Kusenko W16

**Midterm exam 1**

Physics 1B, Spring 2016

Name: Owen Heyninger

, or TA name): Wednesday 4PM, Justin Little

Ple derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem.

13

**Problem 1.**

A rectangular flat-bottom barge with a bottom area $A = 100\,\mathrm{m}^2$ is loaded so that the bottom is at $H = 1\,\mathrm{m}$ below the surface. The density of water is $\rho = 10^3\mathrm{kg/m}^3$, and the water surface is perfectly still.

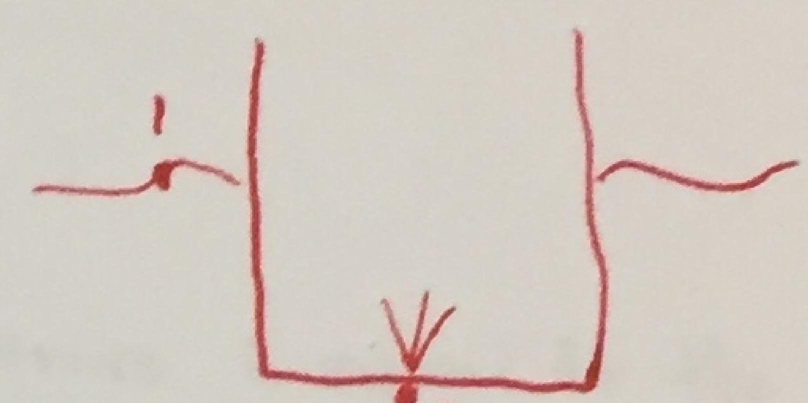
10 (a) Calculate the mass of the barge.

$F_g$ of the barge $= F_b$ $\qquad V_{\text{water displaced}} = 100\,m^2 \cdot 1\,m = 100\,m^3$

$mg = \rho g V$ $\qquad \rho g V =$ weight of water displaced

$m = 10^3\,kg/m^3 \cdot 100\,m^3$

$\boxed{m = 100000\,kg}$ ✓



$\rho g H = \frac{1}{2}\rho v^2$

3 (b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

$P$ at $1\,m = \rho g h = 10^3\,kg/m^3 \cdot 9.81\,m/s^2 \cdot 1\,m = 9.81\times10^3\,Pa$

Volume flow rate through hole = volume flow rate at surface of water in barge

$A_h v_h = A_b v_b \qquad \frac{v_b}{v_h} = \frac{A_h}{A_b} = \frac{\pi(.02\,m)^2}{100\,m^2} = 1.257\times10^{-5} \qquad v_b = (1.257\times10^{-5})v_h$

By Bernoulli's eqn,

$P + \rho g y + \frac{1}{2}\rho v^2 = \text{const.}$

$P_h + \frac{1}{2}\rho v_h^2 = \rho g h + \frac{1}{2}\rho v_b^2$

$9.81\times10^3\,Pa - 10^3\,kg/m^3 \cdot 9.81\,m/s^2 \cdot .05\,m = \frac{10^3\,kg/m^3}{2}(1.257\times10^{-5}\,v_h^2 - v_h^2)$

$\frac{dV}{dt} = \frac{Ah}{t} = \pi r^2 v$

$18.639\,\frac{m^2}{s^2} = v_h^2(-.99987)$

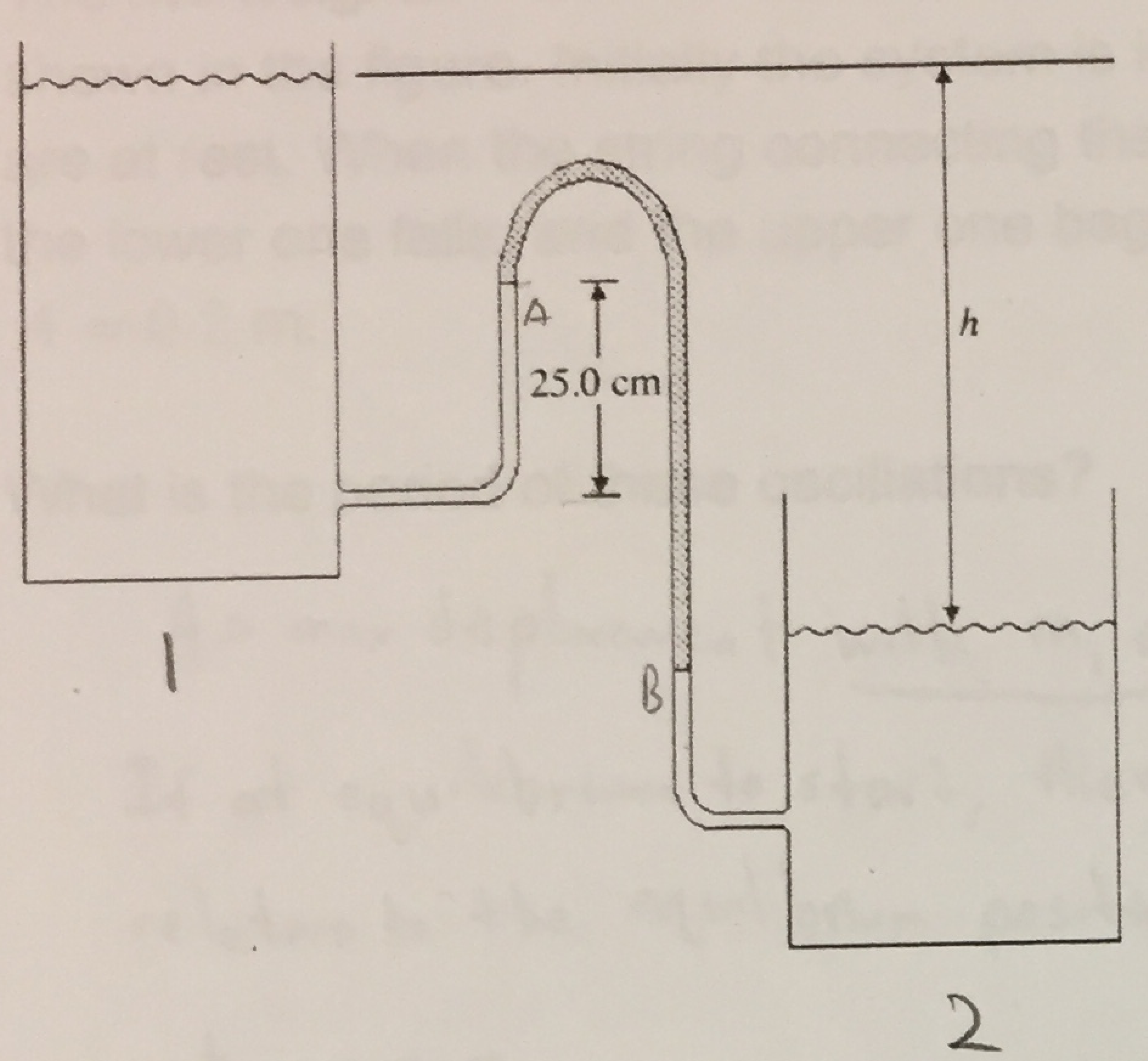
$v_h = 4.317\,m/s$

$t = \cancel{\frac{h}{v_b}} = \frac{.05\,m}{(1.257\times10^{-5})v_h} = \frac{.05\,m}{(1.257\times10^{-5})(4.317\,m/s)} = \boxed{922\,s}$ ✗

12

**Problem 2**

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m³. The manometer contains incompressible oil with a density of 820 kg/m³. What is the difference in elevation $h$ if the manometer reading $m$ is 25.0 cm?




The difference in pressure between the two reservoirs is equal to the pressure difference in the manometer, for the system to be in equilibrium.

Pressure difference in manometer: $P_A - P_B = P_1 - P_2 = \rho_{oil} g d$,

where $d$ = twice the manometer reading since each side is displaced by the amount of the reading.

$\rho_w g h = P_1 - P_2$

$\rho_w g h = \rho_{oil} g d$

$1000 \,\cancel{kg/m^3} \cdot h = 820 \,\cancel{kg/m^3} \cdot (2 \cdot .250\,m)$

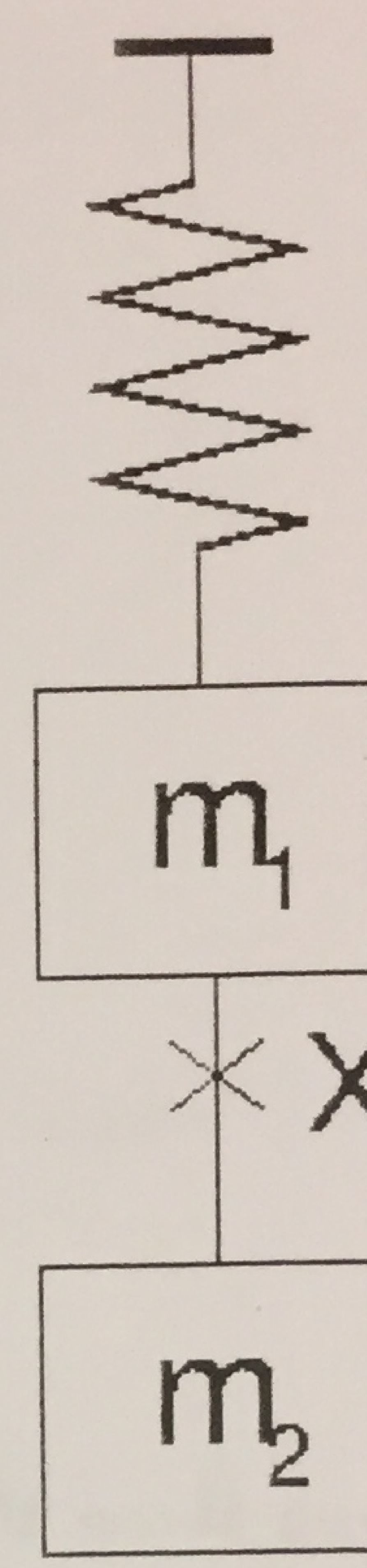
$h = \frac{820}{1000}(2 \cdot .250\,m)$

$\boxed{h = .41\,m}$

20

**Problem 3**

The two weights, $m_1 = 1\ \mathrm{kg}$ and $m_2 = 2\ \mathrm{kg}$ hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.




What is the period of these oscillations?

A = max displacement with $m_1$ on the spring = $x_{max}$

If at equilibrium to start, then $kx_m = m_2 g$ since $x_m$ is relative to the equilibrium position with $m_1$ on the spring.

$kx_m = m_2 g$

$kA = m_2 g$

$k(.2\ m) = (2\ kg)(9.81\ m/s^2)$

$k = 98.1\ N/m$

$T = 2\pi\sqrt{\frac{m}{k}}$ where $m = m_1$

$T = 2\pi\sqrt{\frac{1\ kg}{98.1\ N/m}} = .634\ s$

$\boxed{T = .6\ s}$

20

**Problem 4**

A simple pendulum has a length of 120 cm.

(a) What is its period of oscillations?

$$L = 120\ cm \cdot \frac{1\ m}{100\ cm} = 1.2\ m$$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.2\ m}{9.81\ m/s^2}}$$

$$= 2.198\ s$$

$$\boxed{T = 2.2\ s}$$

(b) What is the period of oscillations inside an elevator moving up with an acceleration 1.2 m/s$^2$

If elevator is accelerating upwards at 1.2 m/s$^2$, objects inside experience an acceleration of 1.2 m/s$^2$ downward. This means the effective acceleration of the restoring force on the pendulum is $a_{eff} = g + 1.2\ m/s^2$.

$$T = 2\pi\sqrt{\frac{L}{a_{eff}}} = 2\pi\sqrt{\frac{1.2\ m}{10.01\ m/s^2}} = 2.18\ s$$

$$\boxed{T = 2.2\ s}$$

$a_{eff}$ replaces $g$ since the denominator is the acceleration due to the restoring force

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

$$g_{mars} = .37\ g = .37 \cdot 9.81\ m/s^2 = 3.63\ m/s^2$$

$$T = 2\pi\sqrt{\frac{L}{g_{mars}}} = 2\pi\sqrt{\frac{1.2\ m}{3.63\ m/s^2}} = 3.613\ s$$

$$\boxed{T = 3.6\ s}$$

20

**Problem 5**

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?

Let the frequencies be of the first harmonic.

Then, $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$.

Both strings have the same $\mu$ and $L$.

Let $\frac{1}{2L\mu} = r$.

$f_1 = r\sqrt{F_1}$ $\quad$ $f_2 = r\sqrt{F_2}$

$F_1 = \frac{f_1^2}{r^2}$ $\quad$ $448.4\ Hz = r\sqrt{F_2}$

$F_1 = \frac{440.0^2 Hz^2}{r^2}$ $\quad$ $F_2 = \frac{448.4^2\ Hz^2}{r^2}$

$F_d$ = tuned tension

$F_d = F_1 = \frac{440.0^2\ Hz^2}{r^2}$

$\frac{440.0^2\ Hz^2}{r^2} = a F_2$

$a = \frac{440.0^2}{r^2} \cdot \frac{1}{F_2}$

$= \frac{440.0^2}{\cancel{r^2}} \cdot \frac{\cancel{r^2}}{448.4^2}$

$a = \frac{440.0^2}{448.4^2} = .9629$

$F_d = .96288 F_2$ $\qquad$ $1 - .96288 = .03712$

$F_d$ is 96.288% of $F_2$, so the musician should decrease the tension by $(100 - 96.288)\% \rightarrow$ $\boxed{\text{decrease tension by } 3.712\%}$ ✓

20

**Problem 6**

Standing waves of frequency 50 Hz are produced on a string that has mass per unit length 0.025 kg/m. With what tension must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.8 m apart?

distance between adjacent nodes $= \frac{\lambda}{2} = .8\,m$

$f = 50\,hz$ $\quad v = \lambda f$ $\quad \lambda = 2 \cdot .8\,m = 1.6\,m$

$v = \sqrt{\frac{F}{\mu}}$ $\quad \sqrt{\frac{F}{\mu}} = \lambda f$

$\frac{F}{\mu} = \lambda^2 f^2$

$F = \mu \cdot \lambda^2 f^2$

$F = .025\,kg/m \cdot (1.6\,m)^2 \cdot (50\,Hz)^2$

$F = 160\,kg\,m/s^2$

$\boxed{F = 160\,N}$ ✓

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| 2 | 12 |
| 3 | 20 |
| 4 | 20 |
| 5 | 20 |
| 6 | 20 |
| TOT | 105 |