Midterm 1

Physics 1A (Lec 5)

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Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	15 20	5	20	20	20	~~100~~ 105

Problem 1

A 9 g bullet is accelerated from rest to a speed of 700 m/s as it travels 25 cm in a gun barrel. Assuming the acceleration is constant, how large was the accelerating force?

$F = ma$

$v^2 = \cancel{v_0^2}^{\to 0} + 2a\Delta x$

$\frac{v^2}{2\Delta x} = a$

$\frac{(700)^2}{2(0.25)} = a$

$a = 980000$

$F = ma$

$F = (0.009)(980000)$

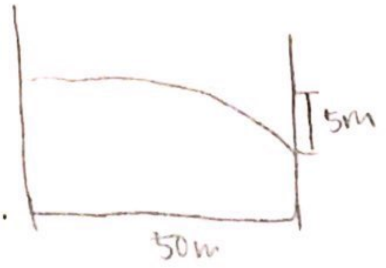
$\boxed{F = 8820N}$

$\frac{20}{20}$

20/20 + 5/20

Problem 2 An arrow shot horizontally hits a wall 50 meters away at a point 5 meters below the height of the bow.

(a) What was the initial velocity?



Δx 50	Δy	-5
v_0 $35\sqrt{2}$	v_0	0
v $35\sqrt{2}$	v	$-7\sqrt{2}$
a 0	a	-9.8
Δt		$\frac{5\sqrt{2}}{7}$ s

$\frac{v - v_0}{a} = t$

$\frac{-7\sqrt{2} - 0}{-9.8} = t$

$V_x = V_{0x}$ bc no acc.

∘ shot horizontally, $V_{0y} = 0$

$v_y^2 = \cancel{v_{0y}^2}^{0} + 2a\Delta y$

$v^2 = 2(-9.8)(-5)$

$v_y = -7\sqrt{2}$

$\Delta x = v_0 t + \cancel{\tfrac{1}{2}a}t^2$

$\frac{\Delta x}{t} = V_{0x}$

$\frac{50}{\frac{5\sqrt{2}}{7}} = V_{0x}$

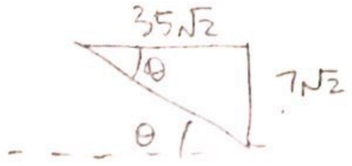
$\boxed{V_0 = 35\sqrt{2} \text{ m/s}}$

✓

(b) What was the angle between the velocity of the arrow and the horizontal when the arrow hit the wall?

$V_{final} = \langle 35\sqrt{2}, -7\sqrt{2} \rangle$

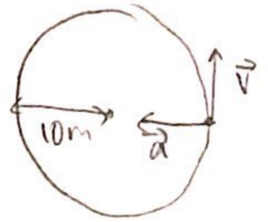
✓



$\tan\theta = \frac{7\sqrt{2}}{35\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{7\sqrt{2}}{35\sqrt{2}}\right)$

$\boxed{\theta = 11.31°}$



$$\frac{v^2 - v_0^2}{2a} = \Delta x$$

$\Delta x =$ $\frac{16-4}{4}$

$\frac{12}{4} = \Delta x$
$= 3\,m$

Problem 3

A particle moves in a circle of radius r = 10 m. During some time interval, its speed varies with time according to $v(t) = a + b/t^3$, where $a = 2$ m/s, $b = 16$ m s^2.

(a) Find the length of the circular path covered between t=1s and t=2 s

@ t = 1s

$v(1) = 2 + \frac{16}{1}$

$= 18\ m/s$

@ t = 2s

$v(2) = 2 + \frac{16}{8}$

$= 4\ m/s$

$V_{final} = 4$

$V_{initial} = 2$

$\Delta t = 1s$

$\Delta x = v_0 t + \frac{1}{2} a t^2$ ✗

$\Delta x = 2 + \frac{1}{2}(2)$

$\boxed{\Delta x = 3\,m}$

$\frac{mv^2}{r}$ $a = \frac{v^2}{r}$

$\frac{V - V_0}{t} = a$

$\frac{4-2}{1} = a$

$a = 2\ m/s^2$

+1

(b) Find the **total** acceleration at time t=2 s.

✗

$a_c = \frac{v^2}{r}$

linear $a(t) = \frac{4b}{t^4}\frac{ms^2}{s^4}$

$b(t)^{-3}$

$-4b(t)^{-4}$

$\frac{m}{s^2}$

$a(2) = \frac{-4(16)}{2^4}$

+4

$v(2) = a + \frac{b}{(2)^3}$

$= 2 + \frac{16}{8}$

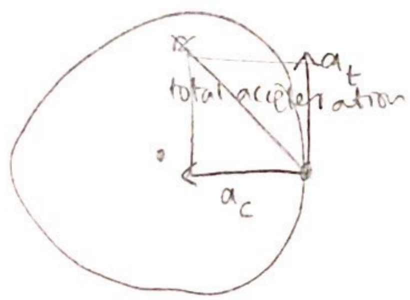
$= 4\ m/s$

$a(2) = -4\ \frac{m}{s^2}$

total acceleration $= a_c + a_t$

$\boxed{= -2.4\ m/s^2}$ ✗

$a_c = \frac{(4\ m/s)^2}{10\ m}$



$a_c = \frac{16}{10}$ ✓

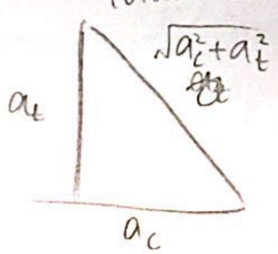
$a_c = 1.6\ m/s^2$

✓

$a_t^2 + a_c^2 = a_t^2$

total acceleration $= \sqrt{a_c^2 + a_t^2}$

$= \frac{4\sqrt{29}}{5}\ m/s^2$



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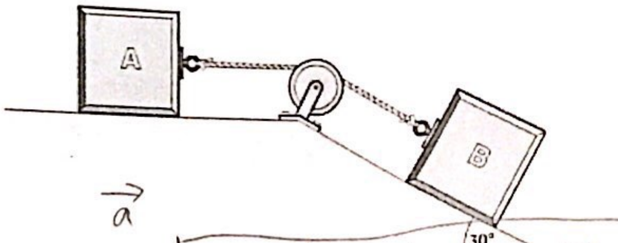
20/20

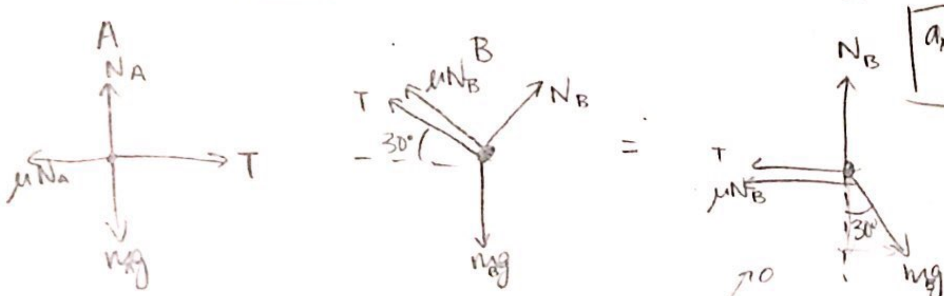
Problem 4

Two boxes with masses m_A and m_B are connected by a cord as shown. The inclined plane makes an angle 30° with the horizontal. The friction coefficient between each box and the surface is μ.



(a) Find the acceleration of the boxes as they slide down.

$$\boxed{a_x = \frac{m_B g(\sin 30 - \mu \cos 30) - m_A \mu g}{m_A + m_B}}$$



$m \overset{\nearrow 0}{a_y} = N_A - m_A g$

$N_A = m_A g$

$m_A a_x = T - \mu m_A g$

$m_A(a_x + \mu g) = T$

$m_B \overset{\nearrow 0}{a_y} = N_B - m_B g \cos 30$

$N_B = m_B g \cos 30$

$m_B a_x = mg \sin 30 - \mu N_B - T$

$-m_B a_x + m_B g \sin 30 - \mu m_B g \cos 30 = T$

$m_A a_x + m_A \mu g = -m_B a_x + m_B g(\sin 30 - \mu \cos 30)$

$a_x(m_A + m_B) = -m_A \mu g + m_B g(\sin 30 - \mu \cos 30)$

(b) Find the tension in the cord connecting the two boxes.

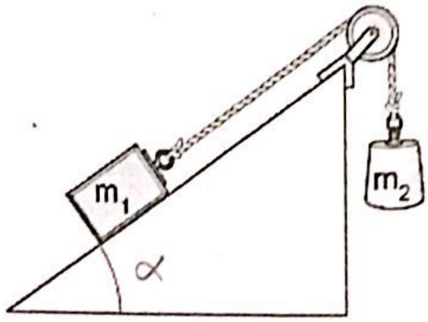
$T = m_A(a_x + \mu g)$

$$T = m_A\left(\frac{m_B g(\sin 30 - \mu \cos 30) - m_A \mu g}{m_A + m_B} + \mu g\right)$$

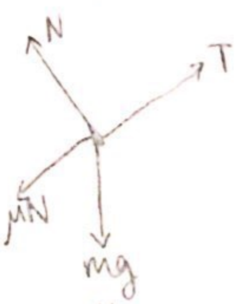
$$\boxed{T = \frac{m_A m_B g(\sin 30 - \mu \cos 30 + \mu)}{m_A + m_B}}$$

Problem 5

The coefficient of friction between the block of mass m_1 and the inclined plane is μ. The angle of the plane with the horizontal is α. What is the maximal mass m_2 for which the system remains at rest? (Express the answer in terms of m_1, μ, α.)



m_1



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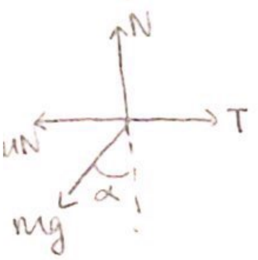

m_2

$m_2 \cancel{a_y}^0 = T - m_2 g$

$T = m_2 g$

continue

$m_1 \cancel{a_x}^0 = T - \mu N - mg\sin\alpha$

$\mu N + mg\sin\alpha = T$

$m \cancel{a_y}^0 = N - mg\cos\alpha$

$N = mg\cos\alpha$

$T = \mu m_1 g\cos\alpha + m_1 g\sin\alpha$

$m_2 g = \mu m_1 g\cos\alpha + m_1 g\sin\alpha$

$m_2 = \mu m_1\cos\alpha + m_1\sin\alpha$

$\boxed{m_2 = m_1(\mu\cos\alpha + \sin\alpha)}$

20

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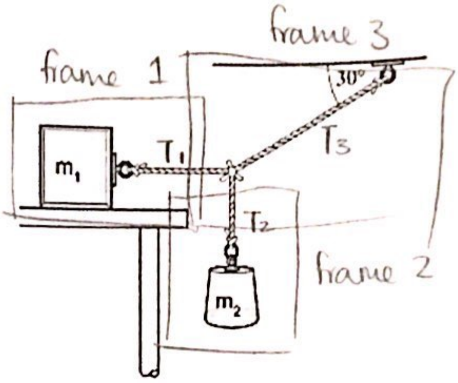
Problem 6

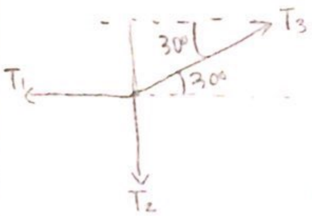
The system shown in the figure is in equilibrium. The coefficient of friction between the block and the table is μ. For a given mass m_1, what is the greatest mass m_2 that can be supported as shown in the figure? Express the answer in terms of m_1, μ, and the angle shown in the figure.



frame 1

frame 2

frame 3



frame 2

$m_2 \overset{0}{a_y} = T_2 - m_2 g$

$\underline{T_2 = m_2 g}$

frame 3

$m\overset{0}{a_x} = T_3 \cos 30 - T_1$

$m\overset{0}{a_y} = T_3 \sin 30 - T_2$

$T_3 \cos 30 = T_1$

$T_3' = \frac{T_1}{\cos 30}$

$T_3 = \frac{T_2}{\sin 30}$

frame 1

$\cancel{m_1} \overset{0}{a_x} = T_1 - \mu N$

$T_1 = \mu N$

$\cancel{m_1} \overset{0}{a_y} = N - m_1 g$

$N = mg$

$\underline{T_1 = \mu m_1 g}$

$\frac{T_1}{\cos 30} = \frac{T_2}{\sin 30}$

$\frac{\mu m_1 g}{\cos 30} = \frac{m_2 g}{\sin 30}$

$\frac{(\sin 30)\,\mu m_1}{\cos 30} = m_2$

$\boxed{(\tan 30)\,\mu m_1 = m_2}$

$\boxed{\frac{\sqrt{3}}{3}\mu m_1 = m_2}$

$\frac{20}{20}$

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