r < a$, $B \cdot 2\pi r = \mu_0 \frac{V\sigma_{c\ell}\pi(r^2-b^2)}{\ell}$

$$\boxed{\Rightarrow B = \frac{\mu_0 V \sigma_{c\ell}(r^2-b^2)}{2r\ell}\hat{\varphi} \quad \text{for } b < r < a}$$

we can split the fraction to get

$B = \left(\frac{\mu_0 V \sigma_{c\ell} r^2}{2r\ell} - \frac{\mu_0 V\sigma_{c\ell} b^2}{2r\ell}\right)\hat{\varphi}$

$B = \left(\frac{\mu_0 V\sigma_{c\ell} r}{2\ell} - \frac{\mu_0 V \sigma_{c\ell} b^2}{2r\ell}\right)\hat{\varphi}$

analysis on back

$2rx$

we see then that the new field is exactly what we had for b $\left(\frac{\mu_0 V\sigma}{2\ell}\right.$

with a component missing $\left(\frac{\mu_0 V \sigma_{ce} b^2}{2r\ell}\right)$ equivalent to the magnetic field giv

in part b, if $r=b$ $\left(\frac{\mu_0 V\sigma_{ce} b^2}{2b\ell} = \frac{\mu_0 V \sigma_{ce} b}{2\ell}\right)$.

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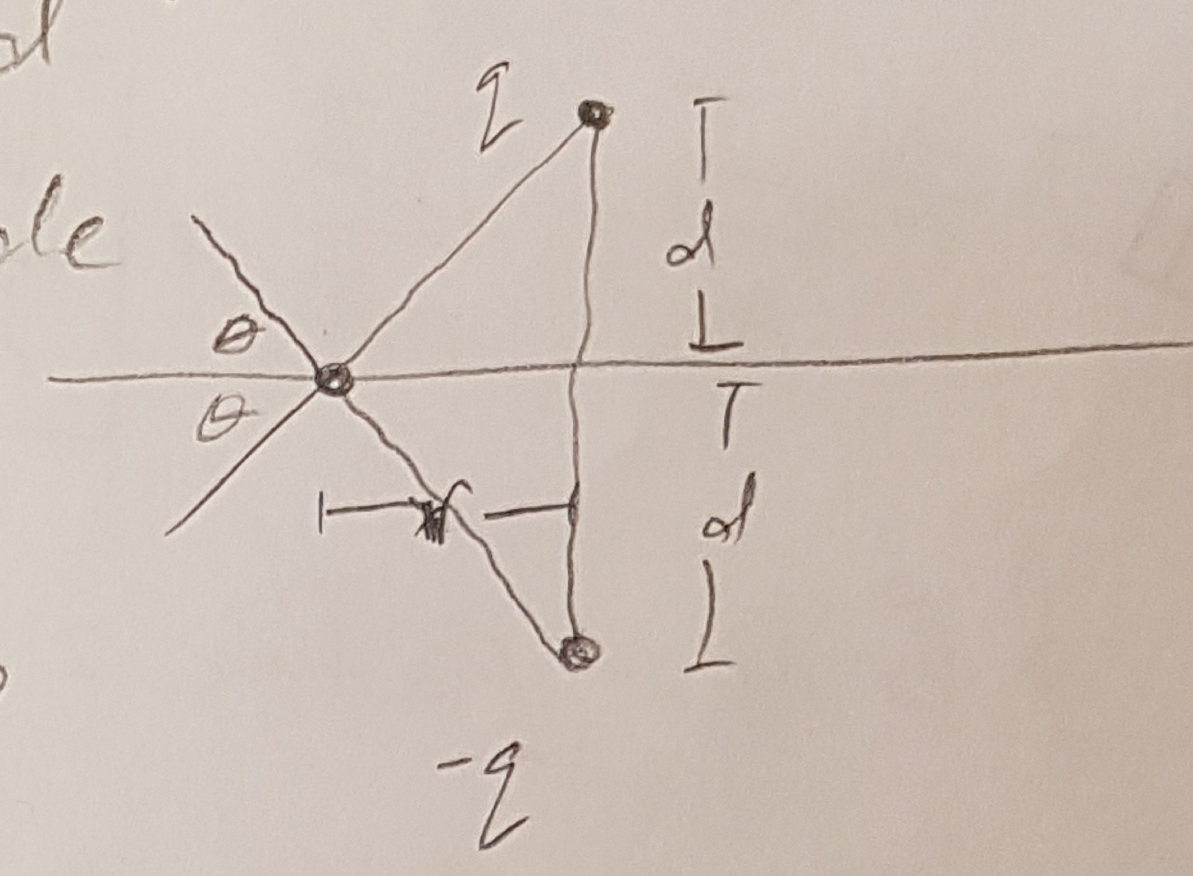
2 a. The image charge should be a distance d below the plane $\boxed{(z=-d)}$ with a magnitude equal and opposite the original $\boxed{(q'=-q)}$

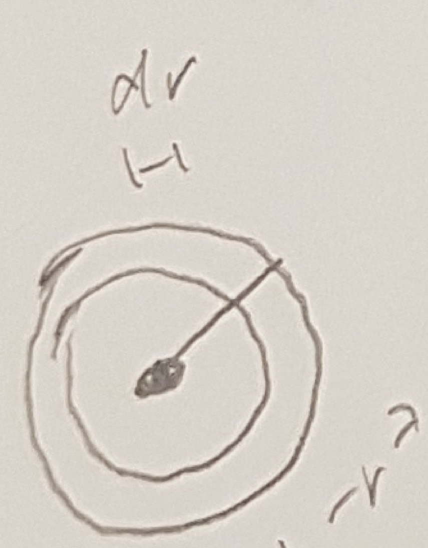


b. The horizontal field components will cancel due to symmetry.

$$E_y = E\sin\theta + E'\sin\theta = \frac{q(-\hat{z})}{4\pi\varepsilon_0(r^2+d^2)}\cdot\frac{d}{\sqrt{r^2+d^2}} - \frac{-q(-\hat{z})}{4\pi\varepsilon_0(r^2+d^2)}\cdot\frac{d}{\sqrt{r^2+d^2}}$$

$$\boxed{E_y = \frac{2qd}{4\pi\varepsilon_0(r^2+d^2)^{3/2}}(-\hat{z})}$$ +10

c. $$\sigma = E_\perp\varepsilon_0 = -\frac{2qd}{4\pi\varepsilon_0(r^2+d^2)^{3/2}}\cdot\varepsilon_0 = \boxed{-\frac{qd}{2\pi(r^2+d^2)^{3/2}}}$$ +5



$$Q = \iint \sigma\, da = \int_0^{2\pi}\int_0^{\infty} -\frac{qd}{2\pi(r^2+d^2)^{3/2}}\, r\,dr\,d\theta$$

$$= -qd\int_0^{\infty}\frac{r}{(r^2+d^2)^{3/2}}dr$$

$u = r^2+d^2$

$\frac{1}{2}du = r\,dr$

$$= -qd\int\frac{1}{2}\frac{du}{u^{3/2}} = -\frac{qd}{2}\cdot(-2u^{-1/2}) = qd\left.\frac{1}{\sqrt{r^2+d^2}}\right|_0^{\infty}$$

$$= qd\left(\frac{1}{\infty} - \frac{1}{\sqrt{0^2+d^2}}\right) = -qd\frac{1}{\sqrt{d^2}} = \boxed{-q}$$ +5

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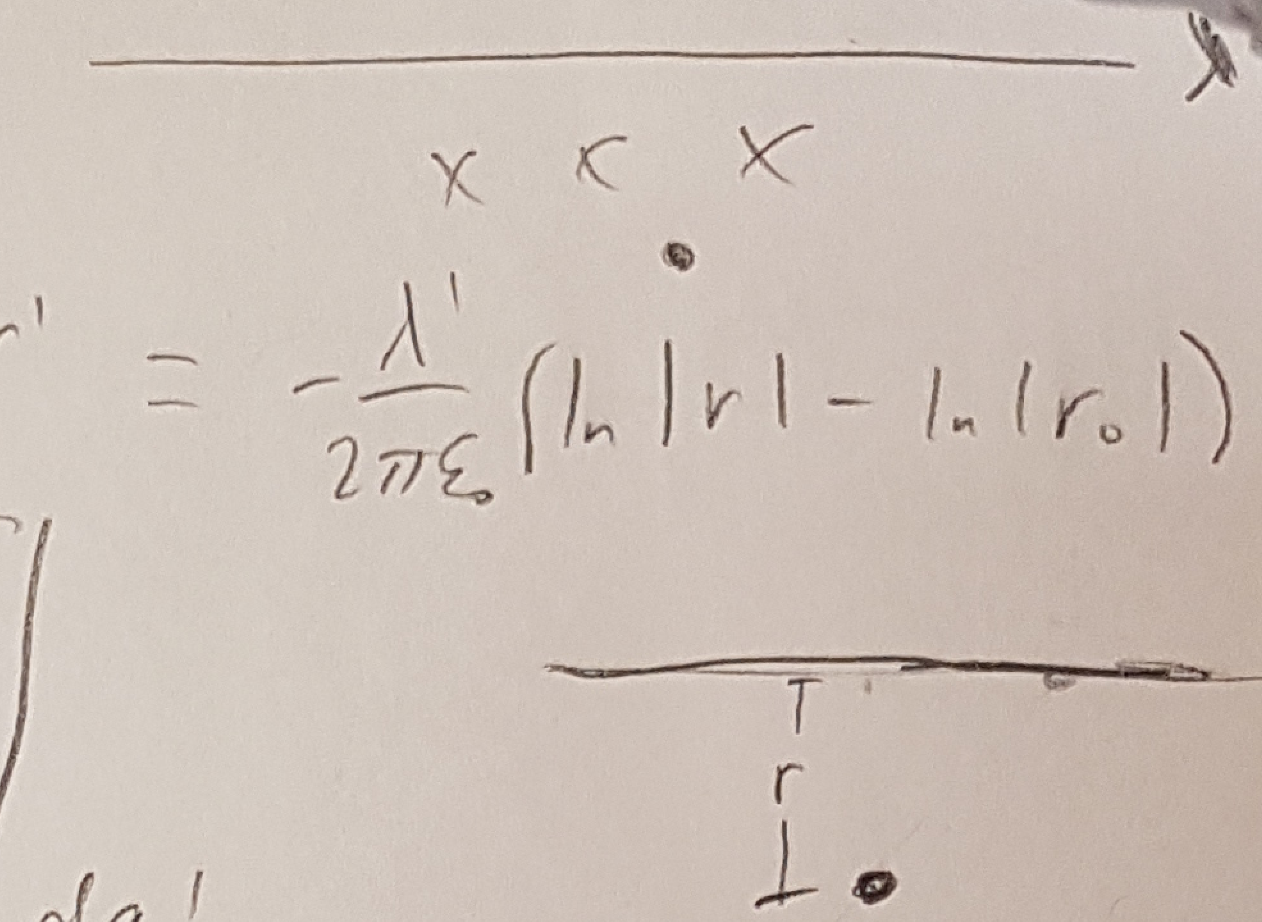
$E = \frac{\lambda'}{2\pi\varepsilon_0 r'}$ let $\varphi(r_0) = 0$



$\varphi'(r) = -\int_{r_0}^{r} \frac{\lambda'}{2\pi\varepsilon_0 r'} dr' = -\frac{\lambda'}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{1}{r'}dr' = -\frac{\lambda'}{2\pi\varepsilon_0}(\ln|r| - \ln|r_0|)$

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$= -\frac{\lambda'}{2\pi\varepsilon_0}\ln\left|\frac{r}{r_0}\right| = \boxed{\frac{\lambda'}{2\pi\varepsilon_0}\ln\left|\frac{r_0}{r}\right|}$

b. $\lambda' = \frac{dq'}{dl'}$ $\lambda = \frac{dq}{dl} = \frac{dq'}{\frac{1}{\gamma}dl'} = \gamma\frac{dq'}{dl'} = \gamma\lambda'$

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So $\boxed{\lambda = \gamma\lambda'}$ where $\gamma = \frac{1}{\sqrt{1-\frac{v_f^2}{c^2}}}$

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$E_\perp = \frac{\lambda}{4\pi\varepsilon_0 r} = \frac{\gamma\lambda'}{4\pi\varepsilon_0 r'} = \gamma E'$

$\varphi(r) = -\int_{r_0}^{r} \cancel{E}dr = -\int_{r_0}^{r}\gamma E'dr' = -\gamma\int_{r_0}^{r}E'dr' = \gamma\frac{\lambda'}{2\pi\varepsilon_0}\ln\left|\frac{r_0}{r}\right| = \gamma\varphi'$

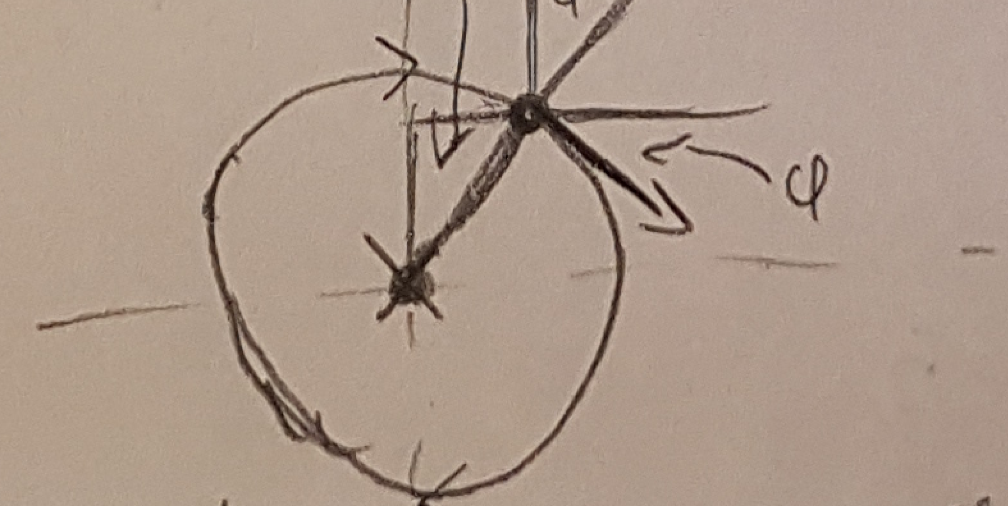
So $\boxed{\varphi(r) = \gamma\varphi'(r) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left|\frac{r_0}{r}\right|}$ ✓

+5

c. $\boxed{I = \lambda v_f = \gamma\lambda' v_f}$ +5 $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0\gamma\lambda' v_f}{2\pi r}\hat{\varphi}$

$\vec{B} = \vec{\nabla}\times\vec{A} = \hat{i}(\partial_y A_z - \partial_z A_y^{0}) - \hat{j}(\partial_x A_z - \partial_z A_x^{0}) + \hat{k}(\cancel{\partial_x A_y - \partial_y A_x}^{0})$

$\vec{B} = \partial_y A_z\,\hat{i} - \partial_x A_z\,\hat{j}$



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$B_x = \frac{\mu_0\gamma\lambda' v_f}{2\pi r}\cdot\cos\varphi\,\hat{i} = \frac{-\mu_0\gamma\lambda' v_f}{2\pi r}\cdot\frac{y}{r}\,\hat{i}$

$B_y = \frac{\mu_0\gamma\lambda' v_f}{2\pi r}\sin\varphi\,\hat{j} = \frac{\mu_0\gamma\lambda' v_f}{2\pi r}\cdot\frac{x}{r}\,\hat{j}$

$$B_x = \partial_y A_z \hat{i} = -\frac{\mu_0 \gamma \lambda' v_f}{2\pi r^2} y$$

$u = x^2 + y^2$

$\frac{1}{2} du = y\,dy$

$$\Rightarrow A_z = -\frac{\mu_0 \gamma \lambda' v_f}{2\pi} \int \frac{y}{x^2+y^2}\,dy = -\frac{\mu_0 \gamma \lambda' v_f}{2 \cdot 2\pi} \int \frac{du}{u} =$$

$$= -\frac{\mu_0 \gamma \lambda' v_f}{4\pi} \ln|u| + \psi(x) = -\frac{\mu_0 \gamma \lambda' v_f}{4\pi} \ln|x^2+y^2| + \psi(x)$$

$$= -\frac{\mu_0 \gamma \lambda' v_f}{4\pi} \ln|r^2| + \psi(x).$$

Clearly the x integral will give the same result, so

$$\boxed{A(r) = -\frac{\mu_0 \gamma \lambda' v_f}{4\pi} \ln|r^2|\,\hat{z}}$$

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d. $$F_\perp = q(\vec{E} + \vec{v}_f \times \vec{B}) = q\left(\frac{\lambda}{2\pi\varepsilon_0 r} - v_f \frac{\mu_0 I}{2\pi r}\right)$$

$$= q\left(\frac{\lambda}{2\pi\varepsilon_0 r} - \frac{\mu_0 \lambda v_f^2}{2\pi r}\right) = q\frac{\lambda}{2\pi r}\left(\frac{1}{\varepsilon_0} - \mu_0 v_f^2\right)$$

$c^2 = \frac{1}{\mu_0 \varepsilon_0}$

$\Rightarrow \mu_0 = \frac{1}{c^2 \varepsilon_0}$

so $$= \frac{q\lambda}{2\pi\varepsilon_0 r}\left(1 - \frac{v_f^2}{c^2}\right) = \frac{q\lambda}{2\pi\varepsilon_0 r}(1-\beta^2)$$

$$\boxed{F_L = \frac{q\lambda'}{2\pi\varepsilon_0 r} \frac{1}{\gamma^2}}$$ where $\lambda' = \gamma\lambda'$

+5