MT2 Physics 1B, S10

Full Name (Printed) [redacted]

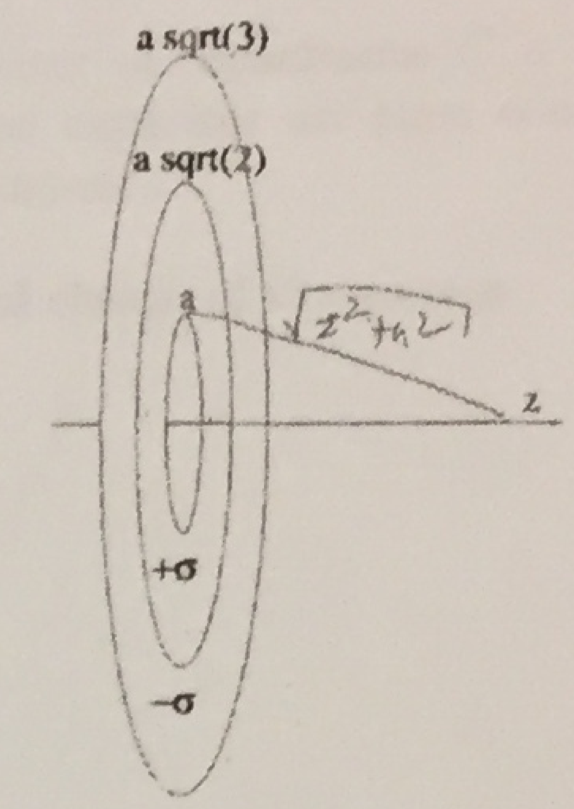
Full Name (Signature) [redacted]

Student ID Number [redacted]

Seat Number ____________

Problem	Grade
1	14 /30
2	17 /30
3*	29 /30
Total	60 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- HINT: Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



1) Consider the washer shown above that lies in the x, y-plane, centered on the origin. From $r = a$ to $r = \sqrt{2}a$, it carries a uniform surface charge density σ. From $r = \sqrt{2}a$ to $r = \sqrt{3}a$, it carries a uniform surface charge density $-\sigma$.

- 1a) (15 points) Find the electric potential (with respect to infinity) at all points on the positive z-axis.

$dq = \sigma 2\pi r dr$

$V = -\int \frac{k\,dq}{r} \Rightarrow \int \frac{k \sigma 2\pi r dr}{r} \Rightarrow -\int \frac{k \sigma 2\pi r dr}{\sqrt{z^2+r^2}} \Rightarrow -(\sigma k 2\pi) \int \frac{r dr}{\sqrt{z^2+r^2}}$

$\Rightarrow \sigma k 2\pi \frac{1}{(z^2+r^2)^{1/2}} \Big|_a^{\sqrt{2}a} - \sigma k 2\pi \frac{1}{(z^2+r^2)^{1/2}} \Big|_{\sqrt{2}a}^{\sqrt{3}a}$

$\Rightarrow \frac{\sigma k 2\pi}{(z^2+2a^2)^{1/2}} - \frac{\sigma k 2\pi}{(z^2+a^2)^{3/2}} - \sigma k 2\pi \frac{1}{(z^2+3a^2)^{3/2}} + \frac{\sigma k 2\pi}{(z^2+2a^2)^{3/2}}$

$\Rightarrow \frac{4\pi\sigma k}{(z^2+2a^2)^{3/2}} - \frac{\sigma k 2\pi}{(z^2+a^2)^{3/2}} - \sigma k 2\pi \frac{1}{(z^2+3a^2)^{3/2}}$

−6

- 1b) (10 points) Find the (vector) electric field at all points on the positive z-axis (be very clear how you obtain each component!)

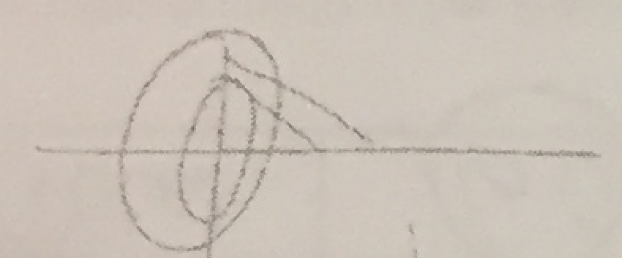
$\vec{E} = -\nabla V$

$\vec{E}_z = -\frac{\partial}{\partial r} \left(\frac{4\sigma\pi k}{(z^2+4a^2)^{1/2}} - \frac{k 2\pi}{(z^2+a^2)^{1/2}} - \frac{\sigma k 2\pi}{(z^2+3a^2)^{1/2}} \right)$

$\Rightarrow \frac{16\sigma\pi k}{3(z^2+2a^2)^{5/2}} - \frac{8k\pi}{3(z^2+a^2)^{3/2}} - \frac{8\sigma k}{(z^2+3a^2)^{3/2}}$

$E_x = 0$ (sym) The vector points in the $+z$ direction

$E_y = 0$



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- 1c) (5 points) Find the electric potential (with respect to infinity) at points along the positive z axis that are very distant from the washer [**Hint:** $(1+x)^n \approx 1 + nx + \frac{1}{2}n(n-1)x^2$]. Discuss the monopole, dipole and quadrapole moments for this charge distribution.

Consider a diffuse, spherically-symmetric charge distribution of radius R and charge Q whose volume charge density varies as r^4 (r being the distance from the center of the distribution).

- 2a) (10 points) Obtain functions that describe the volume charge density of the distribution and $q_{in}(r)$ (the total charge contained in a sphere of radius r, placed concentric with the charge distribution), in terms of Q, R and r, for all points inside and outside the distribution.

$\rho = Cr^4$, $dV \Rightarrow 4\pi r^2 dr$

$q_{in}(r) = \int dq \Rightarrow \int \rho \, dV \Rightarrow \int r^4 (4\pi r^2 dr) \Rightarrow 4\pi \int_0^r r^6 dr \Rightarrow \frac{4\pi r^7}{7}$

$Q_{outside} = \frac{4\pi R^7}{7}$

6

- 2b) (10 points) Find the electric potential for all points inside and outside the distribution **taking the potential on the surface of the sphere, $V(R)$ to be V_0.** Be careful with this one.

$r \leq a$: $V = -\int_0^r \frac{4\pi r^7 \, dr}{28\pi\epsilon_0 r} \Rightarrow -\frac{4\pi r^7}{196\pi\epsilon_0} \Rightarrow -\frac{r^7}{49\epsilon_0}$

$a < r < R$: $V = \int \frac{4\pi r^6}{28\pi\epsilon_0} \Rightarrow -\frac{R^7}{49\epsilon_0} + \frac{r^7}{49} \Rightarrow \frac{1}{49\epsilon_0}\left[r^7 - R^7\right]$

5

$R \geq 0$ ~~$V = 0$~~

- 2c) (10 points) How much work would you have to do to carry a test charge q from $r = 5R$ to $r = R/2$?

$W = \Delta U = q\Delta V$

$\Rightarrow \int_{R/2}^{5R} q\Delta V \Rightarrow \int_{R/2}^{R} q\Delta V + \int_{R}^{5R} q\Delta V$

6

$\Rightarrow \frac{-R^7}{49\epsilon_0} + \frac{R^7}{2^7 49\epsilon_0} + 0$

$\Rightarrow \frac{-R^7}{49\epsilon_0} + \frac{R^7}{2^7 49\epsilon_0}$

3) [HOMEWORK] A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $\frac{1}{2}C$. Compute

- 3a) (5 points) the original charge of the system

$Q = V_0 C$ ✓

- 3b) (10 points) the final potential difference across each capacitor

$Q = Q_1 + Q_2$

$V_1 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$; $C_1 = C$, $C_2 = \frac{C}{2}$

$V_1 = \frac{Q_1 + Q_2}{C_1 + C_2} \Rightarrow \frac{2Q}{3C} \Rightarrow \boxed{\frac{2}{3} V_0 = V_1}$ ✓

- 3c) (5 points) the final energy of the system

$U = \frac{1}{2} CV^2 \Rightarrow \frac{1}{2}\frac{Q^2}{C} \Rightarrow \frac{1}{2}\frac{Q^2}{(Q/V_0)} \Rightarrow \frac{1}{2} QV_0$

-1

- 3d) (5 points) the decrease in energy of the system when the capacitors are connected.

$U = \frac{1}{2} Q V_f \Rightarrow \frac{1}{2} Q\left(\frac{2}{3}V_0\right) \rightarrow \frac{1}{3} QV_0$

Decrease in energy $= \frac{1}{3}QV_0 - \frac{1}{2}QV_0 \Rightarrow -\frac{1}{6} QV_0$ ✓

- 3e) (5 points) Where did the "lost" energy go?

The energy was lost to thermal energy and electromagnetic radiation. ✓