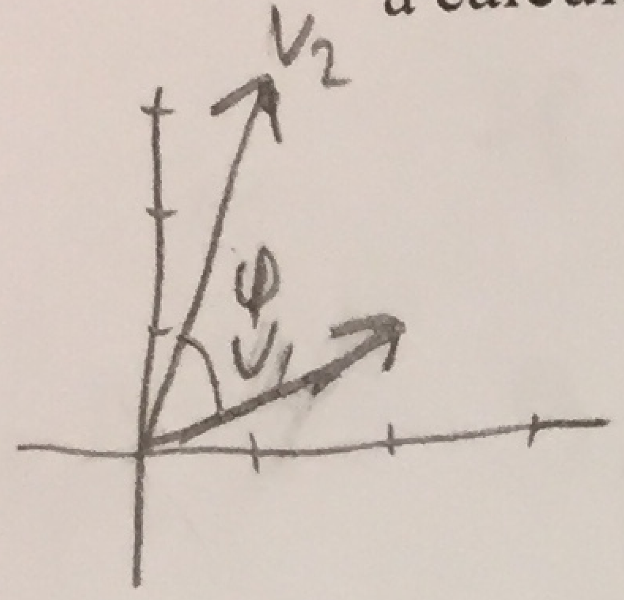
1. The vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ have components: $\mathbf{V}_1 = (2, 1)$; $\mathbf{V}_2 = (1, 3)$.
Find the components of the vector $\mathbf{U} = \mathbf{V}_1 + \mathbf{V}_2$, the modulus $|\mathbf{U}|$, and the angle φ that $\mathbf{U}$ makes with the x-axis.
(You may leave numerical values of trigonometric functions indicated if you do not have a calculator).



$V_1 + V_2 = (2+1 ; 1+3) = \boxed{(3, 4) = U}$ +3

$\boxed{|U| = \sqrt{3^2 + 4^2} = 5 \text{ units}}$ +4

$V_1 \cdot V_2 = |V_1||V_2| \cos\varphi$

$2 \cdot 1 + 3 \cdot 1 = |V_1||V_2| \cos\varphi$

$\cos^{-1}\left(\frac{5}{|V_1||V_2|}\right) = \varphi$

$\cos^{-1}\left(\frac{5}{\sqrt{5} \cdot \sqrt{10}}\right) = \varphi$ $|V_1| = \sqrt{2^2 + 1^2}$

$|V_2| = \sqrt{1^2 + 3^2}$

$\boxed{45^\circ = \varphi}$ +3

2. The acceleration of a particle has components: $\boldsymbol{a}(t) = (\alpha t, \beta)$. The particle starts from rest at the origin at $t = 0$.
Find the position of the particle at time $t = \beta / \alpha$.

V_{0x} and $V_{0y} = 0$

$a_x(t) = \alpha t$ X

$a_y(t) = \beta t$ X

$V_x(t) = \int (a_x(t)\, dt) + V_{0x}$

$V_y(t) = \int a_y(t)\, dt + V_{0y}$

$V_x(t) = \frac{1}{3}\alpha t^3 + 0$

$V_y(t) = \frac{1}{2}\beta t^2 + 0$ +2

$X(t) = \int (V_x(t)\, dt) + X_0$ +2

$y(t) = \int V_y(t)\, dt + y_0$

$X(t) = \frac{1}{12}\alpha t^4 + 0$

$y(t) = \frac{1}{6}\beta t^3 + 0$

$X\left(\frac{\beta}{\alpha}\right) = \frac{1}{12}\alpha\left(\frac{\beta}{\alpha}\right)^4$

$y\left(\frac{\beta}{\alpha}\right) = \frac{1}{6}\beta\left(\frac{\beta}{\alpha}\right)^3$

$= \frac{\beta^4}{12\alpha^3}$

$y\left(\frac{\beta}{\alpha}\right) = \frac{\beta^4}{6\alpha^3}$

$\boxed{\left(\frac{\beta^{\cancel{4}}}{12\alpha^{\cancel{3}}}, \frac{\beta^{\cancel{4}}}{6\alpha^{\cancel{3}}}\right)}$

$R_1 = \frac{V_0^2 \sin\alpha}{g}$ If V_0 is doubled: $R_2 = \frac{(2V_0)^2 \sin\alpha}{g}$ $R_2 = 4R_1$

3. A cannon ball is fired from ground level with initial velocity V_0 (at some angle $0 < \varphi < \pi/2$ above the horizontal) and strikes the ground at a distance L from the starting point. Where does the ball strike the ground if it is fired (at the same angle) with velocity $2V_0$?

$R_{ange} = \frac{V_0^2 \sin(2\varphi)}{g}$

If V_0 is doubled,

$R_2 = \frac{(2V_0)^2 \sin(2\varphi)}{g}$

$R_2 = 4\left(\frac{V_0^2 \sin(2\varphi)}{g}\right)$

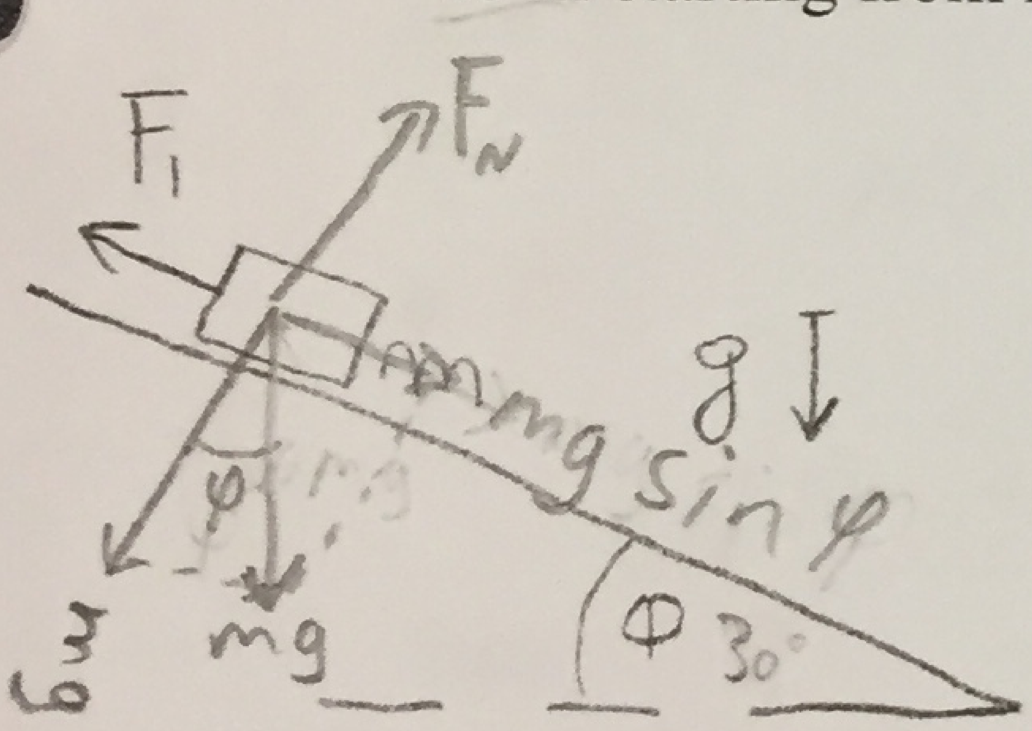
$R_2 = 4R_1$ ✓

The range of fire is quadrupled (x4) by doubling the initial velocity.

$2V_0 \Rightarrow 4L$ ✓

+10

4. A block of mass $m = 1$ kg slides down an incline (making an angle $\varphi = 30°$ with the horizontal); applied to the block there is a constant force $F_1 = 1$ N in the direction opposite to the motion (in addition to the force of gravity and the force exerted by the incline, which has no friction).
Calculate the magnitude of the velocity of the block after it slides down a distance L = 4.5 m starting from rest.



$mg\cos\varphi = F_N$

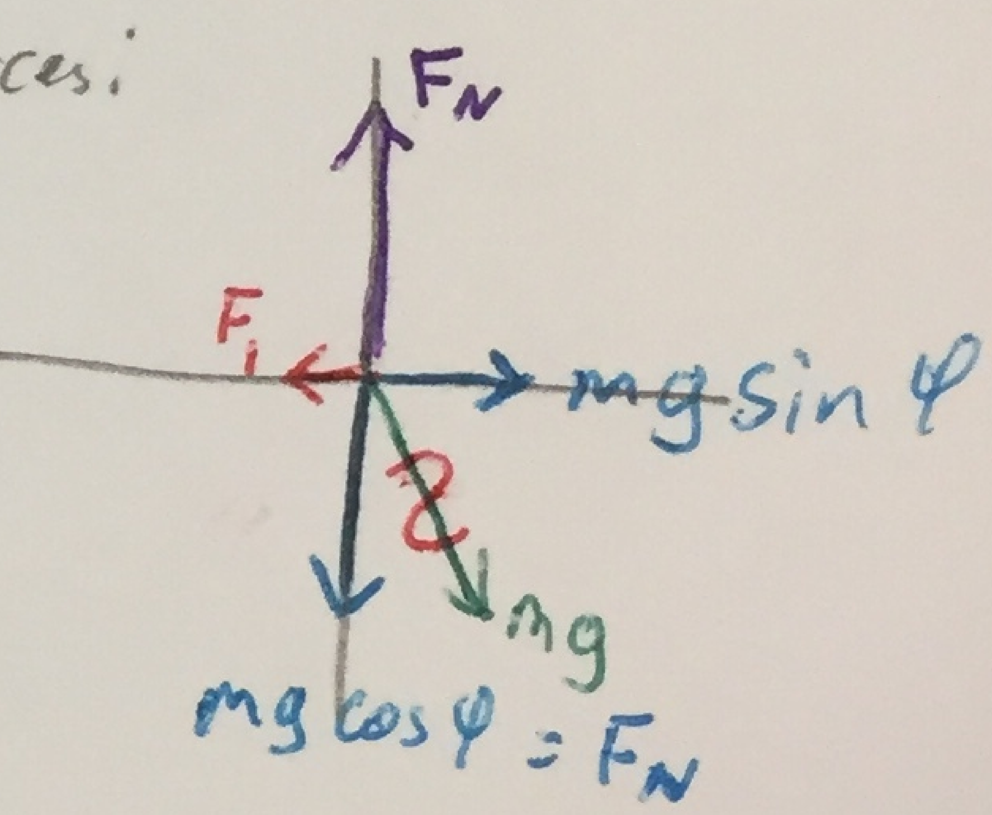
$m = 1\,kg$

$F = mg$

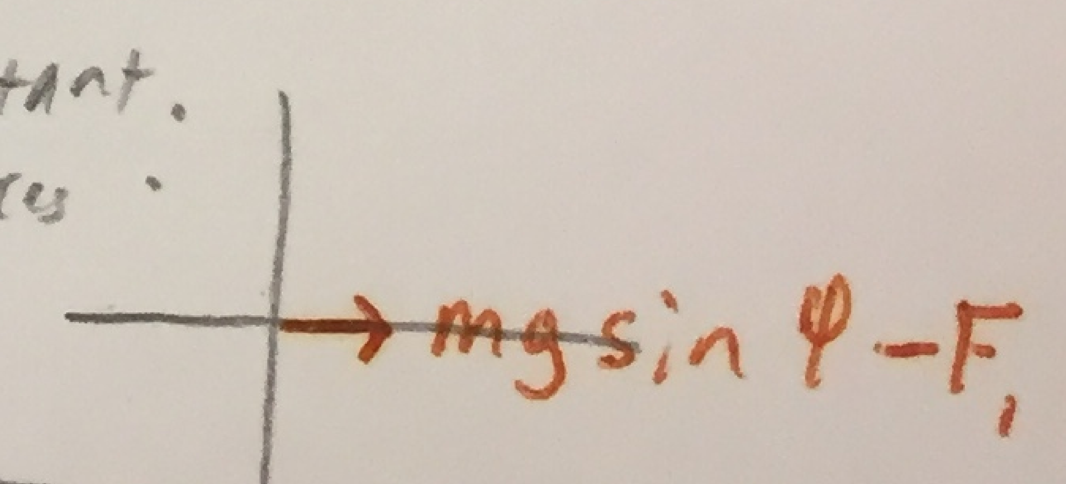
$F = (1\,kg)(9.8\,m/s^2)$

$F = 9.8\,N$

All Forces:



Resultant forces:



$F_{Block} = mg\sin\varphi - F_1$

$F_{Block} = 4.9N - 1N$

$F_{Block} = 3.9\,N$

$F = m \cdot a$

$3.9N = (1\,kg)(a)$

$3.9\,m/s^2 = a$

$V_0 = 0$

$a = 3.9\,m/s^2$

$V(t) = 3.9t + V_0$

$V(t) = 3.9t + 0$

$x(t) = \frac{1}{2}3.9t^2$

$L = \Delta x = 4.5\,m$

$4.5\,m = \frac{3.9}{2}t^2$

$t = \sqrt{\frac{4.5 \cdot 2}{3.9}}$

$t = 1.5\,s$

$V(t) = 3.9t$

$V(1.5) = 3.9(1.5)$

$V(1.5) = 5.85 \Rightarrow \boxed{5.9\,m/s}$

$|\vec{V}_{L=4.5m}| = 5.9$