(25 Pts)

1. At time t = 0, a ballistic missile is launched from the top of a mountain of height z = H located at x = 0 with an initial velocity $\mathbf{v_0}$ that makes an angle θ with respect to the horizontal as shown. At t = 0, a gunner on the ground is located at a distance D from the base of the mountain, and fires a shell vertically upward with an initial speed u.

(9) a. Write the equations for the position $x_M(t)$ and $z_M(t)$ for the missile, and $z_S(t)$ for the shell.

(6) b. Now express the height of the missile and the height of the shell as a function of the x-position of the missile.

(10) c. Show that for the shell to hit the missile, the shell should be fired with an initial vertical speed given by

$$u = \frac{[H\cos\theta + D\sin\theta]}{D} v_0$$

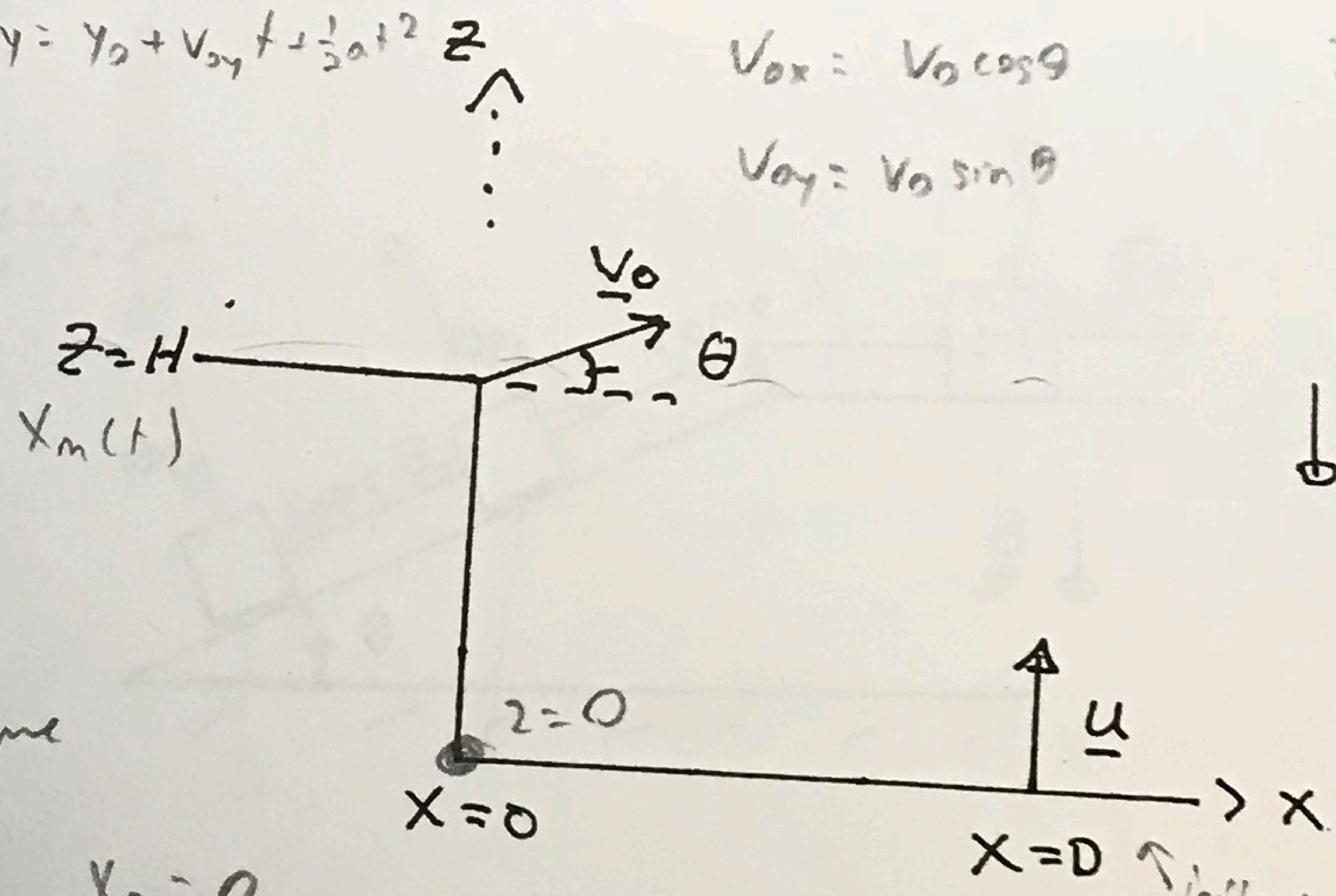
$y = y_0 + V_{0y} t + \frac{1}{2} a t^2$

$V_{0x} = V_0 \cos\theta$

$V_{0y} = V_0 \sin\theta$

function of $X_m(t)$

t should be same



$X_0 = 0$

$x = D$

$z = 0$

(a) $X_m(t) = V_0 \cos\theta t + 0$ initial position

$Z_M(t) = H + V_0 \sin\theta t - \frac{1}{2} g t^2$

$Z_s(t) = ut - \frac{1}{2} g t^2 + 0$ ← initial position

$X_s(t) = D + 0t + 0$

$X_s(t) = D$

+8

(b) $\frac{X_m(t)}{V_0 \cos\theta} = t$

$Z_M(t) = H + V_0 \sin\theta \left(\frac{X_m(t)}{V_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{X_m(t)}{V_0 \cos\theta}\right)^2$

$Z_s(t) = u\left(\frac{X_m(t)}{V_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{X_m(t)}{V_0 \cos\theta}\right)^2$

+8

(c) X and z positions must be equal

$Z_m(t) = Z_s(t)$ and $X_m(t) = X_s(t)$

$X_m(t) = D$

$X_m(t) = V_0 \cos\theta t = D$

$t = \frac{D}{V_0 \cos\theta}$

+9

$H + V_0 \sin\theta t - \cancel{\frac{1}{2} g t^2} = ut - \cancel{\frac{1}{2} g t^2}$

$\frac{H}{t} + \frac{V_0 \sin\theta \cancel{t}}{\cancel{t}} = \frac{u\cancel{t}}{\cancel{t}}$

$\frac{H}{t} + V_0 \sin\theta = u$

$\frac{H}{\frac{D}{V_0 \cos\theta}} + V_0 \sin\theta = u$

$\frac{H V_0 \cos\theta}{D} + V_0 \sin\theta = u$

$\frac{H V_0 \cos\theta}{D} + \frac{D V_0 \sin\theta}{D} = u$

$\frac{V_0(H\cos\theta + D\sin\theta)}{D} = u$

(30 Pts)

2. A block with mass M is pulled along a frictionless horizontal surface by an applied horizontal force F_a as shown. Block M is connected to a massless rope (A) that passes over a massless pulley and connects to a block with mass m_A located on an inclined plane with inclination angle θ as shown. Block m_A is connected to a third block with mass m_B on the inclined plane by a second massless rope (B) as shown.

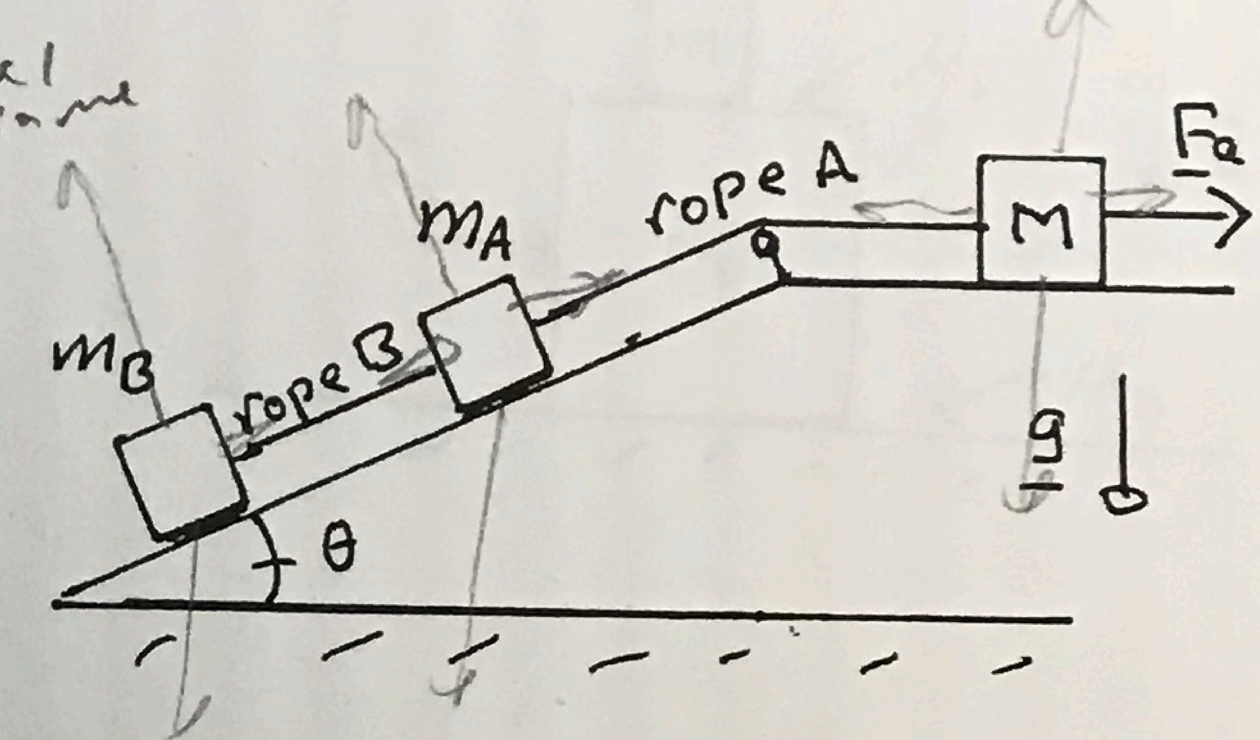
(9) a. Draw free body diagrams for each of the three blocks.

(15) b. Assuming that the three blocks accelerate together, prove that the acceleration is given by

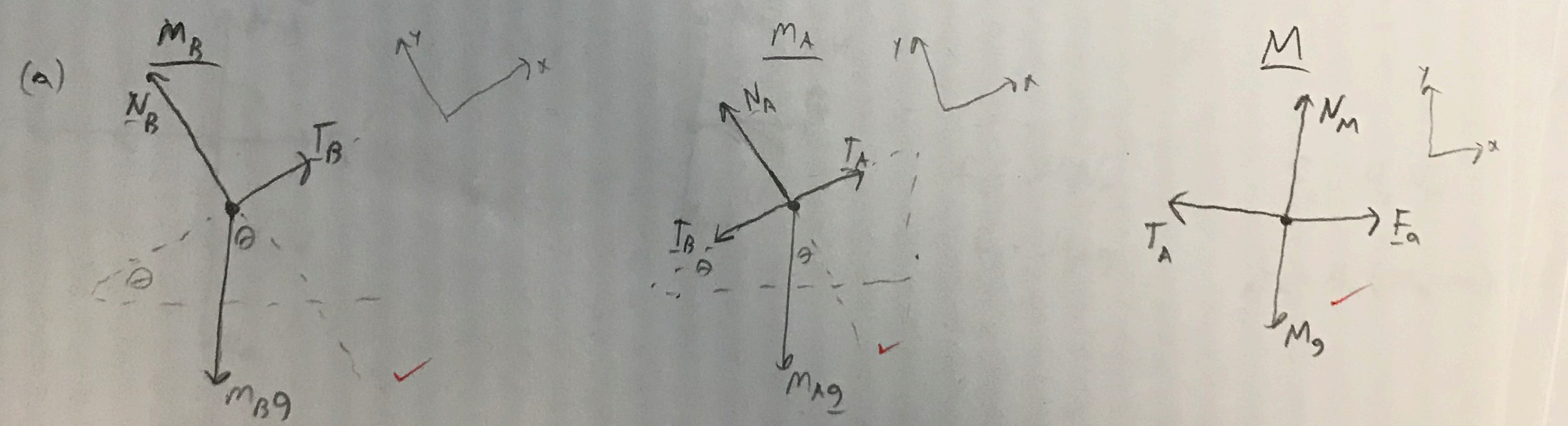
all accel are same

$$a = \frac{F_a-(m_A+m_B)g\sin\theta}{m_A+m_B+M}$$

(6) c. Suppose that rope (A) breaks. Find the acceleration of m_A and m_B and the tension in rope (B).



(a)



(b) m_B: F_x: $T_B - m_B g \sin\theta = m_B a$

F_y: $N_B - m_B g\cos\theta = 0$

$T_B = m_A a + m_B g \sin\theta$

$N_B = m_B g \cos\theta$

m_A: F_x: $T_A - T_B - m_A g \sin\theta = m_A a$

F_y: $N_A - m_A g \cos\theta = 0$

$N_A = m_A g \cos\theta$

M

F_x: $F_a - T_A = Ma$

F_y: $N_M - M_g = 0$

$T_A = F_a - Ma$

mass x accel

$F_a - M_a - (m_B a + m_B g \sin\theta) - m_A g \sin\theta = m_A a$

$F_a - Ma - m_B a - m_B g\sin\theta - m_A g \sin\theta = m_A a$

$F_a - m_B g \sin\theta - m_A g \sin\theta = a(m_A + M + m_B)$

$a = \frac{F - g\sin\theta\,(m_A+m_B)}{m_A + m_B + M}$

(c) m_B: $a = \frac{T_B}{m_B} - g\sin\theta = a(B)$

m_A: $-\frac{T_B}{m_A} - g\sin\theta = a(A)$

-6

27

(30 Pts)

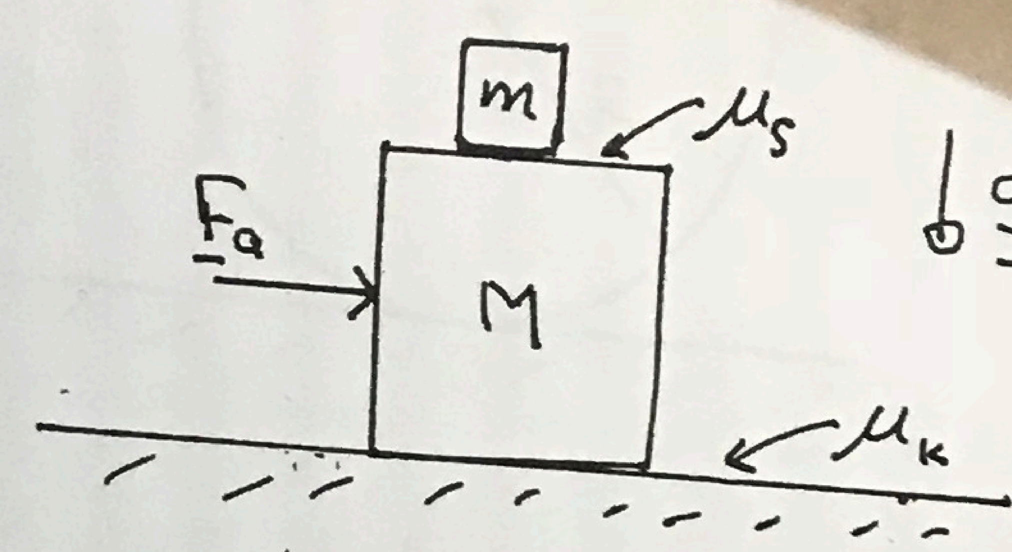
3. A block with mass M is pushed along a rough horizontal surface, with a coefficient of kinetic friction μ_k, by an applied horizontal force F_a as shown. A second block with mass m sits on top of block M. The coefficient of static friction between the surfaces of m and M is μ_s.

(10) a. Draw free body diagrams for the two blocks.

(10) b. If m does not slip relative to M, find the acceleration of the system.

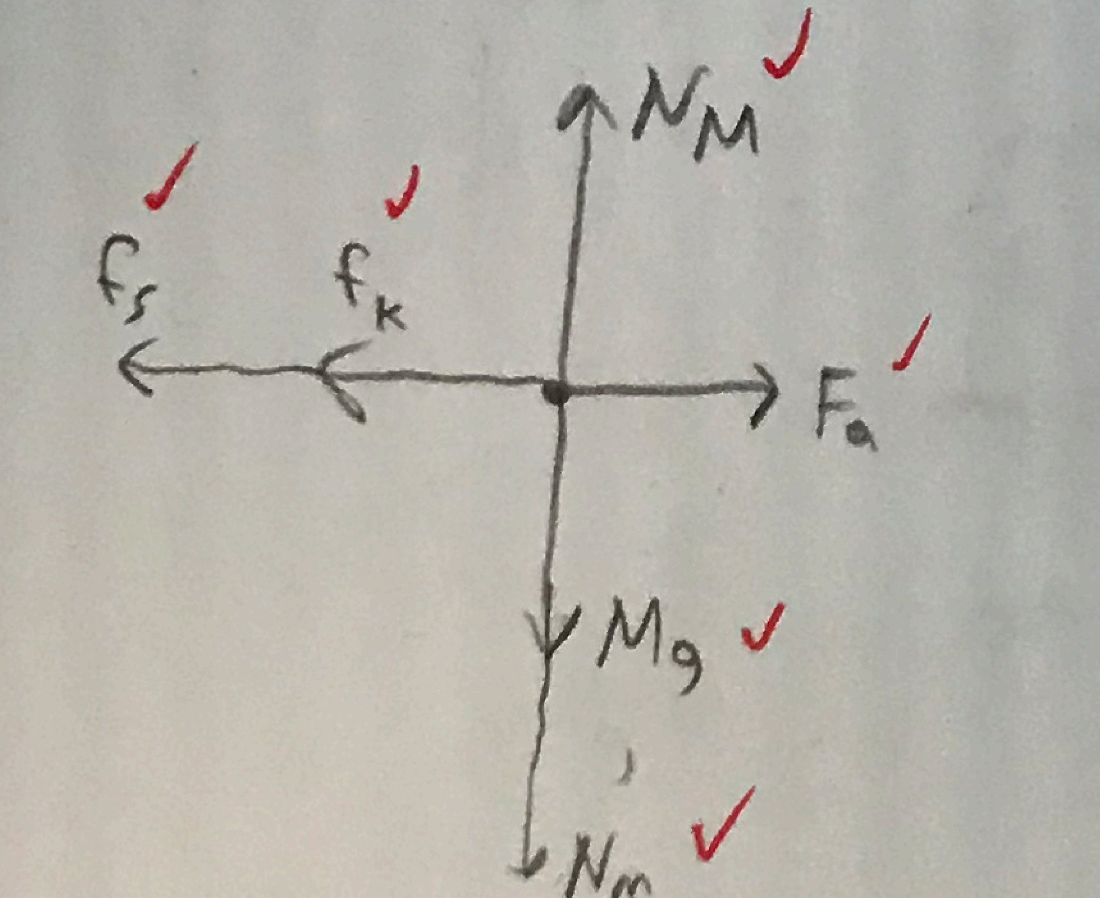
(10) c. Show that the maximum value of the applied force for which m remains at rest relative to M is given by

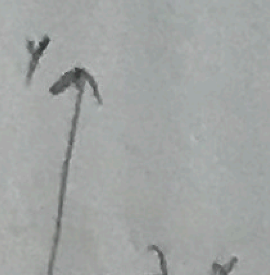
$$(F_a)_{MAX} = (m + M)(\mu_k + \mu_s)g$$



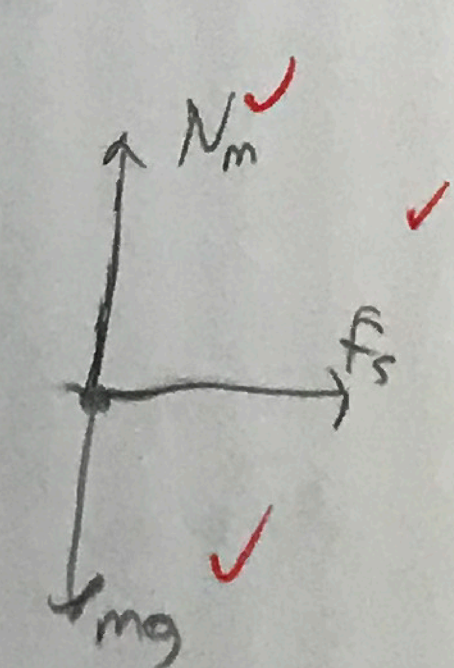
(a) 10

M





m



$f_s = \mu N_m$

$f_s = \mu_s mg$

$f_k = \mu_k N_M$

$f_k = \mu_k (Mg + mg)$

(b) M

7 $F_x = F_a - f_s - f_k = Ma$

$F_y = N_M - Mg - N_m = 0$

$N_M = Mg + N_m$

$N_M = Mg + mg$

$\frac{F_a - f_s - f_k}{M} = a$

$\frac{F_a - \cancel{\mu_s mg} - \mu_k g(M+m)}{M} = a$

m

F_x: $f_s = ma$

F_y: $N_m - mg = 0$

$N_m = mg$

$f_s = \frac{\mu_s mg}{m} = \frac{ma}{m}$

$\mu_s g = a$

max value at max friction

$a = \frac{f_s}{m} = \mu_s g$

(c) m

10 $F_a(max) = Ma + f_s + f_k$

$F_a(max) = \mu_s Mg + \mu_s mg + \mu_k Mg + \mu_k mg$

$F_a(max) = g(m+M)(\mu_k + \mu_s)$

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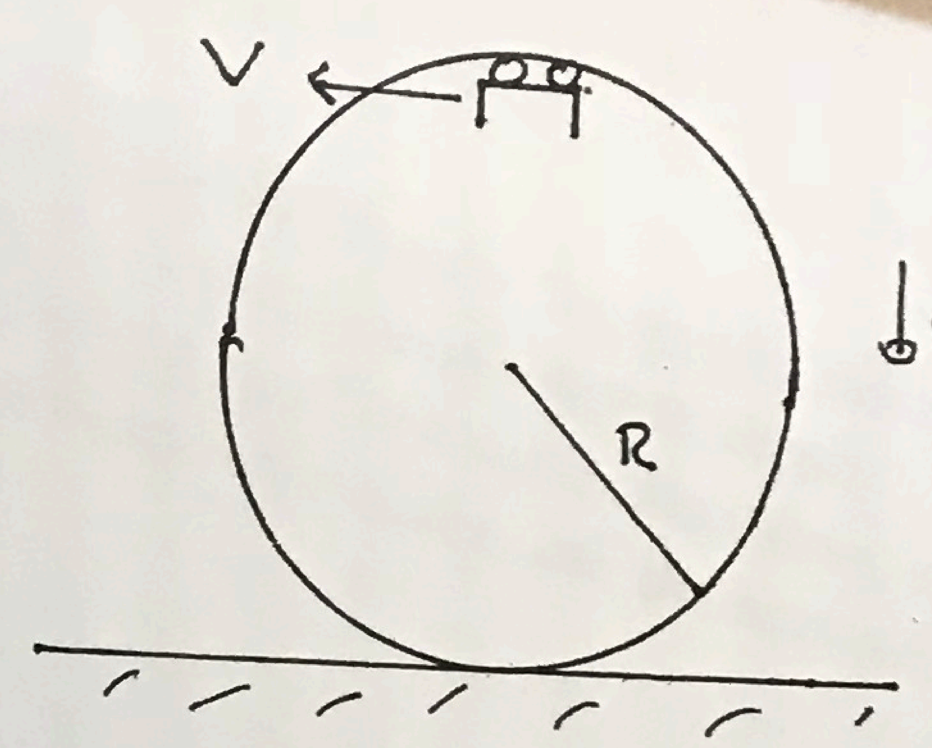
(15 Pts) 15

4. A thrilling roller coaster has a vertical circular loop with a radius R as shown.

(10) a. Find the minimum circular speed at which the roller coaster must be traveling in order that an un –seat-belted passenger does not fall out at the top of the loop. = when $N=0$

(5) b. If the roller coaster travels at this same speed when it is at the bottom of the loop, show that a passenger's weight would be double its normal value.



$V = \omega \cdot r$

(a)

$-ma_c$

mg

N

$N = 0$ because minimum circular speed

$$-mg - N = -\frac{mv^2}{R}$$

$$mg + N = \frac{mv^2}{R}$$

$$mg = \frac{mv^2}{R}$$

$$gR = v^2$$

P

$$v = \sqrt{gR}$$

(b)

N

mg

$$N - mg = \frac{mv^2}{R}$$

$$N - mg = \frac{m\left(\sqrt{gR}\right)^2}{R}$$

5

$$N - mg = \frac{mgR}{R}$$

$$N = 2(mg)$$