PHYSICS 1B

MIDTERM 1

Spring, 2017 Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so please put them, and all cell phones, away. If you need more space, use the backside of the page.

Simon Rufer
Your **Full Name** - Printed Clearly

Your Normal Signature

ber

Problem	**Score**
1	30
2	18
3	20
4	25
Total	93

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$\omega =$

$A = \frac{2}{\cos \omega t}$ $\quad w = 2\pi$

$-4\pi = -2\omega \tan(\omega t)$ $\quad -4\pi = -\frac{2\omega}{\cos \omega t} \sin \omega t$

$x = 2$ $\quad v = -4\pi$

$A\cos(\omega t)$

$-A\omega \sin(\omega t)$.

$-4\pi = -A(2\pi)\sin($

(30 Pts)

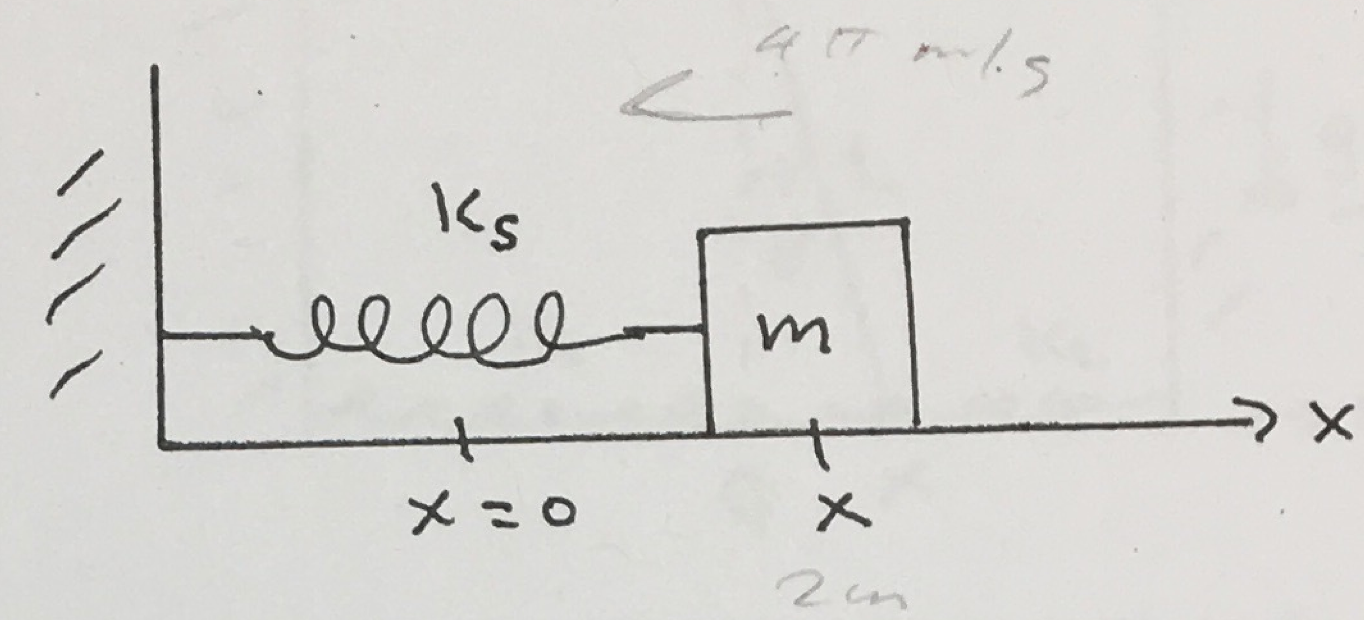
1. A block with mass $m = 2$ kg is attached to a spring with a stiffness constant $k_S = 8\pi^2$ N/m, and is free to oscillate on a smooth horizontal surface about the spring's equilibrium position $x = 0$. At time $t = 0$, the mass is observed to located at $x(0) = 2$ m, and to be moving with a speed $v(0) = 4\pi$ m/s toward $x = 0$. *Leave answers in terms of π and whole numbers.*

(4) a. Find the angular frequency (ω_0) for oscillations about $x = 0$, and the oscillation period.

(10) b. Find the function $x(t)$ that describes the position of the mass as a function of time $t \geq 0$.

(8) c. Find the velocity of the mass for the first time that it passes through $x = 0$. [Recall: $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.]

(8) d. Calculate the total mechanical energy E of the system, and the maximum displacement amplitude. [Recall the relation between E and the amplitude.]



$m = 2$ kg $\quad k_s = 8\pi^2$ N/m $\quad x = 0$ $\quad t = 0$: $x(0) = 2$ m $\quad v_0 = 4\pi$ m/s toward $x = 0$

a) $w = \sqrt{\frac{k_s}{m}} = \sqrt{\frac{8\pi^2}{2}} = \sqrt{4\pi^2} = \boxed{2\pi = w}$

$w = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{w}$ $\quad \boxed{T = 1 \text{ second}}$

$x(t) = A\cos(wt + \phi)$

b) $A = \sqrt{x_0^2 + \frac{v_0^2}{w_0^2}}$ $\quad V_0 = -4\pi$ m/s $\quad w_0 = 2\pi$

$\boxed{x(t) = A\cos(wt + \phi)}$

$= \sqrt{4 + \frac{16\pi^2}{4\pi^2}} = \sqrt{4 + 4} = \sqrt{8}$

$t = 0$: $\sqrt{8}\cos(x) = 2$ $\quad \frac{1}{\sqrt{2}} = \cos x$

$\boxed{x(t) = \sqrt{8}\cos\left(2\pi t + \frac{\pi}{4}\right)}$

0 when $t = \frac{1}{8}$

c) $v(t) = -2\pi\sqrt{8}\sin\left(2\pi t + \frac{\pi}{4}\right)$

when $x = 0$, $t = (2\pi)x = \frac{\pi}{4}$

$t = 1/8$ second

$= -2\pi\sqrt{8}\sin(\pi/2) = \boxed{-2\pi\sqrt{8} \text{ m/sec}} = -wA$

d) $\boxed{\max A = \sqrt{8} \text{ m}}$

$4\pi^2(8)$ $\quad \left(\frac{1}{2}\right) 8\pi^2(8)$

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}m(4\pi^2 8\sin^2(\ldots)) + \frac{1}{2}K(8\cos^2(\ldots))$

$E = \frac{1}{2}k_s A^2 = \frac{1}{2}(8\pi^2)(\sqrt{8})^2 = \boxed{32\pi^2}$

(20 Pts)

2. A pendulum with bob mass m and massless rod of length L hangs under gravity. The mass is attached to two horizontal springs with stiffness constants k_1 and k_2 that are unstretched when the mass hangs straight down. The mass is displaced by a small angle θ, and released to oscillate about the origin $x = 0$. Since $\theta \ll 1$, $\sin\theta \approx \theta$, $x \approx L\theta$, and the motion essentially occurs only in the x-direction.

(12) a. Recalling the $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $\tau = I\alpha$, $\alpha = d^2\theta/dt^2$, and $I = mL^2$ about the pivot point P, or just using $\mathbf{F} = m\,\mathbf{a}$ with $\mathbf{F}$ being only along x, prove that the angular frequency of small oscillations about $x = 0$ is

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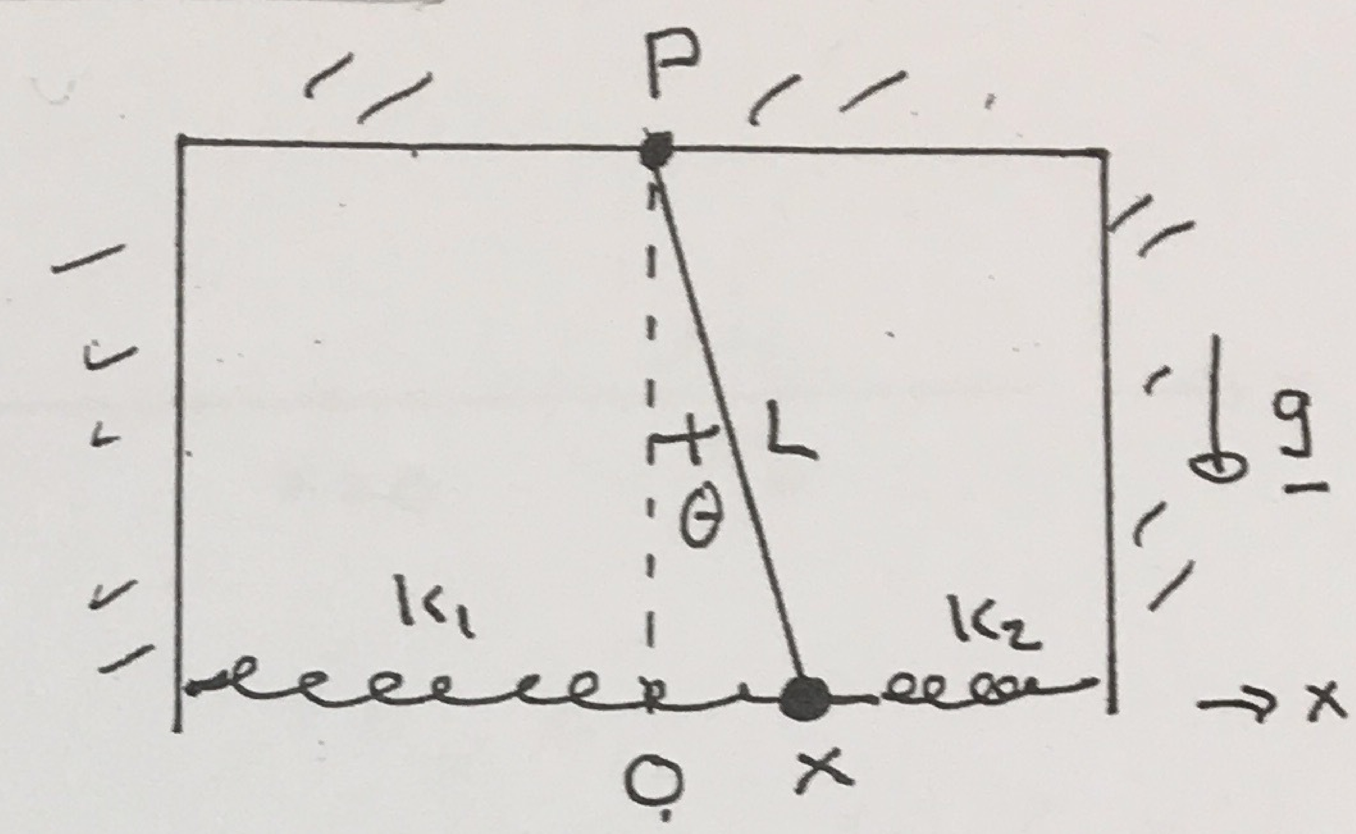
$$\omega_0 = \left[\frac{k_1 + k_2}{m} + \frac{g}{L}\right]^{1/2}$$



(8) b. Now suppose that the horizontal oscillations are driven by a force $F(t) = F_0 \sin(\omega t)$, and that the mass moves through water and experiences a force of friction $f = -b\,v$. Write the equation that now describes the motion (Newton's Second Law or its torque equivalent). If the driving frequency $\omega = \omega_0$, find $x(t)$.

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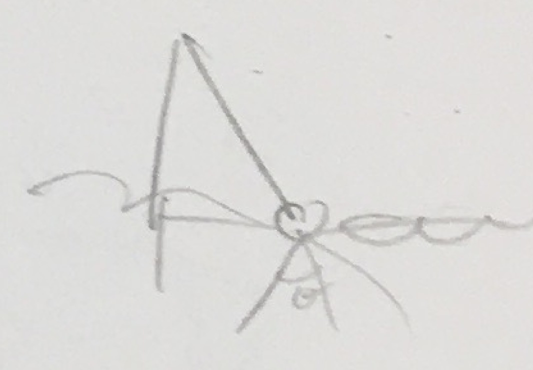
a) $\tau = r \times F \quad \tau = I\alpha$



$-mgl\sin\theta + k_1(l\theta)l + k_2(l\theta)l = I\alpha$

$\frac{1}{ml^2}\quad \theta(mgl + k_1 + k_2(l^2\theta)) = ml^2\ddot{\theta} \Leftarrow$ divide by ml^2

$-\theta\left(\frac{g}{l} + \frac{k_1 + k_2}{m}\right) = \ddot{\theta} \Longleftrightarrow \ddot{x} = \omega_0^2 x$

$\omega_0^2 = \frac{g}{l} + \frac{k_1 + k_2}{m} \qquad \boxed{\omega_0 = \left(\frac{g}{l} + \frac{k_1 + k_2}{m}\right)^{1/2}}$

b) $F(t) = F_0 \sin(\omega t) \qquad f = -bv = -b\omega l$

$v = \omega l$

$\Sigma\tau = I\alpha = r \times F$

$$\boxed{\frac{1}{ml^2}\left[mgl\theta + (k_1 + k_2)(l^2\theta) - b\dot{\theta}l^2 + F_0 \sin(\omega t)\right] = \ddot{\theta}}$$

if $\omega = \omega_0$, then $F_0 \sin(\omega t) = -b\dot{\theta}l^2$ (friction = driving)

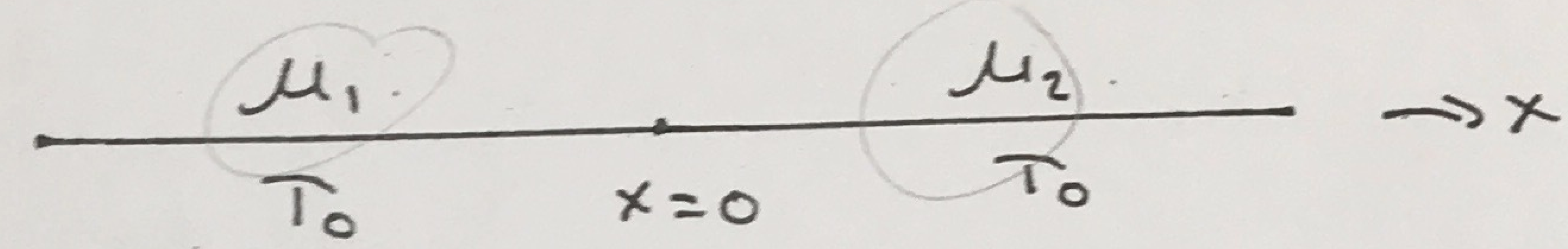
$\int \frac{F_0 \sin(\omega t)}{-bl^2} = \int \dot{\theta}$

$$\boxed{\theta(t) = -\frac{F_0}{bl^2\omega}\cos(\omega t)}$$

(25 Pts)

3. A very long stretched string with equilibrium tension T_0 consists of a piece to the left ($x < 0$) of the origin with mass per unit length μ_1, and a piece to the right ($x > 0$) of the origin with a mass per unit length μ_2. At the origin ($x = 0$), the string is attached to a motor that drives the string in the y-direction with a displacement $y(0, t') = A \sin(\omega t')$.

(6) a. Write (do NOT derive), the partial differential equation (the wave equation) that governs the space and time dependence of the displacement $y(x, t)$ of the string.



(10) b. Find $y(x, t)$ for the waves which are driven by the motor to travel in the positive and the negative x-directions.

T_0, μ_1

$y(0,t) = A\sin(\omega t')$

(9) c. Calculate the time-averaged power that is transported by each wave. Of the two waves, which transports the higher power? [Recall: power = **F** · **v**, and $F_y = -T_0\, \partial y/\partial x$, $v = \partial y/\partial t$; $\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = ½$.]

a) $\boxed{\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2} \qquad c = \sqrt{\dfrac{T_0}{\mu}}}$

$\dfrac{\partial^2 y}{\partial t^2} = \dfrac{T_0}{\mu}\dfrac{\partial^2 y}{\partial x^2}$ 4 $y(x,t) = A\sin(kx - \omega t)$

b) $kc = \omega \qquad c = \sqrt{\dfrac{T_0}{\mu}}$

$y(0,t) = A\sin(\omega t) \qquad k = \frac{\omega}{c}$

right travelling

$y(x,t) = A\sin\left(\frac{\omega}{c}x - \omega t\right)$ $1/c$

left travelling (negative)

$\boxed{y(x,t) = A\sin\left(\omega\left(\sqrt{\frac{\mu_1}{T_0}}x + t\right)\right)}$

$\boxed{\text{Positive: } y(x,t) = A\sin\left(\omega\left(\sqrt{\frac{\mu_2}{T_0}}x - t\right)\right)}$ 9

c) Time-av power

$P = F \cdot v = -T_0\, \partial y/\partial x \ \partial y/\partial t$

right travelling

$P = -T_0\, A\omega\sqrt{\frac{\mu_2}{T_0}}\ \cancel{\sin}\cos(\sim) \cdot A\omega\cos(\sim)$

$\langle P \rangle = -T_0 A^2 \sqrt{\frac{\mu_2}{T_0}}\, \omega^2 \langle \cos^2(\sim) \rangle$ 1/2

$\boxed{\langle P \rangle = \dfrac{-T_0 A^2 \sqrt{\frac{\mu_2}{T_0}}\, \omega^2}{2} = \dfrac{-T_0 A^2 \omega^2}{2c}}$

(dependent on A^2)

left travelling

$P = F \cdot v = -T_0\, \partial y/\partial x \ \partial y/\partial t$ 1/2

$\langle P \rangle = -T_0 \sqrt{\frac{\mu_1}{T_0}}\, \omega A^2 \langle \cos^2(\sim) \rangle$.

$\boxed{\langle P \rangle = \dfrac{-T_0 \sqrt{\frac{\mu_1}{T_0}}\, \omega^2 A^2}{2}}$ 7

The wave with the larger mass/length μ carries the higher power, because the power is dependent on $1/c$

$A/\lambda = C_s$

(25 Pts)

4. A pipe of length L and uniform cross-sectional area is open to the atmosphere at one end, x = 0, and is closed at the other end x = L. The speed of sound in air is C_s, and is related to the compressibility (bulk modulus) $\beta = \rho_0 C_s^2$ where ρ_0 is the equilibrium mass density.

(8) a. State and physically explain the boundary conditions that the longitudinal displacement y (x,t) and the perturbed pressure δP (x,t) must satisfy at the open and closed ends of the pipe. [Recall: $\delta P = -\beta\, \partial y/\partial x$.]

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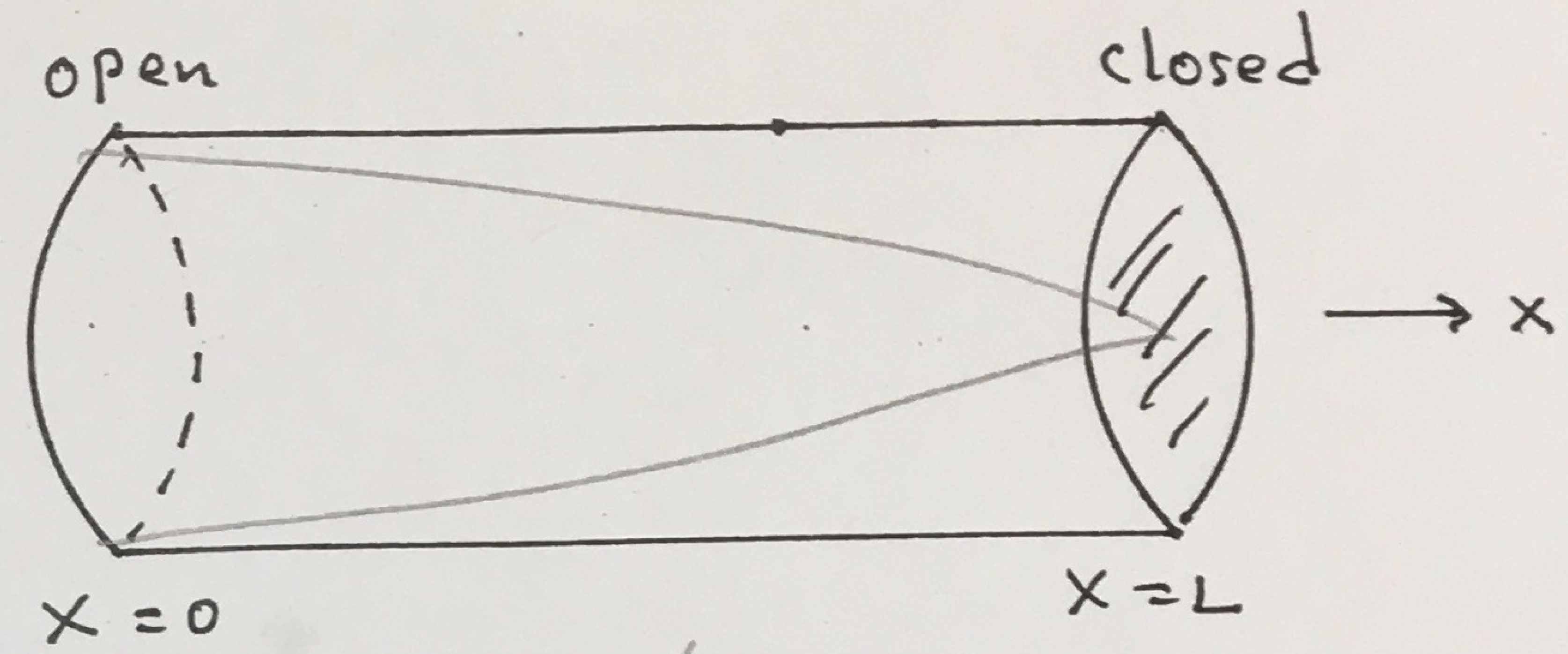
(12) b. Find the solution for y (x,t) for the waves that stand in the pipe (the normal modes), and for the wavenumber k_n, angular frequency ω_n, and ordinary frequency υ_n for the standing waves.



L

C_s

$\beta = \rho_0 C_s^2$

$\lambda \to \frac{\pi}{2}$

$2 \to \frac{3\pi}{2}$

(5) c. The length of the pipe is increased by a small distance $\Delta x \ll L$. The frequency of the n^{th} harmonic is observed to decrease by $\Delta\upsilon$. Show that $\Delta\upsilon$ is approximately given by $\Delta\upsilon \approx \upsilon_n\, \Delta x/L$.

a) At the closed end, $\delta P(x,t)$ must be at a maximum, because at the open end $\delta P(x,t)$ must be 0. This because the pressure of the atmosphere cannot be changed. $y(x,t)$ is at a max at the open end and 0 at the closed end because $\delta P = -\beta\, \partial y/\partial x$, so the two functions are 90° out of phase.

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b) $y(x,t) = [A\sin kx + B\cos kx]\cos\omega t = A\cos kx\cos\omega t$

@ $x=0$, $\cos kx = \max$ ✓ @ $x = L$ $\cos kx = 0$

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$kx = \frac{n\pi}{2}$ $\boxed{k_n = \frac{(2n-1)\pi}{2L}}$ $\omega = kc$ $\boxed{\omega_n = \frac{(2n-1)\pi}{2L} c = \frac{(2n-1)\pi}{2L}\sqrt{\frac{\beta}{\rho_0}}}$

$\omega = 2\pi\upsilon$ $\upsilon = \omega/2\pi$ $\boxed{\upsilon_n = \frac{(2n-1)}{4L} c = \frac{(2n-1)}{4L}\sqrt{\frac{\beta}{\rho_0}}}$

$\boxed{y(x,t) = A\cos\left(\frac{(2n-1)\pi}{2L} x\right)\cos\left(\frac{(2n-1)\pi}{2L}\sqrt{\frac{\beta}{\rho_0}}\, t\right)}$

$\cos kx$ $\cos \omega t$

c) $\Delta x \ll L$ υ decreases $\Delta\upsilon$

$\Delta\upsilon = (\upsilon_n)\frac{\Delta x}{L}$

$c = \lambda\upsilon$ c remains constant

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$\Delta\lambda = \frac{\Delta x}{L}$ wavelength increases by $\frac{\Delta x}{L}$

so, $\Delta\upsilon$ must decrease by a factor of $\frac{\Delta x}{L}$

$\boxed{\therefore \Delta\upsilon = (\upsilon_n)\frac{\Delta x}{L}}$