MECH&AE 101

MECH&AE 105A
Lec 1

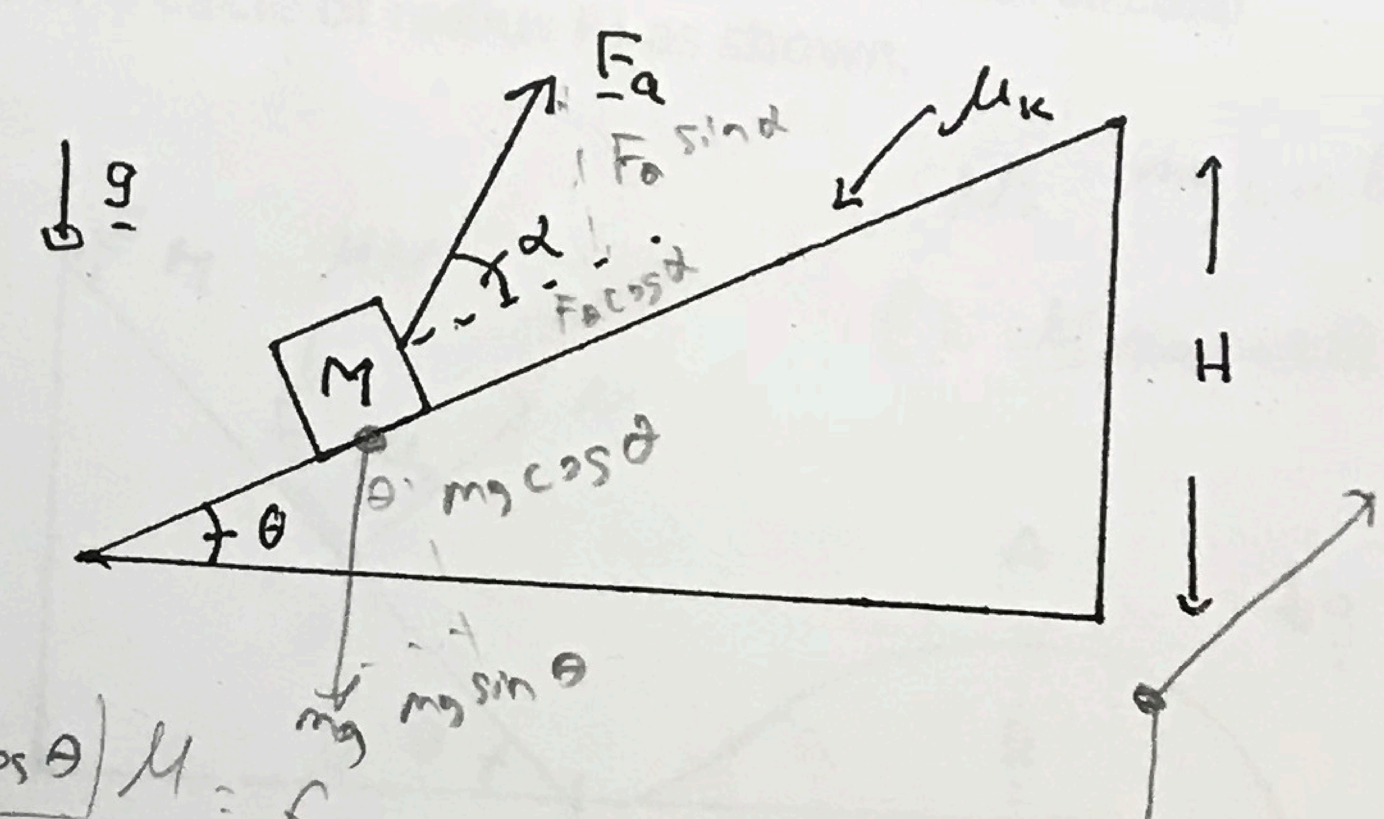
(25 Pts)

1. A block with mass M is pulled up a rough (coefficient of kinetic friction μ_k) inclined plane (inclination angle θ and height H above the ground) at a constant speed by an applied force $\boldsymbol{F_a}$ that makes an angle α with respect to the surface of the plane as shown.

(15) a. Prove that the magnitude of the applied force is

$$|\boldsymbol{F_a}| = \frac{Mg(\sin\theta + \mu_k\cos\theta)}{\cos\alpha + \mu_k\sin\alpha}$$

(10) b. Find the total work done by the applied force in pulling the block from the bottom to the top of the inclined plane.



constant speed = 0

(a) $F = ma = 0$

$N = (mg\cos\theta)\,\mu = f$

same direction as normal

pulling down

f

$F_a\cos\alpha + F_a\,\mu_k\sin\alpha = Mg\sin\theta + Mg\,\mu_k\cos\theta$

$F_a\cos\alpha + F_a\,\mu_k\sin\alpha =$

applied

friction

$F_a \cos\alpha = mg\sin\theta + \mu(F_a\sin\alpha - mg\cos\theta)$.

$F_a\sin\alpha$

$F_a(\cos\alpha - \mu\sin\alpha) = mg\sin\theta - \mu\, mg\cos\theta$

$$|F_a| = \frac{mg(\sin\theta + \mu\cos\theta)}{\cos\alpha + \mu_k\sin\alpha}$$

wrong sign -4

$mg\cos\theta$

$F_a\sin\alpha - mg\cos\theta = N$

$\mu N = f_r$

$\mu(F_a\sin\alpha - mg\cos\theta)$

(b) F · distance = work

$|F_a| \cdot l$

$\sin\theta = \frac{H}{L}$

$L = \frac{H}{\sin\theta}$

$$\left(\frac{mg(\sin\theta - \mu_k\cos\theta)}{\cos\alpha + \mu_k\sin\alpha}\right)\left(\frac{H}{\sin\theta}\right) \cdot \cos\alpha \quad -3$$

AE 105A
Lec 1

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(24 Pts)

2. A skier with mass M starts from rest at the top (height H) of a rough slope (inclined plane with inclination angle θ) with a coefficient of kinetic friction μ_k as shown. At the bottom of the slope (Point B), the snow turns into smooth powder (no friction), and the skier proceeds to the top (Point A) of a small hill at a height R above the ground. The hill has a local circular radius of curvature R at the top (the top is part of a circle of radius R) as shown.

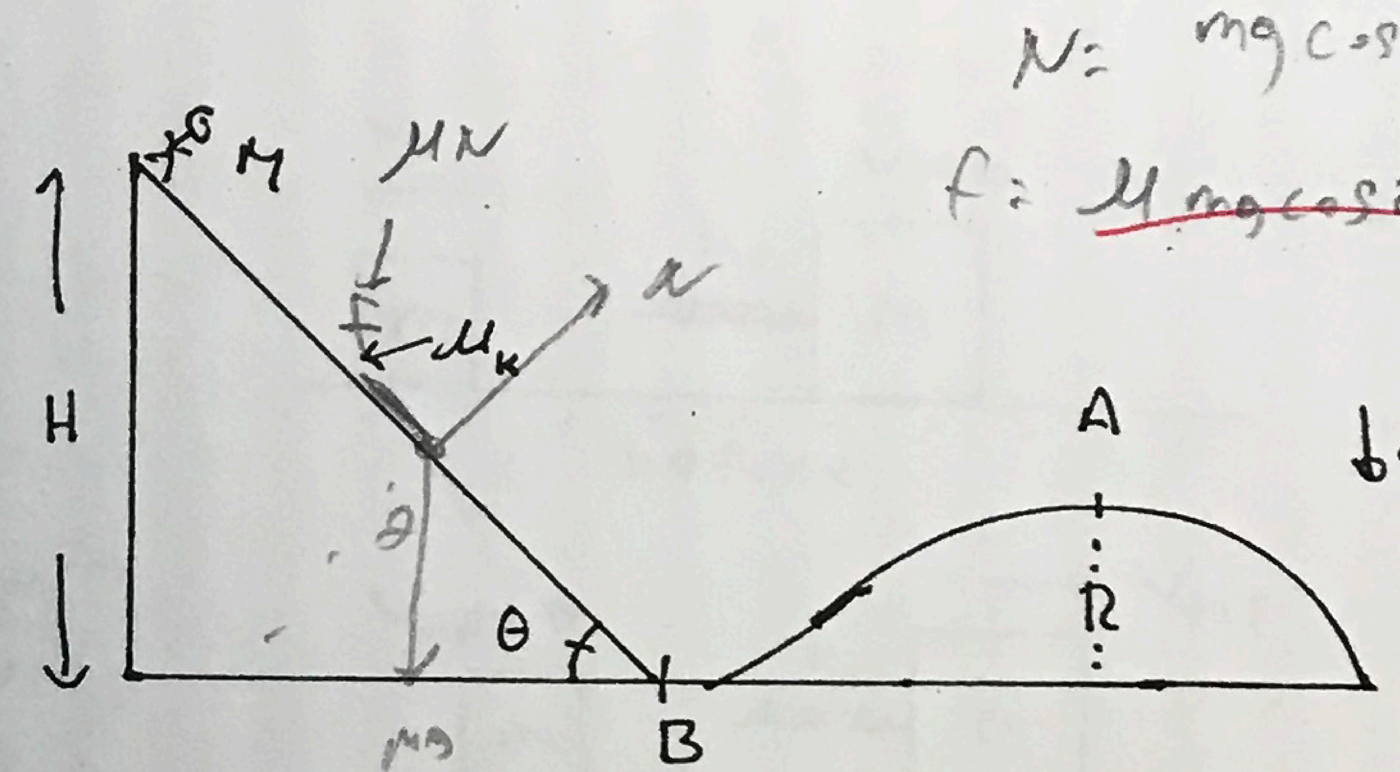
(12) a. Prove that at Point B, the skier's speed is

$$v_B = [2gH(1 - \mu_k \cot\theta)]^{1/2}$$

(12) b. Now find the minimum height H such that the skier will just lose contact with the hill at Point A and take off on a jump.

$N = mg\cos\theta$

$f = \mu\, mg\cos\theta$



(a) 12

$MgH = \frac{1}{2}Mv_B^2 + W_f$

$mgH - W_f = \frac{1}{2}mv_B^2$ $\qquad \frac{M}{m} = (1 - \mu_k \cot\theta)$

$mgH - \mu mgH\cot\theta = \frac{1}{2}mv_B^2$

$F_{\parallel}$: $mg\sin\theta - f_f$

$mg\sin\theta$ $\qquad mg\cos\theta$

$2gH(1 - \mu\cot\theta) = v_B^2$

$(2gH(1 - \mu\cot\theta))^{\frac{1}{2}} = v_B$

$N - mg\cos\theta = 0$

$W_f = \mu mg\cos\theta \cdot \frac{H}{\sin\theta}$

$\sin\theta = \frac{H}{L}$

$L = \frac{H}{\sin\theta}$

(b) 4

$mgH_{min} \rightarrow \frac{1}{2}mv_B^2 \rightarrow$

$\frac{1}{2}mv_B^2$

at point A, $N = 0$

$\cancel{\frac{1}{2}mv^2 = N - mg}$

$\frac{2}{3}GR$

?

$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_f^2 + mgR$

(29 Pts)

3. A block with mass m is moving to the right (positive x-direction) along a smooth horizontal surface with a speed v_0. A second block with mass M is initially at rest, and has a relaxed horizontal spring with stiffness constant k_S. The two blocks collide (one-dimensional), and compress the spring by a maximum distance x_{MAX}. The spring then pushes the blocks apart until they separate at their final speeds v_{mf} and v_{Mf}.

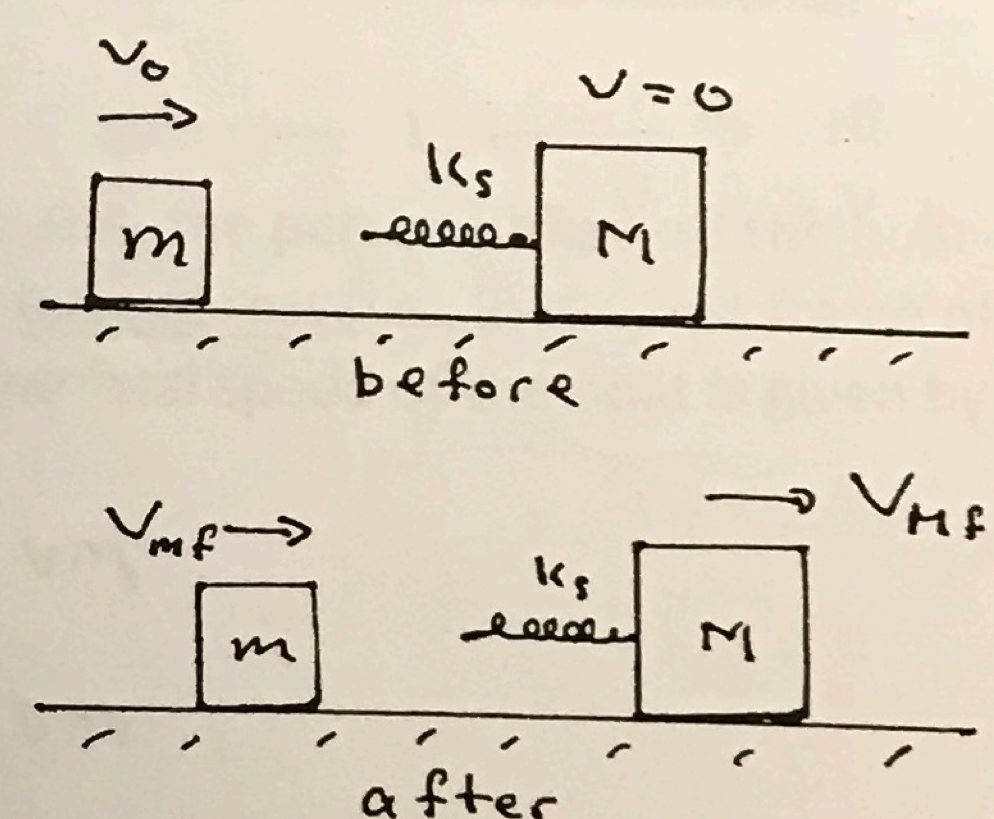
(5) a. Find the speed v_{CM} of the center of mass of the system.

(10) b. The spring's maximum compression occurs when the two blocks are at rest with respect to each other (i.e., $v_m = v_M = v_{CM}$). Show that

$$x_{MAX} = v_0 \left[\frac{mM}{(m+M)k_S}\right]^{1/2}$$

$kx_{max}^2 = V_0^2\left[\frac{mM}{m+M}\right]$

(14) c. Use conservation of momentum and energy to find the final speeds of the two blocks. [No credit will be given for just writing down recalled formulas.]



$V_0^2\left(\frac{m}{m+M}\right)\cdot M$

(a) $mV_0 + M(0) = V_{cm}(m+M)$

$V_0 \frac{m}{m+M} \cdot V_0 M$

$\frac{mV_0}{m+M} = V_{cm}$ +5

$V_{cm} \cdot V_0 M$

(b) $K_i = U + K_f$

$\frac{1}{2}mV_0^2 = U + \frac{1}{2}(m)V_{cm}^2 + \frac{1}{2}MV_{cm}^2$

$x_{max}^2 = \frac{mV_0^2 - (M+m)\left(\frac{(mV_0)^2}{(m+M)^2}\right)}{k}$

$\frac{1}{2}mV_0^2 = U + \frac{1}{2}V_{cm}^2(M+m)$

$x_{max}^2 = \frac{mV_0^2 - (mV_0)^2}{(m+M)k_s}$

$\frac{1}{2}mV_0^2 = \frac{1}{2}kx_{max}^2 + \frac{1}{2}V_{cm}^2(M+m)$

+7

$\frac{mV_0^2 - (M+m)V_{cm}^2}{k} = x_{max}^2$

$mV_0^2 - V_{cm}^2(M+m) = kx_{max}^2$

$x_{max} = V_0\boxed{\sqrt{\frac{mM}{(m+M)k_s}}}$ ✓

(c) go back

(c) $mv_0 + \cancel{M(0)} = mV_{1f} + MV_{2f}$

$mV_0 = mV_{1f} + MV_{2f}$

$\cancel{m}(V_0 - V_{1f}) = \cancel{M}(V_{2f})$

$\cancel{m}(V_0^2 - V_{1f})^2 = \cancel{M}(V_{2f})^2$

$\cancel{\tfrac{1}{2}}mv_0^2 + \cancel{\tfrac{1}{2}M(0)^2} = \tfrac{1}{2}mV_{1f}^2 + \tfrac{1}{2}MV_{2f}^2$

$mV_0^2 = mV_{1f}^2 + MV_{2f}^2$

$m(V_0^2 - V_{1f}^2) = M(V_{2f}^2)$

$V_0 + V_{1f} = V_{2f}$ $\quad$ $mV_0 = mV_{1f} + MV_{2f}$

$V_{2f} - V_0 = V_{1f}$

$mV_0 = mV_{1f} + M(v_0 + V_{1f})$

$mV_0 = mV_{1f} + Mv_0 + MV_{1f}$

$mV_0 - Mv_0 = V_{1f}(m+M)$

$$\frac{V_0(m-M)}{m+M} = V_{1f}$$

+ 14

$mv_0 = m(V_{2f} - V_0) + MV_{2f}$

$mv_0 = mV_{2f} - mV_0 + MV_{2f}$

$V_0(2m) = V_{2f}(m+M)$

$$V_{2f} = \frac{2m(V_0)}{m+M}$$

$\cancel{m}(v_{1f} - v_0)^2 = -\cancel{M}(V_{2f})^2$

$\cancel{m}(V_{1f} - V_0) = -\cancel{M}(v_{2f})$

$V_{1f} + V_0 = V_{2f}$

PHYSICS 1A

FINAL EXAM

Winter, 2017

Dr. Coroniti

There are 225 points on the exam, and you h...
show all your work and reasoning. No cred...
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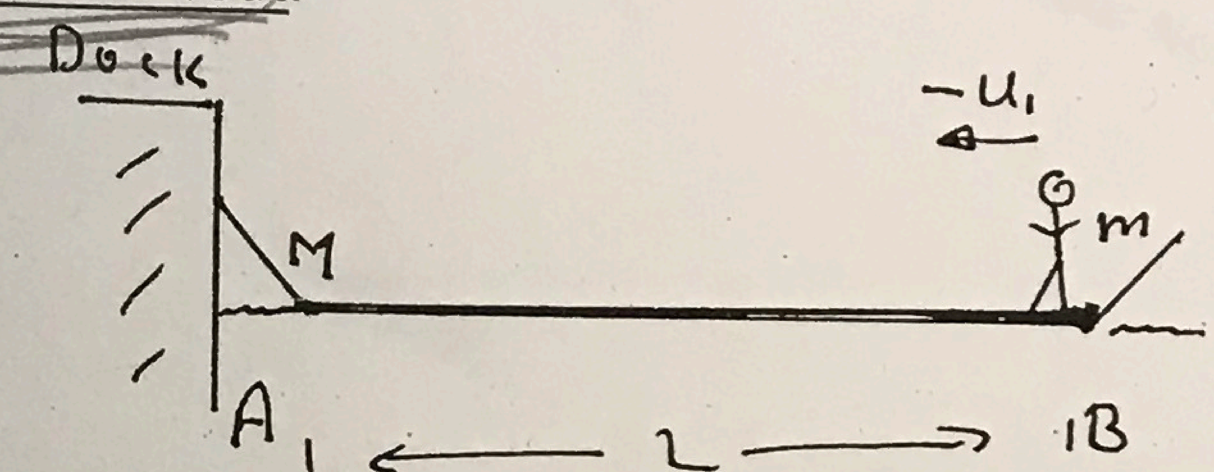
(22 Pts)

4. A boat with a mass M and length L is at rest on the water (frictionless) with the left end (A) touching a dock. A person with mass m starts walking from the right side (B) of the boat toward the dock with a speed $-u_1$ relative to the boat.

(12) a. Find the speed of the boat v_B and the distance Δx that the boat has moved away from the dock when the person reaches left end of the boat (A).



11 (10) b. Without stopping (the boat does not come to rest) at A, the person jumps off the boat toward the dock with a speed $-u_2$ relative to the boat at rest (i.e., in the rest frame of the boat, the person's speed is $-u_2$). Prove that the final speed of the boat is given by

$$v_{Bf} = \frac{m[(m+M)u_1+Mu_2]}{M(m+M)}$$

(a) $m(-u_1) = \cancel{M(v_B)}$

9 $\frac{m}{M}(-u_1) = V_{Bi}$

Initial $Y_{cm} = \frac{(M)(\frac{L}{2}) + m(L)}{M+m}$ = Initial

difference

X_{cm} Final $= \frac{(m)(0) + (M)(\frac{L}{2})}{M+m}$

$\frac{-mL}{M+m} = \Delta x$

(b) $(-u_2)(m) = \cancel{M}(V_{Bf})$

2 $V_{Bf} = \frac{m}{M}(-u_2)$

$V_{Bf} = \frac{m}{M}$

$u_1 = -u_2$

$u_2 = V_{Bi} - u_1$

$\frac{m}{M}(-u_1) - \frac{u_1 M}{M}$

$u_2 = \frac{-u_1(m+M)}{M}$