Midterm 2

Physics 1A (Lec 5) 2020

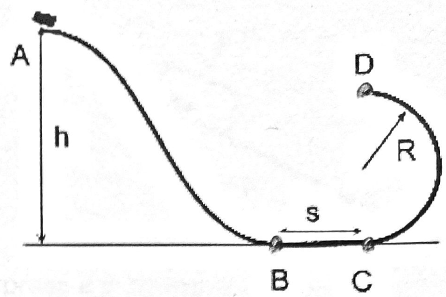
Time to complete the exam: 90 min

Each problem is worth 30 points. If a problem has parts (a) and (b), they are 15 points each. If a problem has parts (a), (b), and (c), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	total
30	30	20	30	30	140

Problem 1

A small block of mass m, initially at rest at point A, slides down a curve leading to a half-circle of radius R. The surface is frictionless, with the exception of a horizontal segment BC of length s, which has the coefficient of friction equal k. (Express all answers in terms of R, a, g, h, s, k, m.)



a) What is the speed at point B?

at point B, $U_g = 0$, $KE = max$

Conservation of Energy

$KE_i + U_{g_i} = KE_f + U_{g_f}$ ($KE_i = 0$, $U_{g_f} = 0$)

$mgh = \frac{1}{2} m v_B^2$

$gh = \frac{1}{2} v_B^2$

$\sqrt{v_B^2} = \sqrt{2gh}$

$\boxed{v_B = \sqrt{2gh}}$

b) What is the minimal height h for which the block reaches the top of the semicircle, point D?

D: mg ↓, N ↓

Top of Circle

$ma_c = mg + N$

$\frac{mv^2}{R} = mg + N$

v = min when $N = 0$

$\frac{\cancel{m}v^2}{R} = \cancel{m}g \rightarrow v^2 = gR$

$v = \sqrt{Rg}$

Energy Conservation

D; $2R$ { R, R

$W_{friction} + KE_i + U_{g_i} = KE_f + U_{g_f}$ ($KE_i = 0$)

$-kmgs + mgh = \frac{1}{2} mv^2 + mg(2R)$

$mgh = \frac{1}{2} m(\sqrt{Rg})^2 + mg(2R) + kmgs$

$mgh = \frac{1}{2} mRg + 2mgR + kmgs$

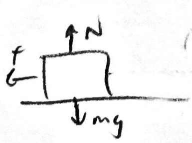
$\cancel{m}\cancel{g}h = \frac{5}{2}\cancel{m}\cancel{g}R + k\cancel{m}\cancel{g}s$

$\boxed{h = \frac{5R}{2} + ks}$

Friction from B → C

$W_{NC} = W_{friction} = -fs$

$= -kmgs$



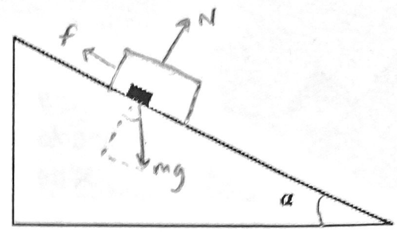
y| $0 = N - mg$

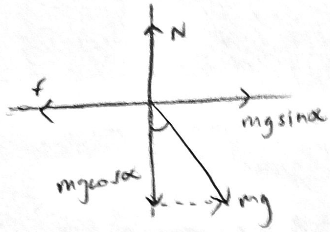
$N = mg$

x| $ma = -f = -kmg$

Problem 2

A body slides over an inclined plane forming an angle of α with the horizon. By taking time lapse snapshots of the motion, a student finds that the relationship between the distance s traveled by the body and the time t is described by the equation $s = C\,t^2$, where C is a constant. Find the coefficient of friction between the body and the plane, and express it in terms of C,α,g.





$y|\ \Sigma F_y = 0 = N - mg\cos\alpha \rightarrow N = mg\cos\alpha$

$f = \mu N = \mu mg\cos\alpha$

$W_{friction} = -fs = -\mu mg\cos\alpha\, s$

$W_{friction} = -\mu mg\cos\alpha\,(Ct^2)$

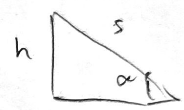
$x|\ \Sigma F_x = ma = mg\sin\alpha - f$

$\cancel{m}a = \cancel{m}g\sin\alpha - \mu \cancel{m}g\cos\alpha$

$a = g\sin\alpha - \mu g\cos\alpha$

$E_i + W_{NC} = E_f$

$mgh - \mu mg\cos\alpha(Ct^2) = \frac{1}{2}mv^2$



$\sin\alpha = \frac{h}{s}$

$h = s\sin\alpha$

$h = (Ct^2)\sin\alpha$

$\cancel{m}g(Ct^2)\sin\alpha - \mu \cancel{m}g\cos\alpha\,(Ct^2) = \frac{1}{2}\cancel{m}v^2$

$g(Ct^2)\sin\alpha - \mu g(Ct^2)\cos\alpha = \frac{1}{2}(2Ct)^2$

$\mu g(Ct^2)\cos\alpha = gCt^2\sin\alpha - \frac{1}{2}(4C^2t^2)$

$s = Ct^2$ position

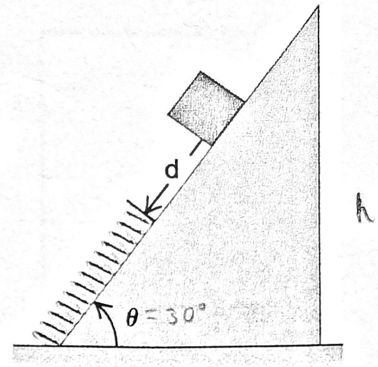
$v = \frac{ds}{dt} = 2Ct$ velocity

$a = 2C$ acceleration

$$\boxed{\mu = \frac{g\cancel{C}\cancel{t^2}\sin\alpha - 2C^{\cancel{2}}\cancel{t^2}}{g\cancel{C}\cancel{t^2}\cos\alpha}}$$

Problem 3

A block of mass m=1 kg slides down an inclined plane making angle θ=30° with the horizontal, lands on a spring with the spring constant k=10 N/m, and bounces back. The initial distance between the block and the end of the undeformed spring is d=1 m. The coefficient of friction is μ=0.4 Find the length x by which the spring is compressed when the block is at the lowest height. Initial speed $v_0 = 0$ m/s



not quite

$$\cancel{KE_i}^{0} + U_{g_i} + U_{s_i} = KE_f + \cancel{U_{g_f}}^{0} + U_{s_f}$$

$$mgh + \frac{1}{2}kd^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

what about friction?

$$mg(d\sin\theta) + \frac{1}{2}kd^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$mg(d\sin\theta) + \frac{1}{2}kd^2 - \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

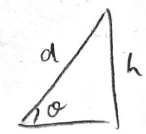
$m = 1$ kg

$k = 10$ N/m

$d = 1$ m

$\mu = 0.4$

$\theta = 30°$



$\sin\theta = \frac{h}{d}$

$h = d\sin\theta$

$$x = \sqrt{\frac{2mg(d\sin\theta) + kd^2 - \cancel{mv^2}^{0}}{k}}$$

Initial speed $v_0 = 0$ m/s

Final speed $v = 0$ m/s

not moving when at its lowest height (full of spring potential energy) elastic

$$x = \sqrt{\frac{2(1\text{ kg})(9.8\text{ m/s}^2)(1\text{ m})(\sin(30°)) + (10\text{ N/m})(1\text{ m})^2}{10\text{ N/m}}}$$

+ 20

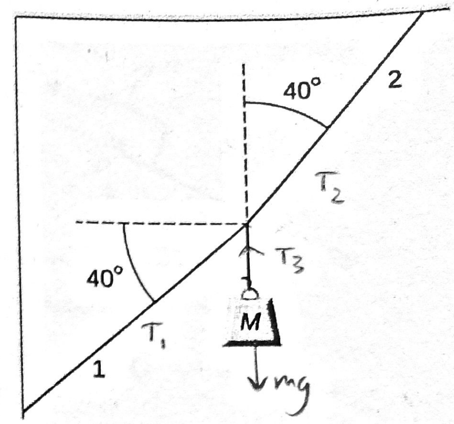
$x = 1.40712$

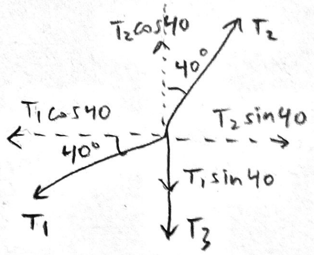
$$\boxed{x = 1.4\text{ m}}$$

Problem 4

A mass M is attached as shown in the figure. Rope 2 will break if the tension exceeds Tmax=1000 N. What is the largest mass M that can be supported?

M.max





$$\underline{T_2 \le T_{max}}$$

$$\underline{m}|\ ma_y = 0 = T_3 - mg$$

$$T_3 = mg$$

$$\underline{x}|\ 0 = -T_1\cos(40°) + T_2\sin(40°) \rightarrow T_2\sin 40° = T_1\cos 40°$$

$$\hookrightarrow T_1 = \frac{T_2\sin 40°}{\cos 40°}$$

$$\underline{y}|\ 0 = T_2\cos(40°) - T_1\sin(40°) - \underset{(mg)}{T_3}$$

$$0 = T_2\cos 40° - \left(\frac{T_2\sin 40°}{\cos 40°}\right)\sin 40° - \cancel{mg}$$

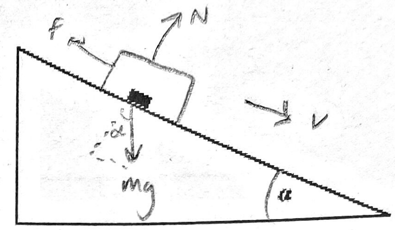
$$+mg \qquad\qquad +\cancel{mg}$$

$$m = \frac{T_2}{g}\left(\cos 40° - \frac{(\sin 40°)(\sin 40°)}{\cos 40°}\right) \qquad \underline{T_2 = T_{max}}$$

$$m = \frac{(1000\ N)}{(9.8\ m/s^2)}\left(\cos 40° - \frac{\sin^2(40°)}{\cos 40°}\right) = 23.1308$$

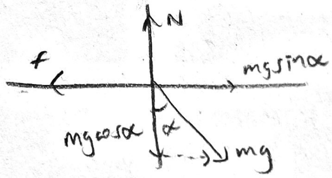
$$\boxed{M_{max} = 23.1\ kg}$$

Problem 5



A box of mass M slides down an inclined plane at a constant speed v if the inclined plane makes angle α with the horizontal.

a) Find the coefficient of friction.



$y| \ \Sigma F_y = 0 = N - mg\cos\alpha \rightarrow N = mg\cos\alpha$

$x| \ \Sigma F_x = 0 = mg\sin\alpha - \mu mg\cos\alpha$

constant speed
$\hookrightarrow a = 0$

$f = \mu N = \mu mg\cos\alpha$

$mg\sin\alpha = \mu mg\cos\alpha$

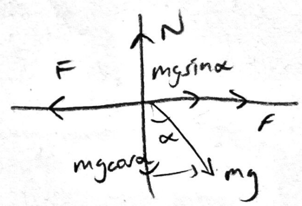
$$\mu = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

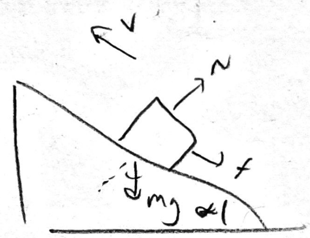
$$\boxed{\mu_k = \tan\alpha}$$

true at constant velocity

b) What power P would be required to pull the box upward on the same plane with the same speed v? in terms of v

$$P = \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = Fv$$





$P = Fv = mg(\mu\cos\alpha + \sin\alpha)v$

$x| \ 0 = f + mg\sin\alpha - F$

$F = f + mg\sin\alpha$

$\hookrightarrow f = \mu N = \mu mg\cos\alpha$

$\mu_k = \tan\alpha$ from above

$y| \ 0 = N - mg\cos\alpha$

$N = mg\cos\alpha$

$P = mg(\tan\alpha\cos\alpha + \sin\alpha)v$

$P = mg\left(\frac{\sin\alpha}{\cancel{\cos\alpha}}\cancel{(\cos\alpha)} + \sin\alpha\right)v$

$F = \mu mg\cos\alpha + mg\sin\alpha$

$P = mg(\sin\alpha + \sin\alpha)v$

$F = mg(\mu\cos\alpha + \sin\alpha)$

$$\boxed{P = mg(2\sin\alpha)v}$$