Final Exam, Phys. 1A, Winter 2015

60 pts tot. (each problem 10 points).
(Show your work ! No credit for just writing down a result.)

Name:

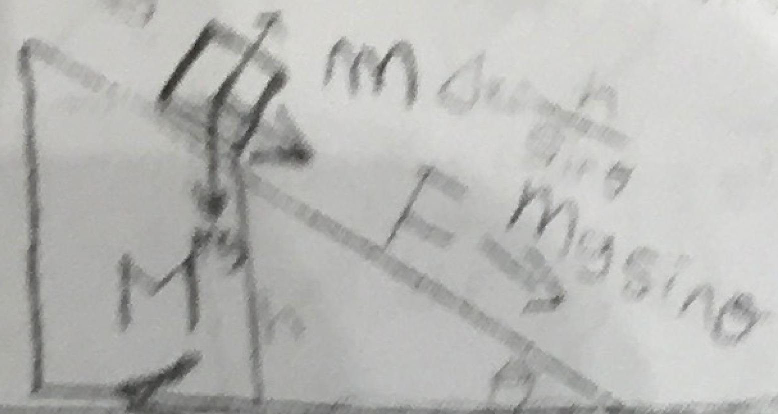
Student ID:

1. 6
2. 9
3. 10
4. 5
5. 6
6. 8

Total: 38

For some of the problems you may have to find some simple moments of inertia. This is part of the problem and will be graded, so show your calculations (I do not expect you to know moments of inertia by heart). If you do not know how to calculate a moment of inertia that you need, you can still do the problem: just leave it indicated as I in your formulas (and state clearly what moment of inertia I stands for).

1. The two blocks can slide without friction. The system starts from rest. The upper block (mass m) drops by a height h before reaching the horizontal surface. Calculate the final velocities of the two blocks along the horizontal surface.



[redacted] $mg\sin\theta$

$\boxed{a_1 = g\sin\theta}$

[redacted]

$\Delta x = V_i t + \frac{1}{2}at^2$

$V_f^2 = 2(g\sin\theta)\left(\frac{h}{\sin\theta}\right) = 2gh$

$\boxed{V_f = \sqrt{2gh}}$ ← top block relative to the bottom block

$mgh = \frac{1}{2}(m+M)V_2^2$

$V_f = \frac{2mgh}{(m+M)}$ ✓

conservation of momentum:

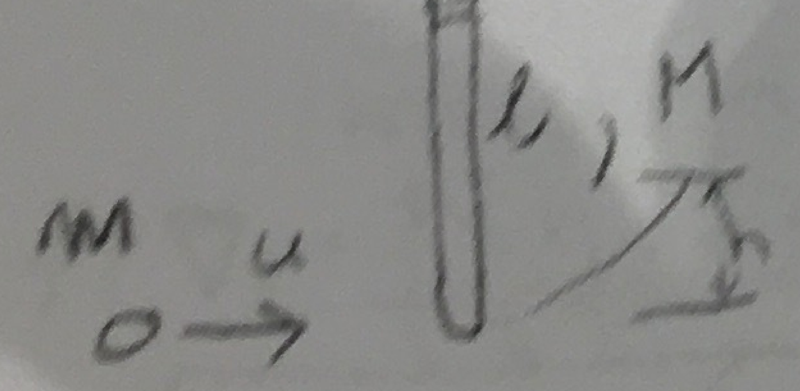
$(m+M)\sqrt{\frac{mgh}{(m+M)}} = m(\;) + MV_x$

$\boxed{V = \frac{(m+M)\sqrt{\;} - m\left(\;-\sqrt{\;}\right)}{M}}$ ← the bottom block

$\boxed{V_t = \sqrt{2gh} - \sqrt{\frac{2mgh}{m+M}}}$ top block on surface ✗

2. A pendulum consists of a uniform rod of mass M and length ℓ, suspended through a pivot at one end. It is initially at rest in equilibrium. A small rubber ball of mass m is flying horizontally with velocity u; it hits the lower end of the pendulum and sticks. Calculate: a) the maximum height reached by the end of the pendulum in its motion after the collision, and b) the mechanical energy lost in this inelastic collision.

$L = \ell m u = I\omega$ ✓



$I_{rod} = \int r^2\,dm = \frac{M}{\ell}\int_{r=0}^{r=\ell} r^2\,d\ell = \frac{M}{\ell}\frac{\ell^3}{3} = \boxed{\frac{1}{3}M\ell^2}$

$dm = \frac{M}{\ell}d\ell$

$I_{ball} = \int_0^m \ell^2\,dm = \ell^2\int_0^m dm = m\ell^2$

$v = \omega r$

$\omega = \frac{v}{\ell}$

$I_{tot} = \frac{1}{3}M\ell^2 + m\ell^2$

$\ell m u = \left(\frac{1}{3}M\ell^2 + m\ell^2\right)\omega$

$\omega = \frac{\ell m u}{\left(\frac{1}{3}M\ell^2 + m\ell^2\right)}$

$mgh = \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}\left[\frac{1}{3}M\ell^2 + m\ell^2\right]\frac{(\ell m u)^2}{\left(\frac{1}{3}M\ell^2 + m\ell^2\right)^2}$

$\boxed{h = \frac{1}{2mg}\frac{(\ell m u)^2}{\left(\frac{1}{3}M\ell^2 + m\ell^2\right)}}$

$\frac{1}{2}mu^2 = \frac{1}{2}I\omega^2 + E_{lost}$

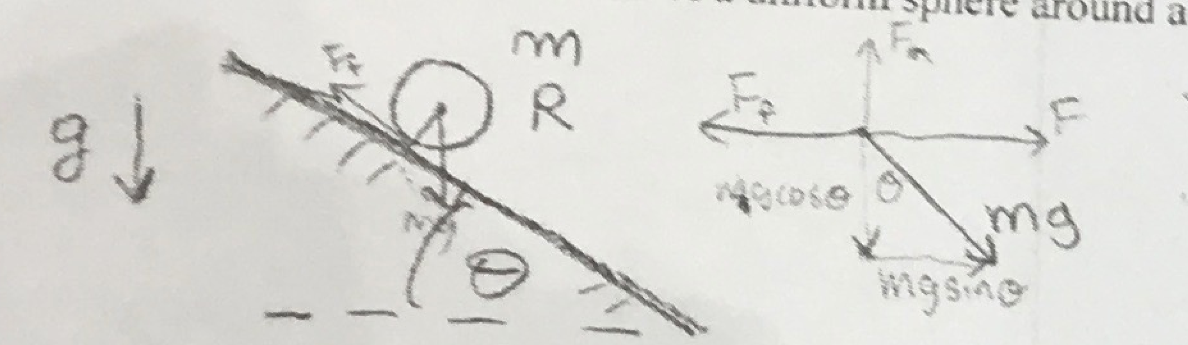
$\frac{1}{2}mu^2 = \frac{1}{2}\frac{(\ell m u)^2}{\left(\frac{1}{3}M\ell^2 + m\ell^2\right)} + E_{lost}$

$\boxed{E_{lost} = \frac{1}{2}mu^2 - \frac{1}{2}\frac{m^2u^2}{\left(\frac{1}{3}M + m\right)}}$ ✓

2

h_{max}

3. A uniform ball (mass m , radius R) rolls down a slope (without slipping); the slope makes an angle θ with the horizontal.
a) Find the acceleration of the CM of the ball along the slope.
b) What is the minimum coefficient of static friction μ_s such that the ball does not slip ?
(The moment of inertia of a uniform sphere around a diameter is $I = 2/5\ m\ R^2$)



$g\downarrow$

$\tau = F_f \cdot d = I\alpha$

$= F_f R = \frac{2}{5} mR^2 \left(\frac{a}{R}\right)$

$F_f = \frac{2}{5} ma$

$F_f = \frac{2}{5} ma$

$ma = mg\sin\theta - F_f$

$\mu mg\cos\theta = \frac{2}{5} m \left(\frac{5}{7} g\sin\theta\right)$

$ma = mg\sin\theta - \frac{2}{5} ma$

$\boxed{\mu_s = \frac{2}{7}\frac{\sin\theta}{\cos\theta}} = \boxed{\frac{2}{7}\tan\theta}$

$\frac{7}{5} ma = mg\sin\theta$

$\boxed{a = \frac{5}{7} g\sin\theta}$

because my particles are moving in a straight line

4. Consider a closed system of 3 interacting particles (means: there is no external force on the system, but the particles do exert forces on each other). Using Newton's laws, prove that:
a) The total momentum of the system is conserved.
b) The total angular momentum of the system is conserved.

$L = rmv$ if mv is conserved as shown below, then multiplying everything by r would also be conserved

$m_1 V_1 + m_2 V_2 = m_1 V_1 + m_2 V_2$

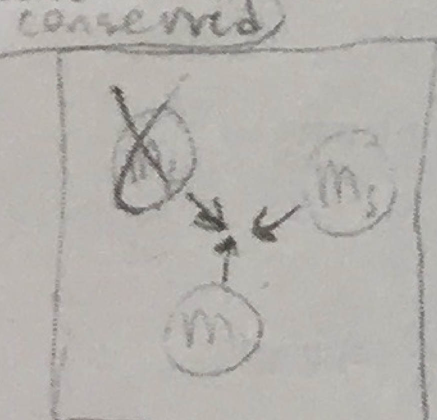
$F = ma$ (newtons 2nd)

$F_{A on B} = -F_{B on A}$ (New tons third)

Energy is conserved

$W = \Delta KE = \int F dx$

$\boxed{P = mv}$



$L = I\omega = \vec{r} \times m\vec{v}$

(3 colliding particles)

$F_{m_1} = -(F_{m_1 by m_2} + F_{m_1 by m_3})$

$m_1 a_1 = -m_2 a_2 - m_3 a_3$

(take integral of both sides)

$\int m_1 a_1 dt = \int (-m_2 a_2 - m_3 a_3) dt$

1.) $m_1 V_1 = -m_2 V_2 - m_3 V_3$

Substituting equations 2 & 3 into equation 1:

$m_1 V_1 = -(-m_1 V_1 - m_3 V_3) - (-m_1 V_1 - m_2 V_2)$

$a = \frac{dV}{dt}$

$F_{m_2} = -(F_{m_2 by m_1} + F_{m_2 by m_3})$

$m_2 a_2 = -m_1 a_1 - m_3 a_3$

$\int m_2 a_2 dt = \int (-m_1 a_1 - m_3 a_3) dt$

2.) $m_2 V_2 = -m_1 V_1 - m_3 V_3$

$m_1 V_1 = m_1 V_1 + m_3 V_3 - (-m_1 V_1 - m_2 V_2 + m_3 V_3)$

$m_1 V_1 = m_1 V_1 + m_2 V_2 - m_3 V_3$

$\boxed{m_1 V_1 = m_1 V_1}$

momentum is conserved

$F_{m_3} = -(F_{m_3 by m_1} + F_{m_3 by m_2})$

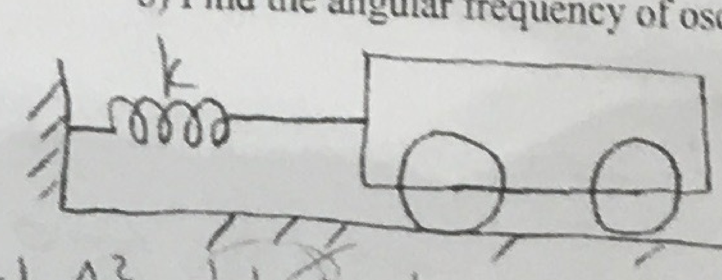
$m_3 a_3 = -m_1 a_1 - m_2 a_2$

$\int m_3 a_3 dt = \int (-m_1 a_1 - m_2 a_2) dt$

3.) $m_3 V_3 = -m_1 V_1 - m_2 V_2$

3

5. A wagon is attached to a spring and can otherwise roll on a horizontal surface. The spring has stiffness K and is massless. The wagon consists of 4 wheels which are uniform disks (mass m, radius R) and the body, of mass M (i.e. the total mass of the wagon is M + 4 m).
a) The wagon is started from rest at a position where the spring is stretched by Δ. Find the maximum velocity of the CM of the wagon during the subsequent motion.
b) Find the angular frequency of oscillation (ω) for this system.



$I_{disk} = \int r^2 dm, \quad dm = \frac{m}{\pi R^2} dA$

$I_{disk} = \frac{m}{\pi R^2} \iint r^2 dA = \frac{m}{\pi R^2} \int_0^{2\pi}\int_0^R r^3 dr d\theta$

$= \frac{m}{\pi R^2} \left[\frac{R^4}{4}\right](2\pi) = \frac{mR^2}{2}$ ✓ 1

$E_1 = \frac{1}{2} k\Delta^2 = \frac{1}{2} kx^2 + \frac{1}{2} Mv^2 + 4\left(\frac{1}{2} mv^2\right) + 4\left(\frac{1}{2} I\omega^2\right)$

$\frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{4} mv^2$

$\frac{1}{2} k\Delta^2 = \frac{1}{2} Mv^2 + 2mv^2 + mv^2 \rightarrow \frac{1}{2} k\Delta^2 = v^2\left(\frac{1}{2} M + 3m\right)$ 2

$\frac{1}{2} k\Delta^2 = \frac{1}{2} Mv^2 + 3mv^2$

$\boxed{v = \sqrt{\frac{k\Delta^2}{2\left(\frac{1}{2}M + 3m\right)}}}$ ✓

$\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{k}{M + 4m}}}$

$(M + 4m)\ddot{x} = -kx$

$\ddot{x} = -\frac{k}{M + 4m} x \rightarrow = \omega^2 \rightarrow \boxed{\omega = \sqrt{\frac{k}{M + 4m}}}$

6. Two pucks of masses m_1 and m_2 can slide without friction on a horizontal surface. Initially, m_1 has velocity u and m_2 is at rest. For a head on (i.e. this is a 1-D problem) *elastic* collision:
a) calculate the final velocities of the two masses;
b) give limiting forms of your result for the cases $m_1/m_2 >> 1$, $m_1/m_2 << 1$, $m_1 = m_2$.

$V_{a_2} = \left(\frac{m_A - m_B}{m_A + m_B}\right) V_{a_1}$

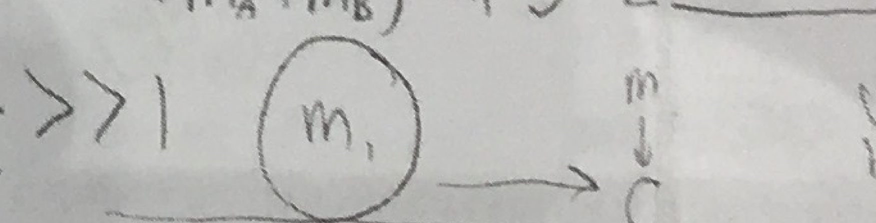
$V_{B_2} = \left(\frac{2m_A}{m_A + m_B}\right) V_{a_1}$ ✓

$\boxed{V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u}$ ✓ 4

$\boxed{V_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) u}$

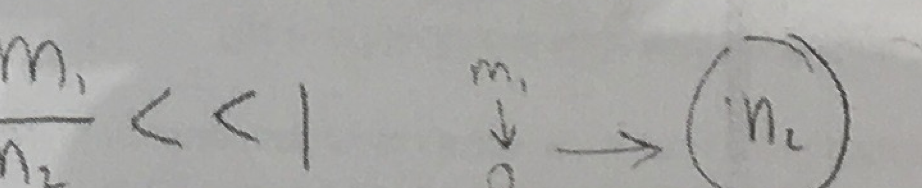
give values.

$\frac{m_1}{m_2} >> 1$



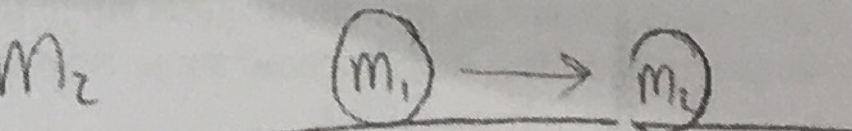
When they collide, if m_1 is much bigger than m_2, m_1 will go a little bit slower than it was before, & in same direction. m_2 will go in same direction as m_1 but faster.

$\frac{m_1}{m_2} << 1$



m_1 will go slightly slower and in opposite direction. m_2 will go very slowly in direction m_1 was initially going in.

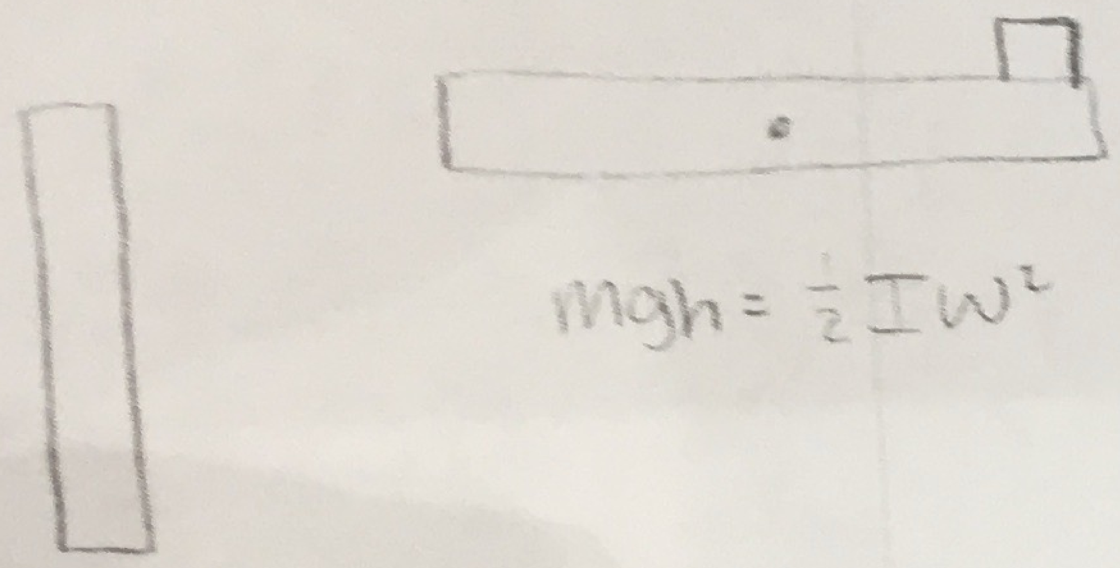
$m_1 = m_2$



m_1 will stop and m_2 will continue to travel forward at velocity m_1 was traveling at.

4

[redacted]



$(m+M)$

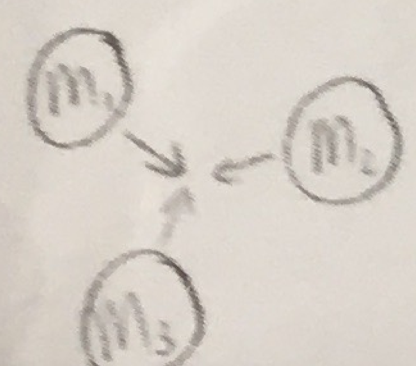
$mgh = \frac{1}{2} I \omega^2$

b) $\frac{1}{2} m u^2 = \frac{1}{2} I \omega^2 + E_{lost}$

$\frac{1}{2} m u^2 = \frac{1}{2} \frac{(\ell m u)^2}{(\frac{1}{3} M \ell^2 + m \ell^2)} + E_{lost}$

$= \frac{1}{2} \frac{m^2 u^2}{(\frac{1}{3} M + m)} + E_{lost}$

$= \frac{m^2 u^2}{(\frac{2}{3} M + m)} + E_{lost}$



$F_{m_1} = -(F_{m_1 \text{ by } m_2} + F_{m_1 \text{ by } m_3})$

$m_1 a_1 = -(m_2 a_2 + m_3 a_3)$

$\boxed{m_1 v_1 = -m_2 v_2 - m_3 v_3}$

$m_2 a_2 = -(m_1 a_1 + m_3 a_3)$

$\boxed{m_2 v_2 = -m_1 v_1 - m_3 a_3} \leftarrow m_1 v_1 = -m_2 v_2 - m_3 v_3$

$m_3 a_3 = -(m_1 a_1 + m_2 a_2)$

$\boxed{m_3 v_3 = -m_1 v_1 - m_2 v_2}$

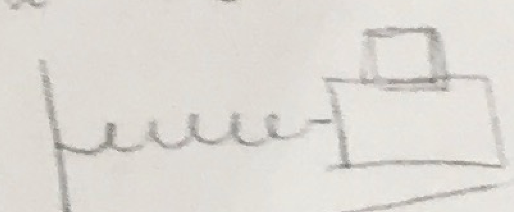
$m_1 v_1 = -m_2 v_2 - m_3 v_3$

[redacted]

$mgh = \frac{1}{2} I \omega^2$

$mgh = \frac{1}{2} \cancel{\left[\frac{1}{3} M \ell^2 + m \ell^2\right]} \frac{(mu)^2}{[\frac{1}{3} M \ell + m \ell]}$

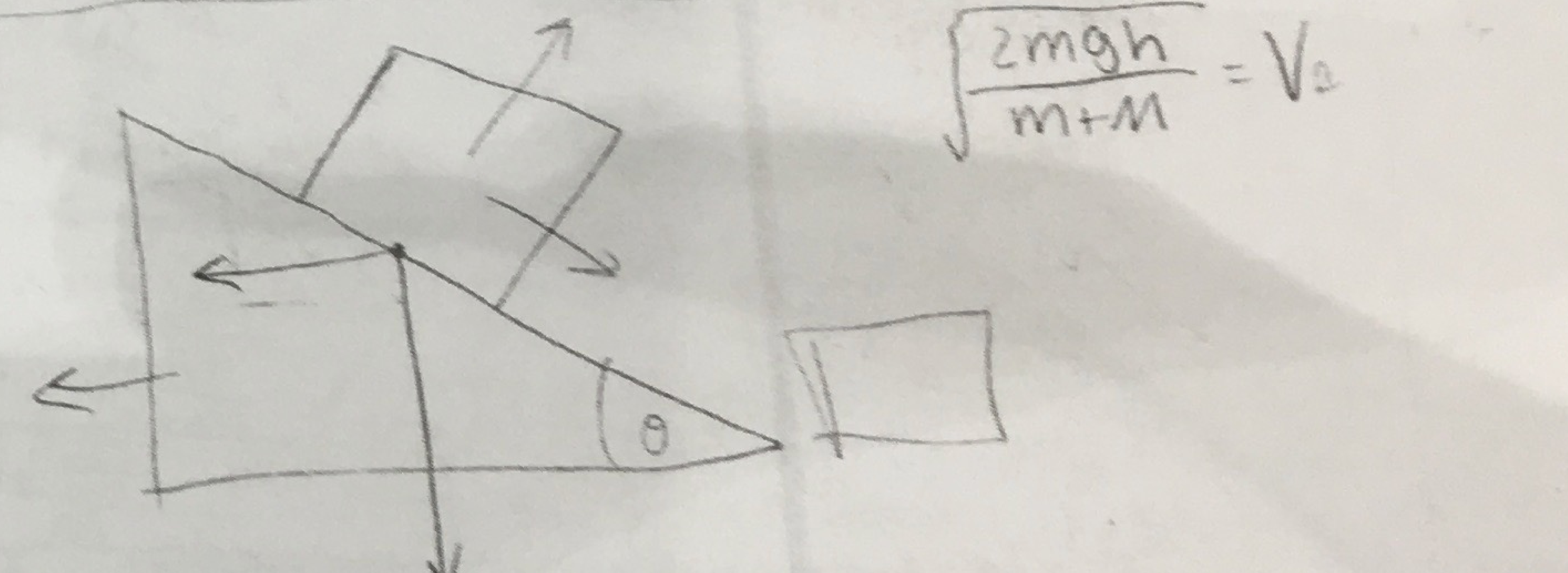
$mgh = \frac{1}{2} \frac{(\ell m u)^2}{(\frac{1}{3} M \ell^2 + m \ell^2)}$



$h = \frac{1}{2mg} \frac{(\ell m u)^2}{(\frac{1}{3} M \ell^2 + m \ell^2)}$

$mgh = \frac{1}{2}(m+M) V_f^2$

$\sqrt{\frac{2mgh}{m+M}} = V_0$



$(m+M)\sqrt{\frac{2mgh}{(m+M)}} = m\sqrt{2gh} + MV_x$

$$\frac{(m+M)\sqrt{\frac{2mgh}{(m+M)}} - m\left(\sqrt{2gh} - \sqrt{\frac{2mgh}{m+m}}\right)}{M}$$

$$\frac{m\star + M\star - m\sqrt{2gh} + m\star}{M}$$

$$\frac{2m\star + M\star - m\sqrt{2gh}}{M}$$

30, 30   30 22   30   30

Physics 1A

March 21, 2012

Final

Name: [redacted] UID: [redacted]

Discussion Section: 1D

30/30

$\frac{ML^2}{3}$

1) A homogeneous bar of length L can rotate on a horizontal frictionless plane around a vertical axis passing through one of its ends (moment of inertia $I = ML^2/3$). It is at rest until a small bullet ~~of mass m with velocity v~~ perpendicular to the bar hits it in the mid point, ~~bouncing back with velocity $v' = -v$~~/3. Given that the mass M of the bar is 9 times larger than the mass m of the bullet, calculate the final kinetic energies of the bullet and of the bar. Determine if the collision was elastic or not. Huh?

mass of the bullet = m

mass of the bar = 9m

$L_i = L_f$

$L_i(\text{bullet}) = mv\left(\frac{L}{2}\right)$

$L_f = I\omega + mv'\left(\frac{L}{2}\right)$

$= \frac{1}{3}mL^2\omega + m\cdot\frac{-v}{3}\cdot\frac{L}{2}$

$= \frac{1}{3}\cdot 9mL^2\omega - \frac{mvL}{6}$

$= 3mL^2\omega - \frac{mvL}{6}$

$\therefore L_i = L_f$

$\therefore \frac{mvL}{2} = 3mL^2\omega - \frac{mvL}{6}$

$\therefore \frac{mvL}{2} + \frac{mvL}{6} = 3mL^2\omega$

$\therefore \frac{3mvL + mvL}{6} = 3mL^2\omega$

$\therefore \frac{4mvL}{6} = 3mL^2\omega$

$\therefore \frac{4mvL}{6}\cdot\frac{1}{3mL^2} = \omega$

$\therefore \omega = \frac{4v}{18L}$

$\therefore \omega = \frac{2v}{9L}$

Final kinetic energy of the bar

$= \frac{1}{2}I\omega^2$

$= \frac{1}{2}\cdot\frac{1}{3}mL^2\cdot\left(\frac{2v}{9L}\right)^2$

$= \frac{1}{2}\cdot\frac{1}{3}\cdot 9m\cdot L^2\cdot\frac{4v^2}{81L^2}$

$= \frac{m2v^2}{[redacted]}$ [redacted]

$= \frac{2mv^2}{27}$

Kinetic energy of the bullet (Final) $= \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(\frac{-v}{3}\right)^2$

$= \frac{1}{2}\frac{mv^2}{9}$

$= \frac{mv^2}{18}$

In an elastic collision, kinetic energy is conserved,

$K_i = \frac{1}{2}mv^2 + 0$ of the bar

$K_f = \frac{1}{2}I\omega^2 + K_{BF}$ bullet after collision

$= \frac{2mv^2}{27} + \frac{mv^2}{18}$

$= \frac{4mv^2 + 3mv^2}{54}$

$= \frac{7mv^2}{54}$

$K_i \neq K_f$

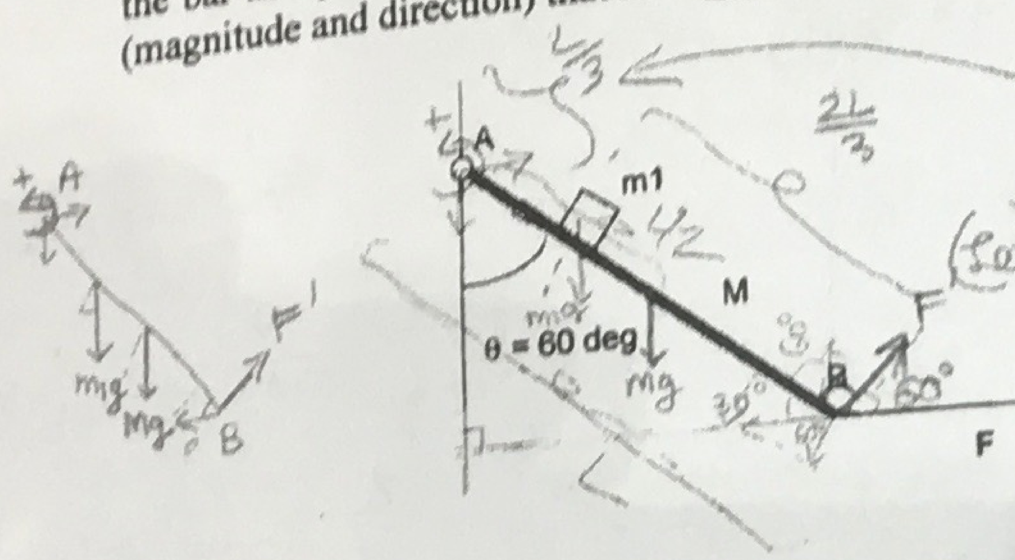
The collision is not elastic as the KE is not conserved before and after the collision

3 18 27
3 6 9
2 2 3
3 1 3
1 1

$2mv^2 \times 2$
$mv^2 \times 3$

30/30

2) A homogeneous bar of length L and mass $M = 20$ kg (moment of inertia $I = ML^2/3$) is hinged to a horizontal axis passing through its end A. On the bar at a distance $D = 2L/3$ from B is fixed a small body of mass $m_1 = 10$ kg. Calculate the magnitude of the horizontal force F to be exerted on B that is required to maintain the bar in equilibrium at an angle $\theta = 60$ degrees from the vertical and the force (magnitude and direction) that is exerted on A in these conditions. VIP!



For purpose of statics calculations only!

$\frac{L}{3} + \frac{2L}{3}$

$\frac{3L}{3}$

$L - \frac{2L}{3}$

$\frac{3L-2L}{3} = \frac{L}{3}$

L

$m_1 = 10$ kg

$m_{Bar} = 20$ kg

$F_x = 0$

(a) $\Sigma\tau = 0$ (see diagram for reference)

$\therefore -m_1 g \cdot \cos 30° \cdot \frac{L}{3} - Mg\cos 30° \cdot \frac{L}{2} + F'L = 0$

$\therefore -10 \times 9.8 \times \cos 30° \cdot \frac{L}{3} - 20 \times 9.8 \times \cos 30° \cdot \frac{L}{2} + F'L = 0$

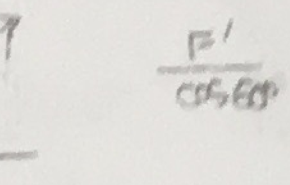
$\therefore -28.290L - 84.87L + F'L = 0$

$\therefore F'L = 28.280L + 84.87L$

$\therefore F'\cancel{L} = 113.1607\cancel{L}$

$\therefore F' = 113.1607$ N

$\therefore F' = F\cos 60°$



$\frac{F'}{\cos 60°} = F$

$\therefore F = \frac{F'}{\cos 60°}$

$= 226.32$ N

(b) $\Sigma F_x = 0$

$F - A_x = 0$

$\therefore F = A_x = 226.32$ N

$\Sigma F_y = 0$

$A_y - m_1 g - Mg = 0$

$\therefore A_y = m_1 g + Mg$

$= g(m_1 + M)$

$= 294$ N

$A_x = 226.32$ N, $A_y = 294$ N under given circumstances.

$A = \sqrt{A_x^2 + A_y^2} = 371.021$ N

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

$= 52.41°$

3) (a) What is the escape speed (in m/s) on a spherical asteroid whose radius is 525km and whose gravitational acceleration at the surface is 2.7 m/s^2?
(b) How far from the surface will a particle go if it leaves the asteroid surface with a radial speed of 1000m/s? (in m)
(c) With what speed will an object hit the asteroid if it is dropped from 1000km above the surface? (in m/s)

(a) For Escape speed,

$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$ . . (1)

$\frac{1 km}{1000 m} = \frac{525 km}{5x}$

where M is the mass of the asteroid, m is the mass of the object

$g = \frac{GM}{r^2}$

$\therefore 2.7 = \frac{6.67\times10^{-11} \cdot M}{(525000)^2}$

$M = 1.116\times10^{22}$ kg

$\therefore$ From (1),

$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r}$

$\therefore \frac{1}{2}v^2 = \frac{GM}{r}$

$\therefore v^2 = \frac{2GM}{r}$

$\therefore v = \sqrt{\frac{2GM}{r}} = 1683.746$ m/s

+10

Alternative method!

(without G)

Have the value memorized and question is not explicit that I cannot use G

$g \cdot \frac{r^2}{G} = M$

$M = \frac{gr^2}{G}$

$v = \sqrt{\frac{2G \cdot \frac{gr^2}{G}}{r}}$

$= \sqrt{2gr}$

$= \sqrt{2\times2.7\times525000}$

Same!

(b) $\frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r} = -\frac{GM\cancel{m}}{r'}$

$r = 525000$ m

$\therefore \frac{1}{2}v^2 - \frac{GM}{r} = -\frac{GM}{r'}$

$\therefore \frac{1}{2}(1000)^2 - 1417500 = -\frac{GM}{r'}$

$\Rightarrow -917500 = \frac{-7.44\times10^{11}}{r'}$

$\therefore r' = \frac{-7.44\times10^{11}}{-917500}$

$r' = 811103.5422$ m

Height from the surface $= r' - r$

$= 286103.5422$ m

+10

(c) $E_i = E_f$

$-\frac{GM\cancel{m}}{r'} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r}$

$\therefore -\frac{GM}{r'} = \frac{v^2}{2} - \frac{GM}{r}$

$r' = 525000 + 1000000 = 1525000$ m

$r = 525000$ m

$\therefore -487991.8033 = \frac{v^2}{2} - 1417500$

$\therefore \frac{v^2}{2} = 929508.1967$

$\therefore v^2 = 1859016.393$

$v = 1363.46$ m/s

+10

4) Inside a recreation vehicle, a small ball of mass m is resting on a shelf at height 1.5 m from the floor. At a given instant $t=0$ the vehicle accelerates forward with constant acceleration $a = 3\ m/s^2$.

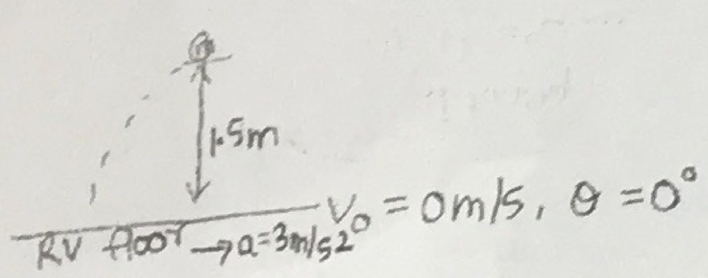
VIP!

a) At $t = 0$ the ball starts to fall with zero initial velocity from the top of the shelf. In which point does the ball touch the floor of the RV?

b) Assuming an elastic bounce, where does it touch the second time?

NO(X) motion, only Y motion

(Relative to ground, not RV



$V_0 = 0 m/s,\ \theta = 0°$

(a) $y = y_0 + V_0 \sin\theta t - \frac{1}{2}gt^2$

$\therefore\ 0 = 1.5 + 0 - \frac{1}{2}gt^2$

$\therefore\ \frac{1}{2}gt^2 = 1.5$

$\therefore\ 4.9t^2 = 1.5$

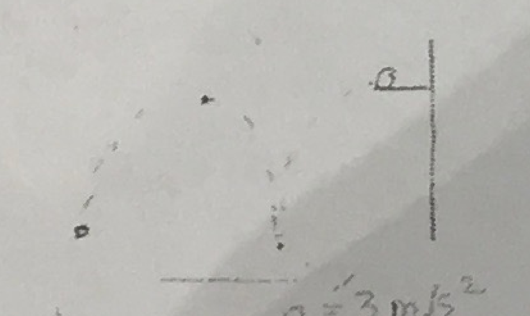
$\therefore\ t = 0.553s$

$x = x_0 + v_0t + \frac{1}{2}at^2$

$= 0 + 0 + \frac{1}{2}\times3\times(0.553)^2$

$= 0.4587m$

(b)



$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$2gh = v^2$

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$V = \sqrt{2gh} = 5.42\ m/s$

(velocity when the ball hits the floor for the first time)

the energy cons in all frames

$V_0 = 5.42 m/s$

$V = 0 m/s$

$a = 9.8 m/s^2$

$\therefore\ V = V_0 + at$

$\therefore\ 0 = 5.42 - 9.8t$

$\therefore\ 5.42 = 9.8t$

$\therefore\ t = 0.553s$

$\therefore\ t_i$ (time spent in the second bounce) $= 2\times0.553$

$= 1.106s$

$x = x_0 + v_0t + \frac{1}{2}at^2$

$= 0 + 0 + \frac{1}{2}\times3\times1.106^2$

$= 1.837m$

$\therefore\ x_2 = 1.837 + 0.4587$

$= 2.295\ m$

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5) A block of mass m_1=3kg slides on a frictionless horizontal plane with velocity v_1= 8m/s. In front of it another block of mass m_2 = 5 kg is moving in the same direction with velocity v_2 = 2 m/s. A spring (elastic constant k = 1000 N/m) is fixed on the back of m_2.

a) What is the velocity of the center of mass?
b) When the blocks collide, what is the maximum compression of the spring?

8m/s 2m/s
m_1 m_2
3kg 5kg

(a) $P_1 = m_1 v_1$
$= 3kg \times 8 m/s$
$= 24 \ kg \cdot m/s$

$P_2 = m_2 v_2$
$= 5 kg \times 2 m/s$
$= 10 \ kg \cdot m/s$

$V_{cm} = \frac{\sum m_i v_i}{m_i}$
$= \frac{24+10}{(3+5)}$
$= 4.25 m/s$

10

(b) $P_i = P_f$

$P_i = m_1 v_1 + m_2 v_2$
$= 24 + 10$
$= 34 \ kg \cdot m/s$

$\therefore P_f = (m_1 + m_2) V_f$
$= 8 V_f$

$\therefore 34 = 8 V_f$

$\therefore V_f = 4.25 m/s$

$\therefore E_i = E_f$

$\therefore \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} k x^2 + \frac{1}{2}(m_1+m_2) v_f^2$

$\therefore m_1 v_1^2 + m_2 v_2^2 = kx^2 + (m_1+m_2) v_f^2$

$\therefore (3 \times 8^2) + (5 \times 2^2) = 1000 x^2 + [(3+5) \times 4.25^2]$

$\therefore (3 \times 64) + (5 \times 4) = 1000 x^2 + (8 \times 18.0625)$

$\therefore 192 + 20 = 1000 x^2 + 144.5$

$\therefore 212 = 1000 x^2 + 144.5$

$\therefore 67.5 = 1000 x^2$

$\therefore x^2 = 0.0675$

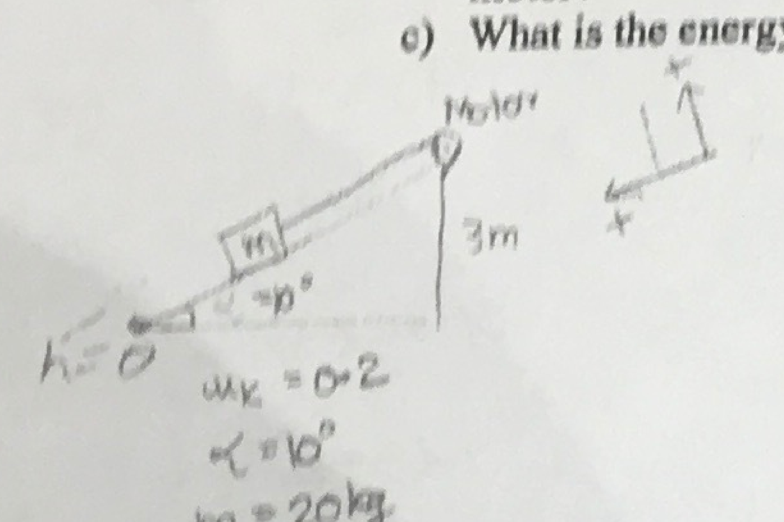
$\therefore x = 0.256 \ m$

20

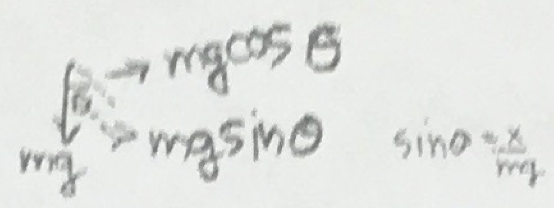
70

6) A small body of mass $m = 20$ kg can move upwards on an inclined plane which forms an angle $\alpha = 10$ degrees with the horizontal. The dynamic friction coefficient between the body and the surface of the inclined plane is $\mu_k = 0.2$. The body is attached to a motor by a massless rope. After an initial phase, the motor puts out a constant power $P = 250$ W.

a) What is the limit velocity of the body?
b) If the body gets to a height $h = 3$ m, what is the total work done by the motor?
c) What is the energy dissipated by friction?



$\mu_k = 0.2$
$\alpha = 10°$
$m = 20kg$



$\sin\theta = \frac{x}{mg}$

$f_k = \mu_k N$
$= \mu_k mg \cos\theta$

(a) Power = Force x Velocity,

$\therefore$ $250W = (mg\sin\theta + \mu_k mg\cos\theta)v$

$\therefore$ $250 = (34.035 + 38.604)v$

$\therefore$ $v = 3.44 m/s$

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(b) $W_{motor} = mgh + W_{friction}$

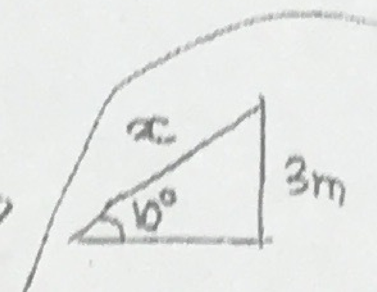
$= mgh + f \cdot x$ — $W = F \times D$

$= 20 \times 9.8 \times 3 + (\mu_k mg\cos\theta) \cdot x$

$= 588 + (38.604 \times 17.27)$

$= 1254.94 J$

12



$x \sin 10° = 3$

$\sin 10° = \frac{3}{x}$

$x = \frac{3}{\sin 10°}$

$= 17.27 m$

(c) Energy dissipated by friction = $W_{friction}$ (work done by friction)

$W_{friction} = \mu_k mg\cos\theta \times x$ [see part (b)] 6

$= 666.10 J$