(30 Pts)

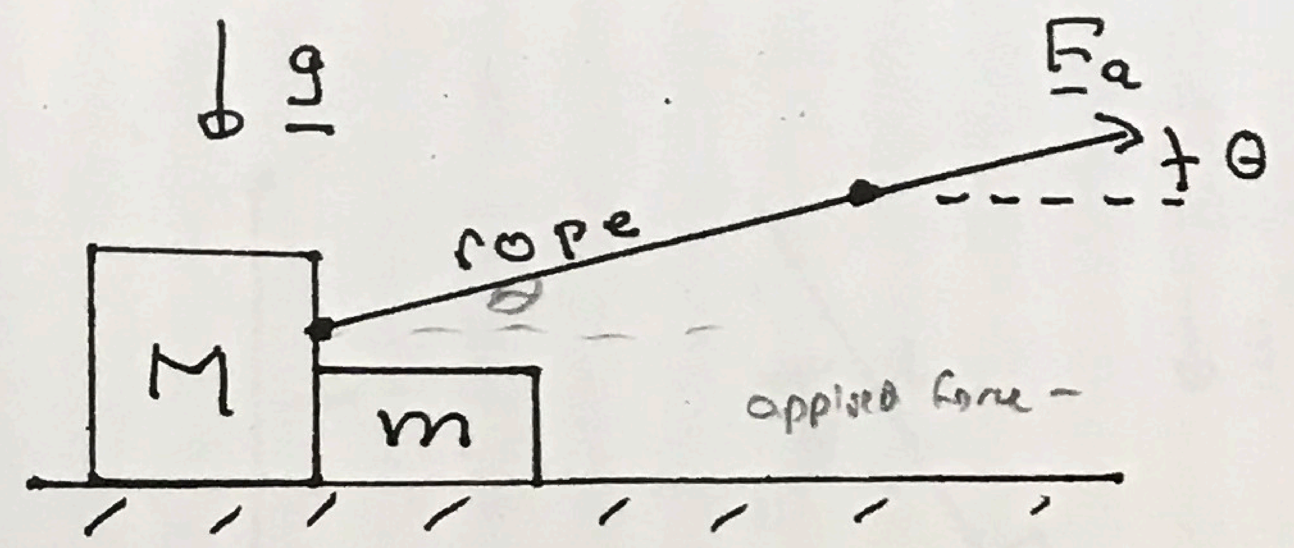
1. A block with mass M is free to slide along a smooth (frictionless) horizontal surface. A massless rope is attached to M, and is pulled with a constant force F_a that makes an angle θ with respect to the horizontal x-axis as shown. A second block with mass m is pushed along the surface by M. Gravity acts downward with acceleration g.

(10) a. Draw free body diagrams that exhibit all of the forces that act on M and m.

(10) b. Find the acceleration of the system, and the force of contact between M and m.

(10) c. Now suppose that the horizontal surface is rough with a coefficient of static friction μ_S, and that the two masses are at rest. Show that the maximum value of the force $|F_a|_{MAX}$ such that the system just remains at rest is given by

$$|F_a|_{MAX} = \frac{\mu_S(m+M)g}{\cos\theta + \mu_S \sin\theta}$$



(C) $F_a\cos\theta$ $-\mu_s mg - \mu_s(Mg - F_a\sin\theta) = 0$

$F_{Mm} - \mu mg = 0$

$F_{Mm} = \mu mg$

$F_a\cos\theta + \mu(F_a\sin\theta - Mg) - \mu mg = 0$

$F_a\cos\theta - \underbrace{\mu_s mg}_{\text{friction } m} - \underbrace{\mu_s(Mg - F_a\sin\theta)}_{\text{friction } M} = 0$

$F_a\cos\theta + \mu_s F_a\sin\theta = \mu_s mg + \mu_s Mg$

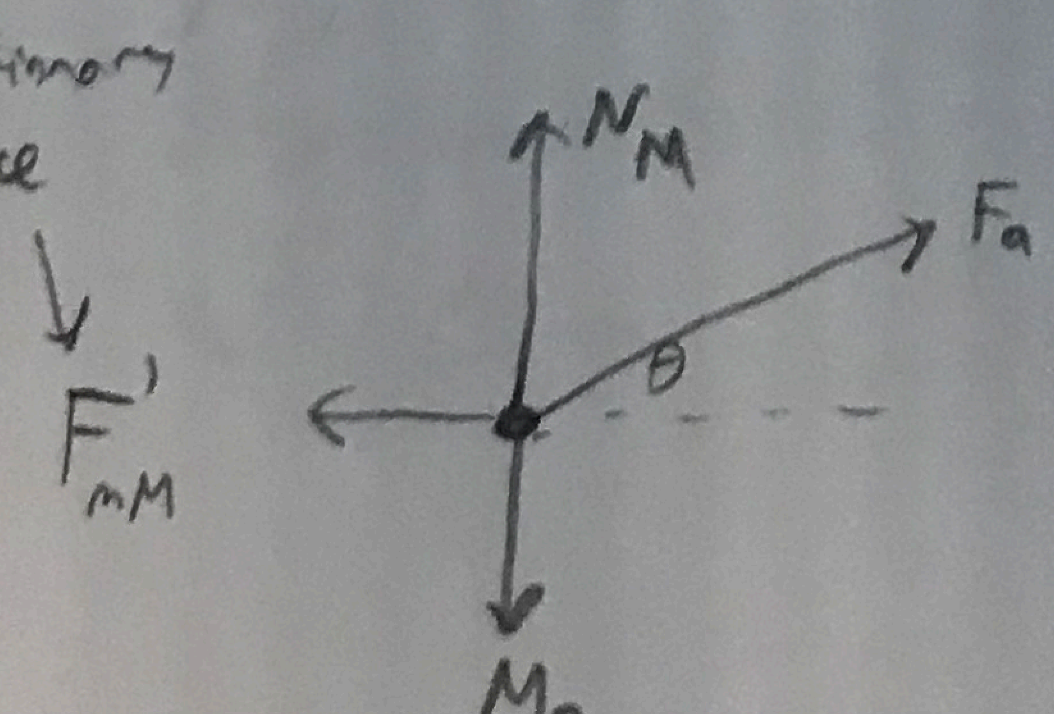
$F_a = \dfrac{\mu_s g(M+m)}{\cos\theta + \mu_s\sin\theta}$ ✓

+10

(a)

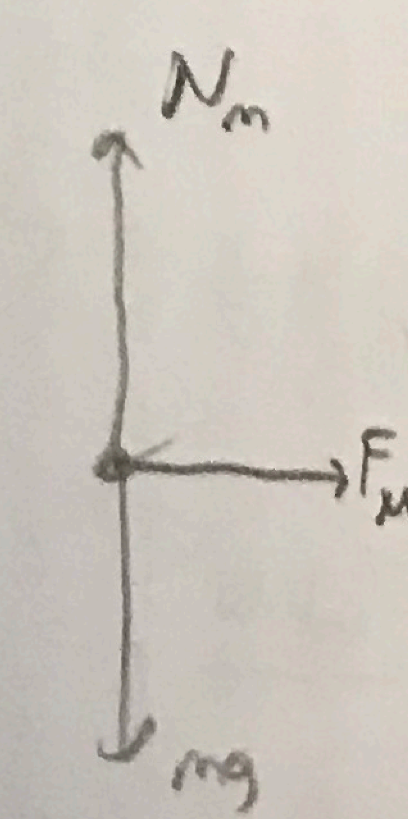
mass M

reactionary force



+10

mass m



mass M

(b) ΣF_x: $F_a\cos\theta - F'_{mM} = Ma$

ΣF_y: $N_M + F_a\sin\theta - Mg = Ma$

mass m

force of contact

ΣF_x: $F_{Mm} = ma$

ΣF_y: $N_m - mg = 0$

$F_a\cos\theta - ma = Ma$

$F_a\cos\theta = (M+m)a$

+10

$F_{Mm} = m\left(\dfrac{F_a\cos\theta}{M+m}\right)$

(25 Pts)

2. A pendulum with bob mass m and length L hangs vertically at rest under gravity on a massless string. A small particle with mass M initially moves horizontally with speed v_0 as shown. The particle has an elastic, one-dimensional collision with the pendulum mass. After the collision, the pendulum mass swings upward through an angle θ as shown.

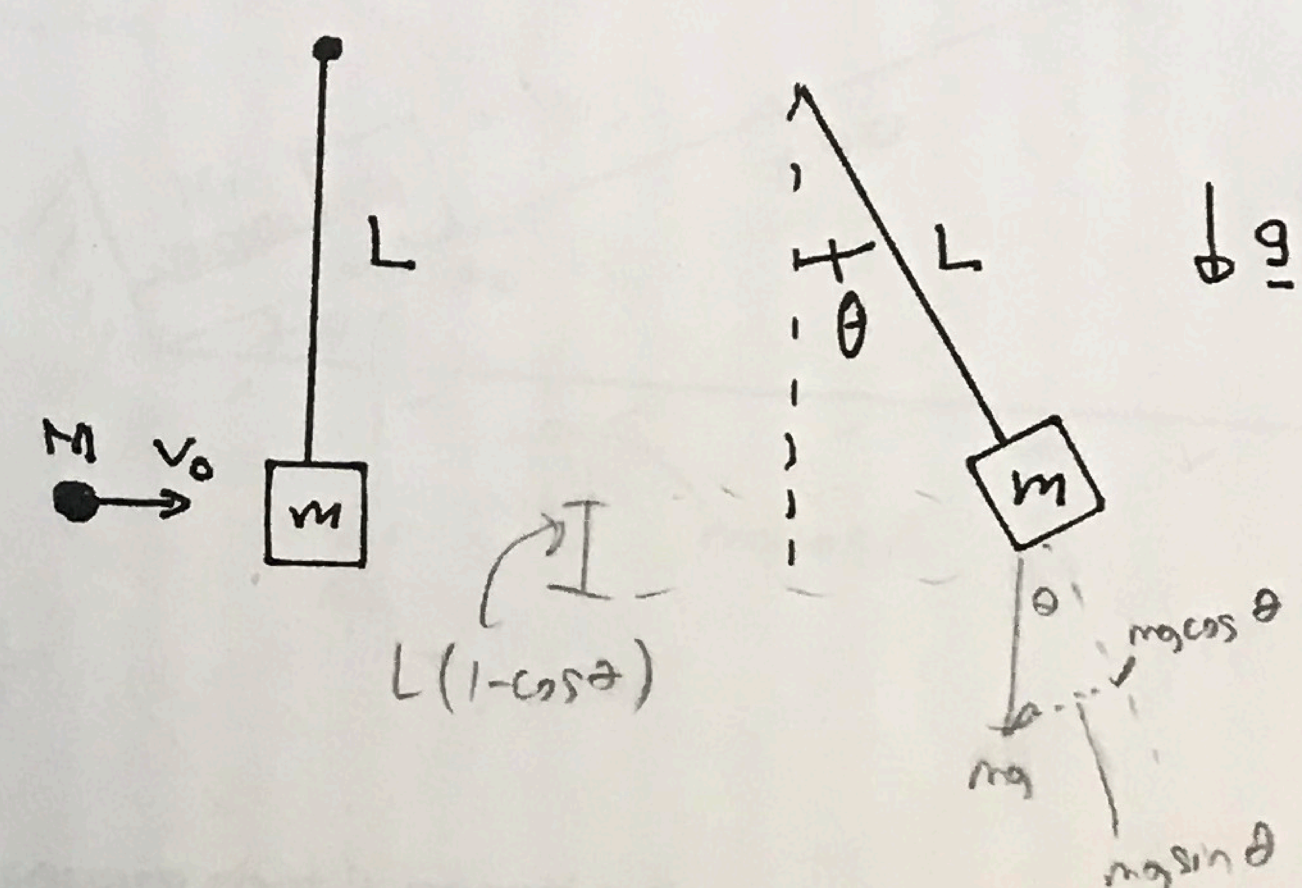
(9) a. Find the speed v_{mf} of mass m immediately after the collision ($\theta = 0$), and the speed $v_m(\theta)$ of the pendulum after it has swung upwards to θ .

(9) b. Show that the tension $T\ (\theta)$ in the string when the pendulum is at an angle θ is given by

$$T(\theta) = \frac{mv_m^2(\theta)}{L} + mgcos\theta$$

(7) c. The tension is observed to vanish when $\theta = \pi/_2$. Show that particle M's initial speed is

$$v_0 = \frac{m+M}{2M}\sqrt{2gL}$$



$V_0 = 2gL\left(\frac{m+M}{2M}\right)^2$

$\left(\frac{MV_0^2 - 2mgL(1-\cos\theta)}{m}\right)^{\frac{1}{2}} = V_m(\theta)$

(a)

~~$Mv_0 = (m+M)V_f$~~

~~$V_{mf} = \frac{M}{m+M}V_0$~~

$\frac{1}{2}Mv_0^2 = \frac{1}{2}mV_{mf}^2$

$\left(\frac{MV_0^2}{m}\right)^{\frac{1}{2}} = V_{mf}$

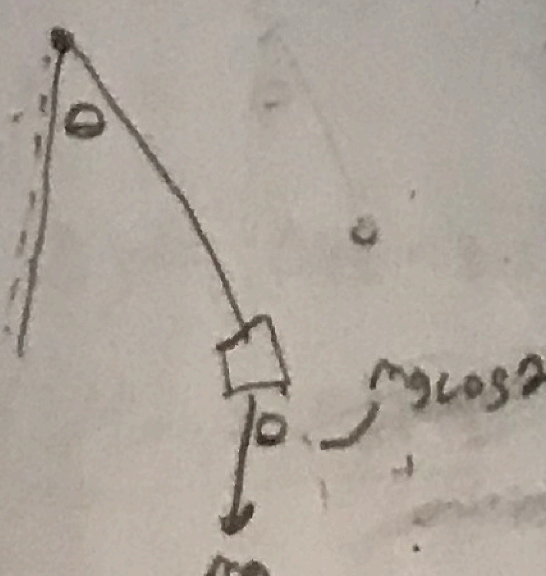
$\sqrt{\frac{M}{m}}V_0 = V_{mf}$

+3

$\frac{1}{2}MV_{mf}^2 = \frac{1}{2}mV_m^2(\theta) + mgL(1-\cos\theta)$

$\left(\frac{MV_{mf}^2 - 2mgL(1-\cos\theta)}{m}\right)^{\frac{1}{2}} = V_m(\theta)$

(b)



$T(\theta) =$ centripetal force $+ mg\cos\theta$

$T(\theta) = \frac{mV_m^2(\theta)}{L} + mg\cos\theta$

+9

$V_m(\frac{\pi}{2}) = \left(\frac{MV_{mf}^2 - 2mgL}{m}\right)^{\frac{1}{2}}$

$V_m(\frac{\pi}{2}) = (V_{mf}^2 - 2gL)^{\frac{1}{2}}$

(c) $T(\frac{\pi}{2}) = \frac{mV_m^2(\frac{\pi}{2})}{L} + mg\cos(\frac{\pi}{2})$ +3

$mV_{mf}^2 - 2mgL = 0$

$\frac{M}{m}V_0^2 = 2gL$

$V_0 = \sqrt{2gL}\sqrt{\frac{m}{M}}$?

$\frac{M^2}{(m+M)^2}V_0^2 - 2gL = 0$

$V_0^2 = 2gL\frac{(M+m)^2}{M^2}$

$V_0 = \sqrt{2gL}\left(\frac{M+m}{M}\right)$

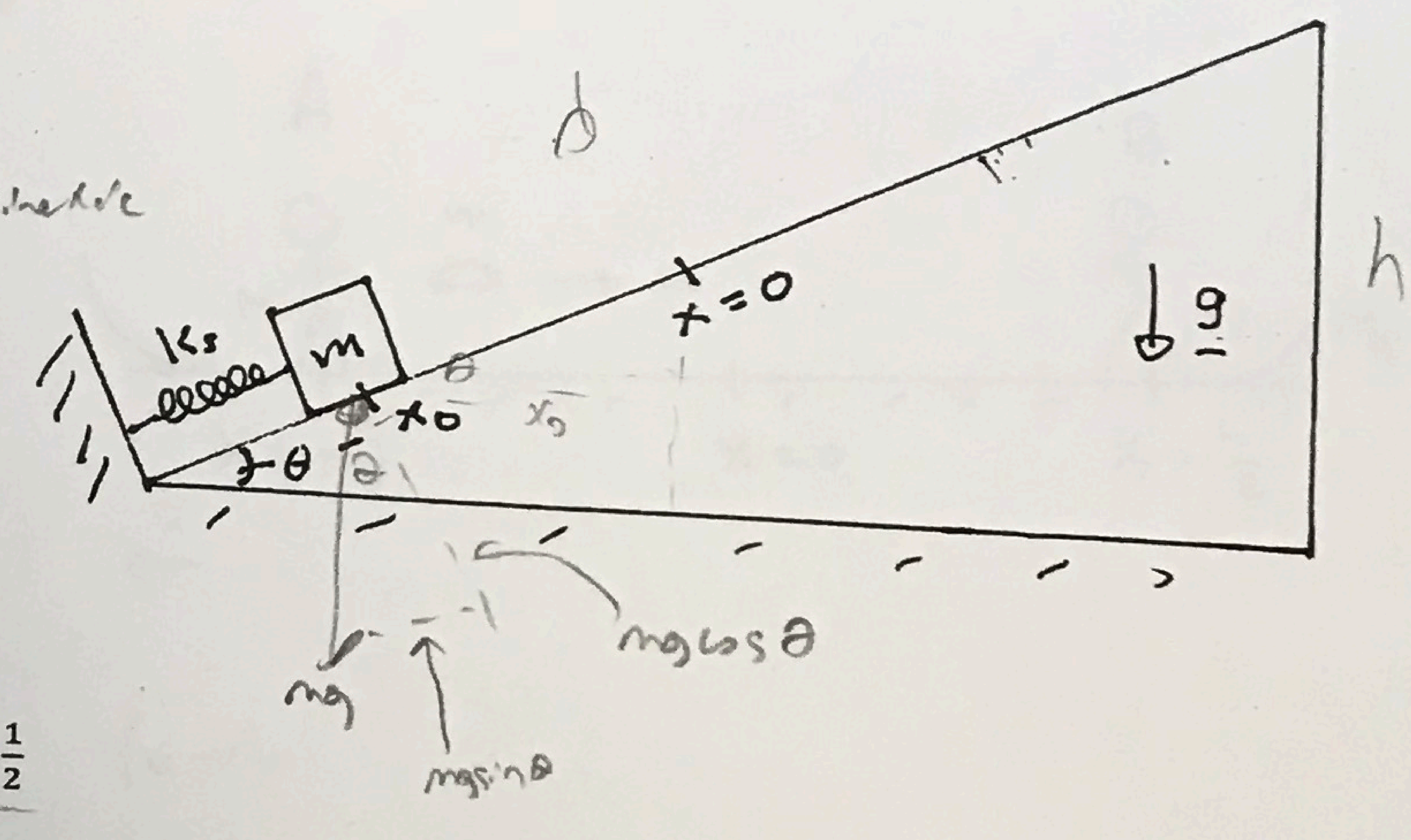
18

(30 Pts)

3. A spring with stiffness constant k_s has one end attached at the bottom of a fixed inclined plane (inclination angle θ). The spring is initially compressed by a distance x_0 from its equilibrium (uncompressed) position at x = 0 as shown. A block with mass m is attached to the other end of the spring, and is initially held at rest. Measure distances x along the inclined plane from the origin x = 0.

(15) a. Assume that the surface of the plane is frictionless. The block is released, and move up the plane. How far does the block travel up the plane before coming to rest? ← no kinetic

(15) b. Now assume that the surface of the plane is rough with a coefficient of kinetic friction μ_K. After the block is released, show that it will pass the origin x = 0 with speed



$$v = [\frac{k_s x_0^2}{m} - 2gx_0(\sin\theta + \mu_K\cos\theta)]^{\frac{1}{2}}$$

What happens if the quantity under the square root is negative?

(a) $E_i = \frac{1}{2}k(-x_0)^2$ $E_F = mgh + \frac{1}{2}k(d - x_0)^2$ $\sin(\theta) = \frac{h}{d}$

7

distance up plane → $d = \frac{h}{\sin\theta}$

$\frac{1}{2}kx_0^2 = mgh$

$\frac{Kx_0^2}{2mg} = h$

$d = \frac{kx_0^2}{2mg\sin\theta}$

(b) $E_i = \frac{1}{2}k(-x_0)^2$ $E_f =$

11

$\Delta W = \Delta KE + \Delta U$ ✓

$x\sin\theta$

$(\mu mg\cos\theta)(-x_0) = \frac{1}{2}mv^2 + mgh - \frac{1}{2}kx^2$

✓

$\mu mg\cos\theta(-x_0) = \frac{1}{2}mv^2 + mgx_0\sin\theta - \frac{1}{2}kx^2$

$2\mu_k g\cos\theta(-x_0) = v^2 + 2gx_0\sin\theta - \frac{kx_0^2}{m}$

$\frac{kx_0^2}{m} - 2\mu_k g\cos\theta x_0 - 2gx_0\sin\theta = v^2$

✓ $\left[\frac{Kx_0^2}{m} - 2gx_0(\sin\theta + \mu_k\cos\theta)\right]^{\frac{1}{2}} = V$

12

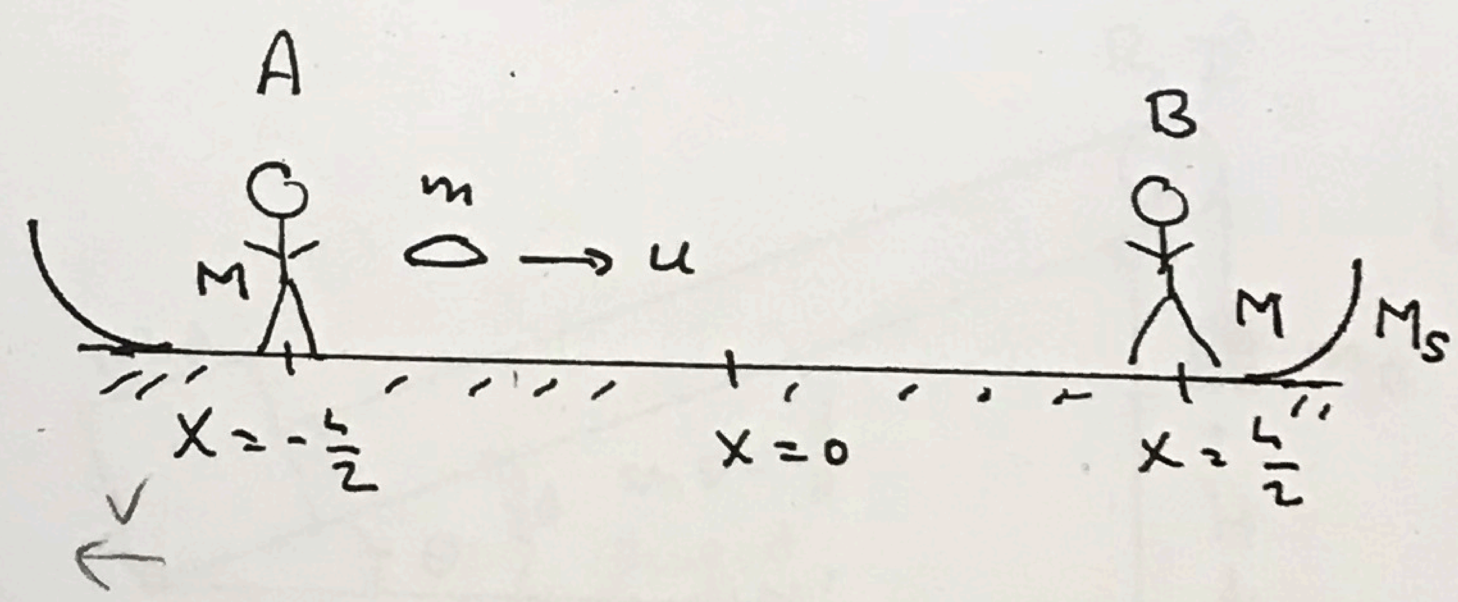
(25 Pts)

4. A sled with mass M_S and length L is at rest on a frozen (frictionless) lake. Person A with mass M is standing still on the sled at one end (x = - L/2), and Person B with mass M is standing still at the other end (x = + L/2) as shown. Person A throws a football with mass m toward Person B. The football moves with speed u relative to the sled. Person B catches the football. The final state of the system is at rest.

(8) a. In what direction and at what speed does the sled and the passengers move after the football is thrown by A and before the football is caught by B ?

(7) b. Through what distance has the sled and its passengers moved after Person B catches the football?

(10) c. Now find the distance that the sled and passengers have moved by analyzing the motion of the system's center of mass.



(u-v)

(a) 6 The sled and passengers move to the left ✓

$P_i = 0 = P_f$ momentum = conserved ✓

$m(u - v_f) = (M + M + M_s)(-v_f)$

$m(u-v) = (2M + M_s)(-v_f)$

$mu - mv_f = -2Mv_f - M_s v_f$

$\Rightarrow v_s = \frac{mu}{2M+M_s+m}$

u-v

$m(m) =$

(b) 0

$P_i = 0 = P_f$

$(2M + m + M_s)v = mu$

(c) 6 initial $X_{cm} = \frac{M(-\frac{L}{2}) + M(\frac{L}{2}) + M_s(0) + m(-\frac{L}{2})}{M+M+M_s+m}$ ✓ = final $X_{cm} = \frac{M(-\frac{L}{2}+\Delta x) + M(\frac{L}{2}+\Delta x) + M_s(\Delta x) + m(\frac{L}{2}+\Delta x)}{M+M+M_s+m}$

$-m(\frac{L}{2}) = M_s(\Delta x) + m(\frac{L}{2}) + m\Delta x$

$-2m(\frac{L}{2}) = \Delta x(M_s + m)$

$\Delta x = -2m(\frac{L}{2})$

(30 Pts)

5. Block A with mass m_A is located at the base of a frictionless inclined plane, and is attached to a massless rope that passes over a pulley with radius R and moment of inertia I about an axis through its center of mass. The rope is attached to a second block B with mass $m_B > m_A$ that hangs straight down under gravity, and is initially at a height H above the ground as shown. Assume that the rope does not slip on the pulley. Block B is then released, and the system is free to move.

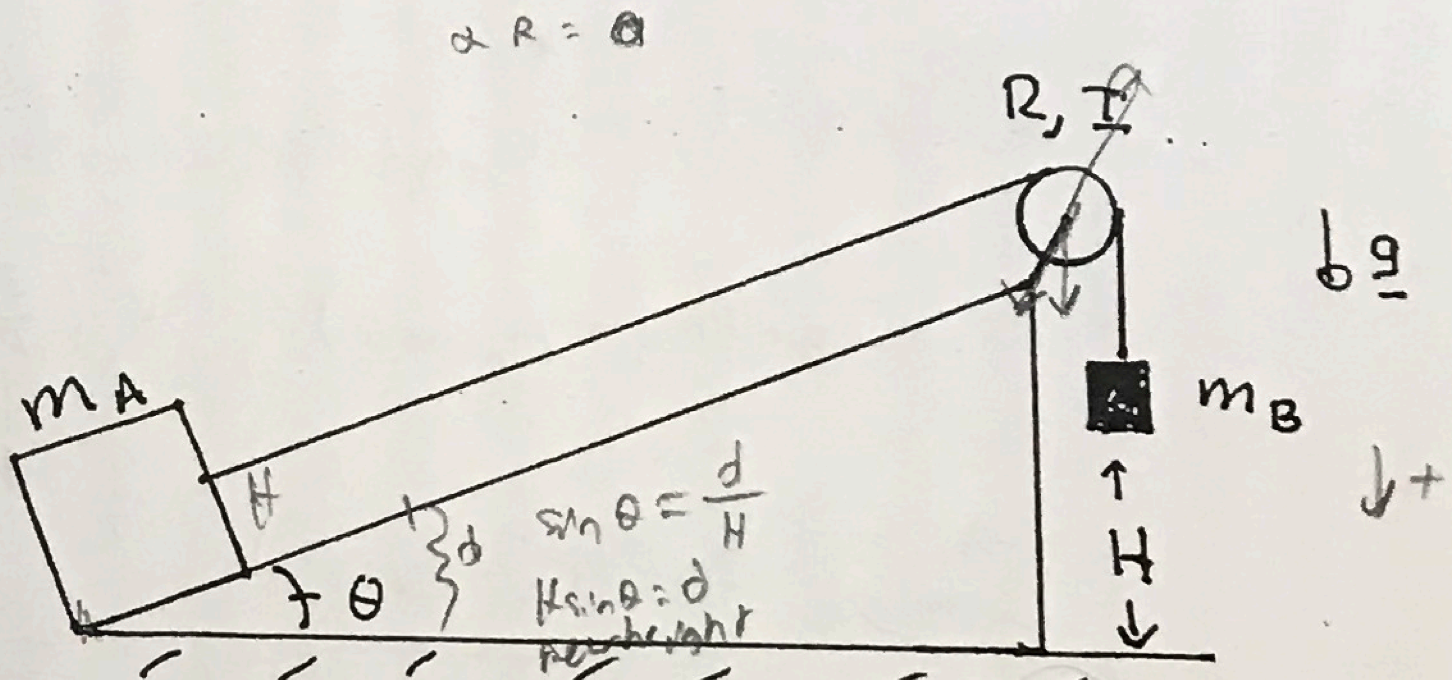
$\omega R = v$

$\alpha R = a$

(10) a. Sketch free body diagrams for blocks A and B and for the pulley.

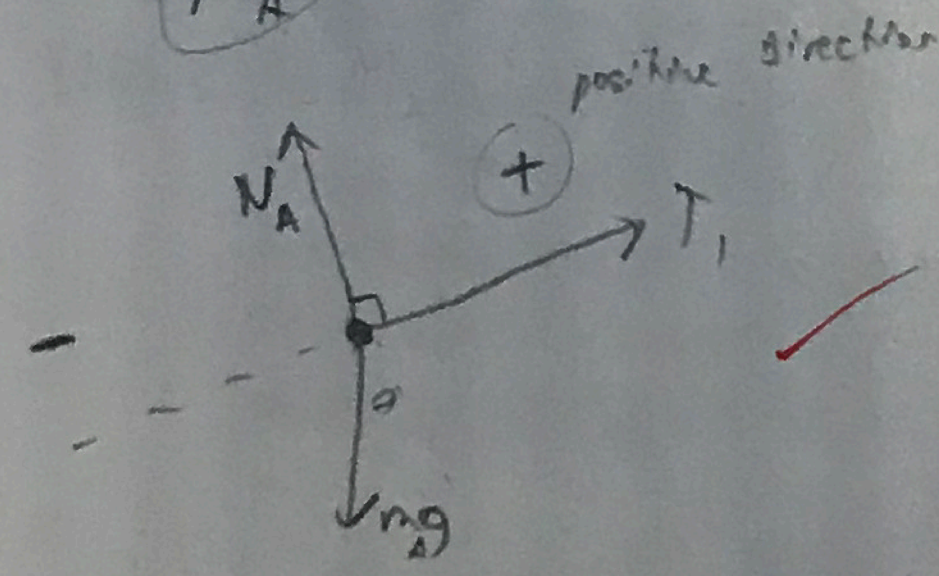
(10) b. Find the acceleration of the system after block B is released and before it hits the ground.

(10) c. Use conservation of energy to find the speed of block B just before it hits the ground. on back

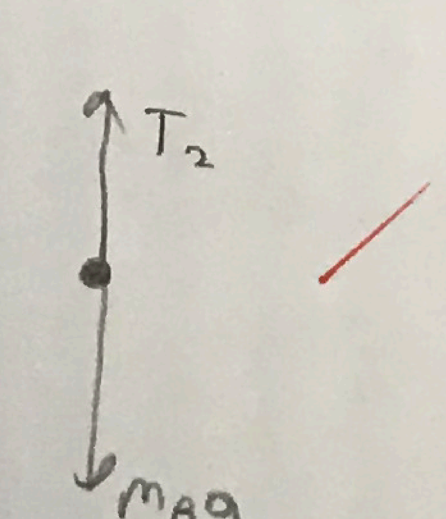


(a)

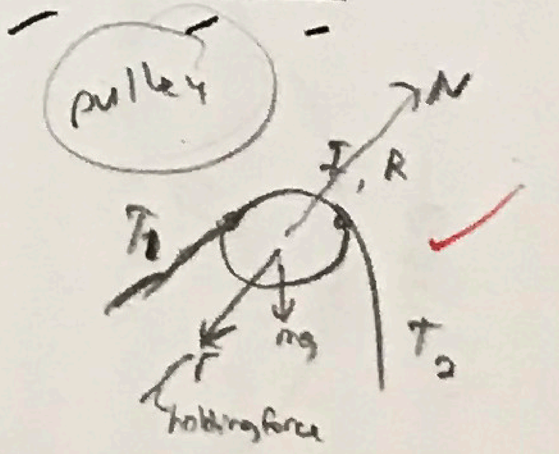
M_A



M_B



pulley



$\tau = I\alpha = r \times F$

$I\alpha = \tau = R(T_2 - T_1)$

$I\alpha = R(T_2 - T_1)$

$\alpha = \dfrac{R(T_2 - T_1)}{I}$

$\dfrac{a}{R} = \dfrac{R(T_2 - T_1)}{I}$

$a = \dfrac{R^2(T_2 - T_1)}{I}$

(b) $T_1 - m_A g \sin\theta = m_A a$

$N_A - m_A g \cos\theta = 0$

$T_1 = m_A a + m_A g \sin\theta$

$m_B g - T_2 = m_B a$

$T_2 = m_B(g - a)$

$Ia = R^2([m_B(g-a)] - [m_A a + m_A g \sin\theta])$

$Ia = R^2[m_B g - m_B a - m_A a - m_A g \sin\theta)$

$Ia = R^2 m_B g - R^2 m_B a - R^2 m_A a - R^2 m_A g \sin\theta$

$Ia + R^2 m_B a + R^2 m_A a = R^2(m_B g - m_A g \sin\theta)$

$a(I + R^2 m_B + R^2 m_A) = R^2(m_B g - m_A g \sin\theta)$

$a = \dfrac{R^2(m_B g - m_A g \sin\theta)}{I + R^2 m_B + R^2 m_A} =$

$$\boxed{a = \dfrac{m_B g - m_A g \sin\theta}{\frac{I}{R^2} + m_B + m_A}}$$

$\omega \times r = v$

$E_i \qquad E_f \qquad \omega = \frac{v}{r}$

(c) $m_B g h = \frac{1}{2} I \omega^2 + \frac{1}{2} m_B v^2 + m_A g h \sin\theta + \frac{1}{2} M_A v^2$

$2 m_B g h = I\omega^2 + m_B v^2 + 2 m_A g h \sin\theta + M_A v^2$

$2 m_B g h = I\left(\frac{v^2}{R^2}\right) + m_B v^2 + 2 m_A g h \sin\theta + M_A v^2$

$2 m_B g h - 2 m_A g h \sin\theta = I\left(\frac{v^2}{R^2}\right) + m_B v^2 + m_A v^2$

$2gh(m_B - m_A \sin\theta) = v^2\left(\frac{I}{R^2} + m_B + m_A\right)$

$$\left(\frac{2gh(m_B - m_A \sin\theta)}{\frac{I}{R^2} + m_B + m_A}\right)^{\frac{1}{2}} = V$$

V= speed of block b

(30 Pts)

6. A sphere with mass M, radius r, and moment of inertia about a diagonal axis I = 2/5 Mr^2 is initially held at rest against a clamped spring with stiffness constant k_S that has been compressed by a distance D (x = - D) from its equilibrium position (x = 0) as shown. The surface between x = - D and x = 0 is smooth (frictionless). At time t = 0, the clamped spring is released. The sphere immediately starts to slip without rolling, and loses contact with the spring when it reaches x = 0. At x = 0, the surface becomes rough and remains rough, and the sphere immediately starts to roll without slipping. The sphere continues to roll without slipping as it moves up a vertical circular track with radius R >> r.

ωR = v
αR = a

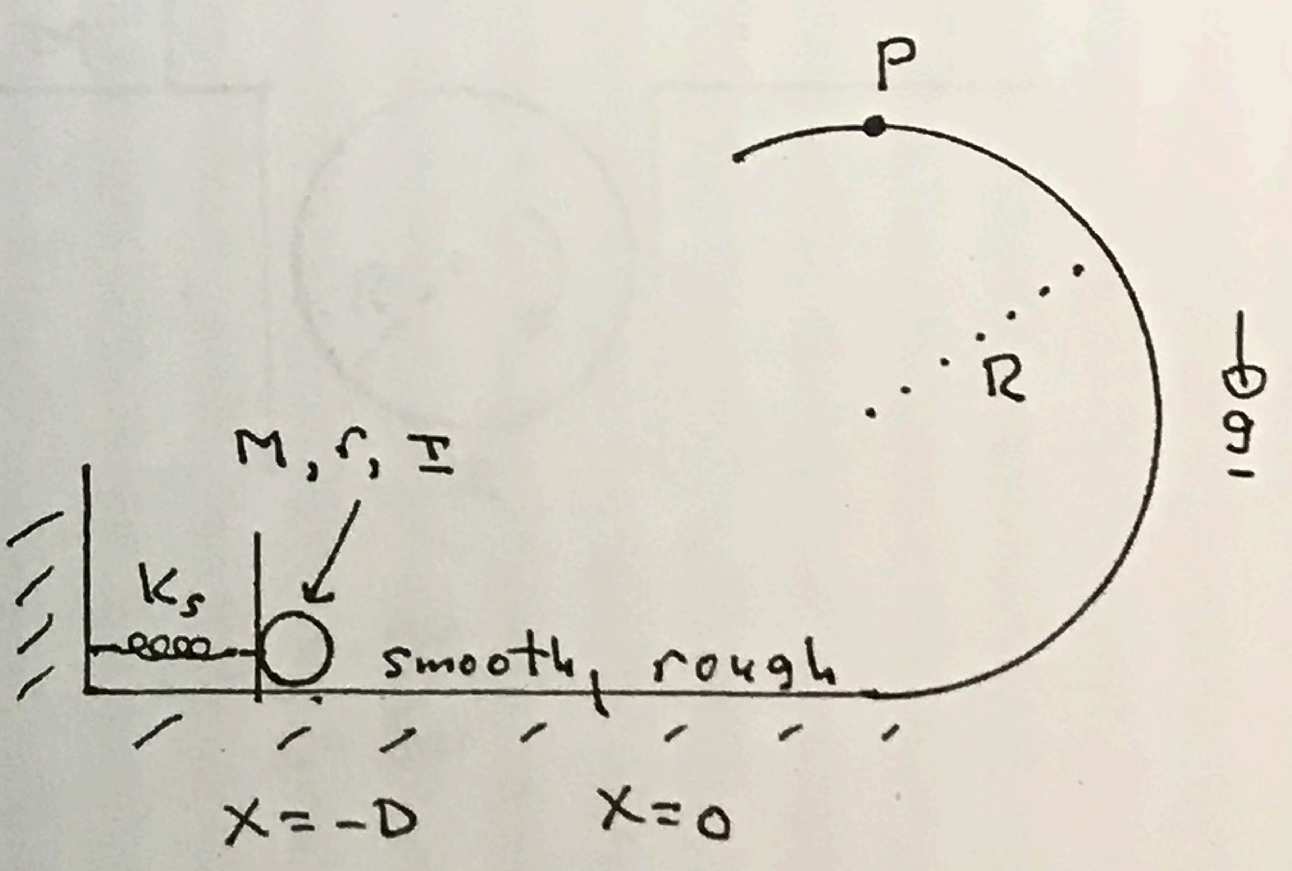
(5) a. Find the horizontal speed v_0 of the sphere just before it reaches x = 0.

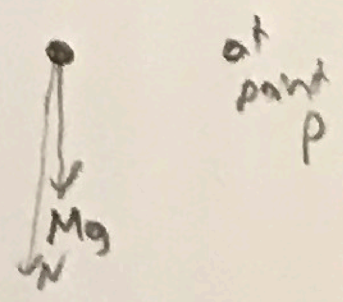
(12) b. Show that the sphere's center of mass speed v_f just after it passes x = 0 is $v_f = {}^5/_7\, v_0$. [Hint: Recall the impulse momentum and angular momentum theorems.]

P $V_0 = \frac{7}{5} V_f$ mv =   L   r × mv

(13) c. Show that the minimum distance D_{min} through which the spring must be compressed in order for the sphere to reach the top of the track (Point P) without falling off is given by

$$D_{min} = \left[\frac{189}{25}\frac{MgR}{k_S}\right]^{1/2}$$





(a) $\frac{1}{2}k(-D)^2 = \frac{1}{2}mV_0^2$

$$\boxed{\sqrt{\frac{KD^2}{m}} = V_0}$$

✓

(b) $\Delta p = J$ $J = F \cdot t$ $\omega r = v$ $\frac{v}{r} = \omega$

$\Delta p = mv_0$

−10

$\frac{1}{2}mv_0^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mV_f^2$

$mv_0^2 = \left(\frac{2}{5}Mr^2\right)\left(\frac{V_f^2}{r^2}\right) + mV_f^2$

$V_0^2 = \frac{2}{5}V_f^2 + V_f^2$

$V_0^2 = \frac{7}{5}V_f^2$

$\frac{5}{7}V_0 = V_f$

2   3   2   2

(c) $\frac{1}{2}k(-D_{min})^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mg(2R)$

−9

$-Mg - N = -\frac{Mv^2}{R}$

$Mg = \frac{Mv^2}{R}$

$\sqrt{gR} = V$

$\omega R = v$

$\frac{v}{R} = \omega$

$\omega = \frac{\sqrt{gR}}{R}$

$K(-D_{min})^2 = mv^2 + I\omega^2 + 4mgR$

$= 5MgR + I\omega^2$

$= 5MgR + \left(\frac{2}{5}MR^2\right)\left(\frac{gR}{R^2}\right)$

$= \boxed{\left[\frac{27}{5}\frac{MgR}{K_s}\right]^{\frac{1}{2}}}$

(25 Pts)

7. A block with mass M slides along a smooth horizontal surface with an initial speed of v_0. A cylinder with radius R and moment of inertia I is free to rotate about its axis of symmetry in the direction shown, but is initially at rest. When the block first makes contact with the rough surface of the cylinder, the block slips on the cylinder. However, the friction is large enough that slipping stops before the block loses contact with the cylinder and moves to the dashed position with a final speed v_f.

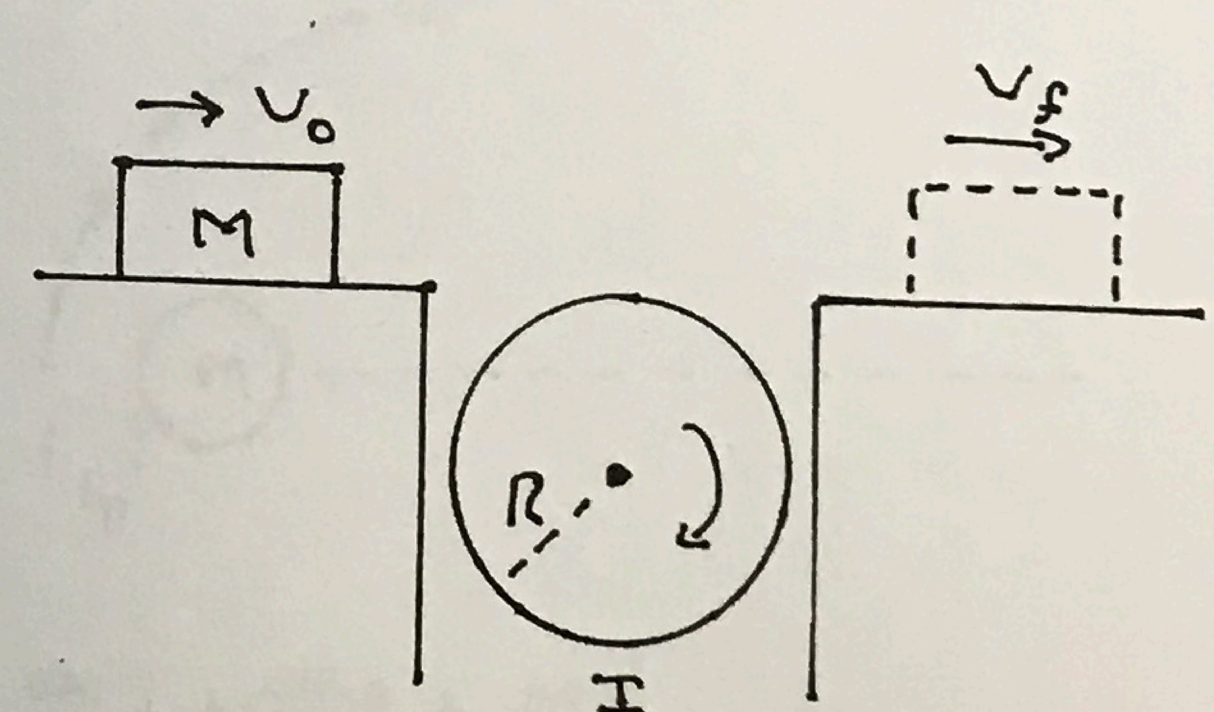
(15) a. Find the final angular frequency $\leftarrow \omega$ of the cylinder and the final speed of the block. [Hint: Is the system's angular momentum conserved?] Ω

(10) b. Show that the difference between the initial (E_i) and the final (E_f) total mechanical energies of the system is given by

$$E_f - E_i = -\frac{Mv_0^2}{2}\frac{1}{(1+\frac{MR^2}{I})}$$

$I + I(mR^2)$

What happened to the lost energy?



$L = r \times mv$

$L = I\omega$

(a) $L = I\omega$ $\qquad E_i = \frac{1}{2}Mv_0^2$ $\qquad E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2$

9

$L_0 = 0 = L_f$ $\qquad P_i = Mv_0$ $\qquad P_f =$ $\qquad mv_0 = I\omega_f + \cancel{mv_f}$

$\underset{P_0}{\downarrow} mv_0 = \underset{L_f}{\downarrow} mvr + \cancel{mv_f}$ P_f $\qquad$ There is a torque here $\qquad \frac{m(v_0 - v_f)}{I} = \omega_f$

initial momentum = mv_0 $\qquad L = r \times P$

final momentum = mv_f

initial angular momentum = 0

final angular momentum = $I\omega$

angular momentum is not conserved here

$\Delta E = \Delta K + \Delta U$

$(f_r)(R)$

(b) $E_i = \frac{1}{2}Mv_0^2$ $\qquad E_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}(I + mR^2)\omega_f^2$

$\frac{1}{2}Mv_f$

Spring, 2017

PHYSICS 1B

There are 100

show all

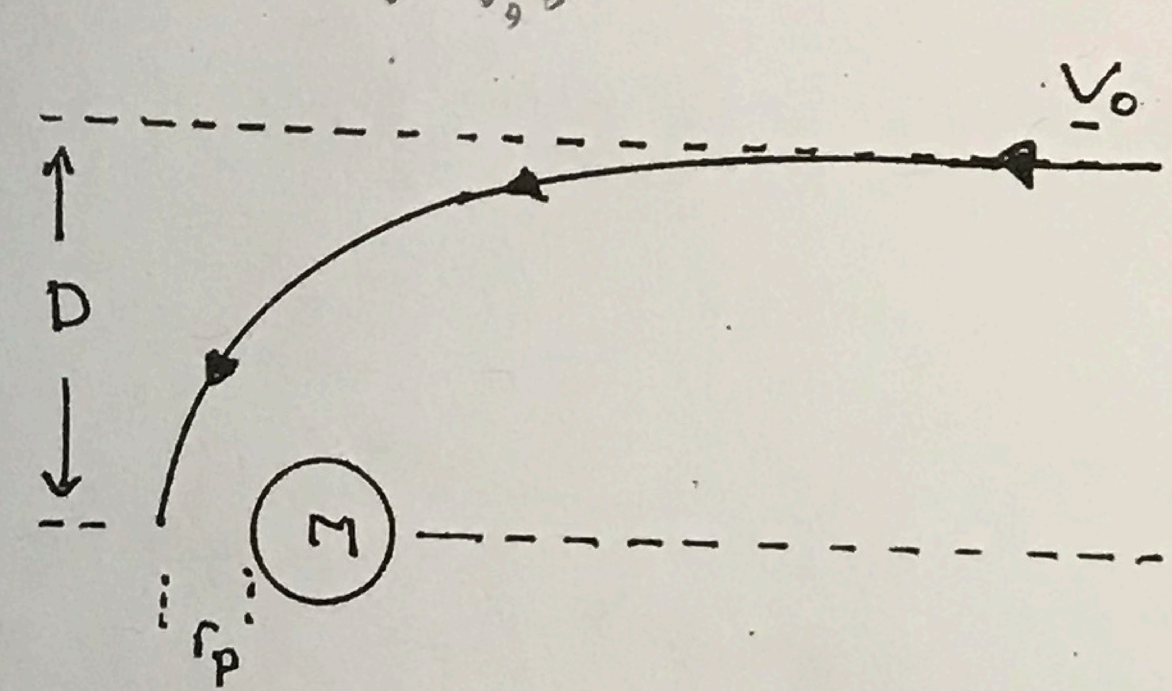
$F = \frac{GMm}{r^2}$

$U = \frac{-GmM}{r}$

(30 Pts)

8. A comet of mass m starts at a very large distance (essentially $r \to \infty$) from the Sun (mass = M) with a velocity v_0 such that, if the comet were to travel in a straight line, it would pass the Sun at a distance D as shown. The Sun's gravity bends the comet's trajectory so that the comet's closest approach to the Sun (perihelion) is at a radial distance r_P. At perihelion, the comet's velocity is perpendicular to the radius vector from the Sun. [Recall: in cylindrical coordinates (r, θ) the velocity in the orbital plane is $\boldsymbol{v} = v_r\hat{\boldsymbol{r}} + v_\theta\hat{\boldsymbol{\theta}}$ where $v_r = \dot{r}$ and $v_\theta = r\dot{\theta}$. At perihelion, $\dot{r} = 0$.]

$V = V_\theta \hat{\theta}$



(5) a. From Newton's law of Universal Gravitation, prove that the angular momentum of the comet about the Sun is conserved.

(7) b. From conservation of the angular momentum, find the comet's velocity at perihelion.

(10) c. From conservation of energy,

show that the perihelion distance $r_P = -\frac{GM}{v_0^2} + |(\frac{GM}{v_0^2})^2 + D^2|^{\frac{1}{2}}$.

(8) d. For a Sun-grazing comet, the perihelion distance equals the radius R of the Sun, so that the comet crashes into the sun (and burns up). Show that the condition for a comet to be Sun-grazing is that $D \le R[1 + \frac{2GM}{Rv_0^2}]^{1/2}$, and thus the effective collision cross-section of the Sun is equal to πD^2.

$L = r \times P$

prove that τ is constant

(a) 0 $\frac{GMm}{r^2} = F$

$\tau = r \times F$

$\frac{dA}{dt} = \frac{1}{2}r^2 \frac{d\theta}{dt}$

$\frac{dA}{dt} = \frac{1}{2}r^2 w$

equal areas in equal time

$\frac{dA}{dt}(2m) = mr^2 w$

$\underbrace{2m\frac{dA}{dt}}_{\text{this is a constant so angular momentum is conserved}} = Iw = L$

(b) $\frac{1}{2}mv_0^2 = \cancel{\frac{1}{2}mv_p^2 + \frac{GMm}{D}}$

$\left[\frac{2(\frac{1}{2}mv_0^2 + \frac{GMm}{D})}{m}\right]^{\frac{1}{2}} = V_p$

$E_i = \frac{1}{2}mv_0^2$

(b) ~~$L = mr \times v = Iw$~~

~~$mDv_0 =$~~