MT2 Physics 1A(3), W17

Full Name (Printed) HE KAI LIM

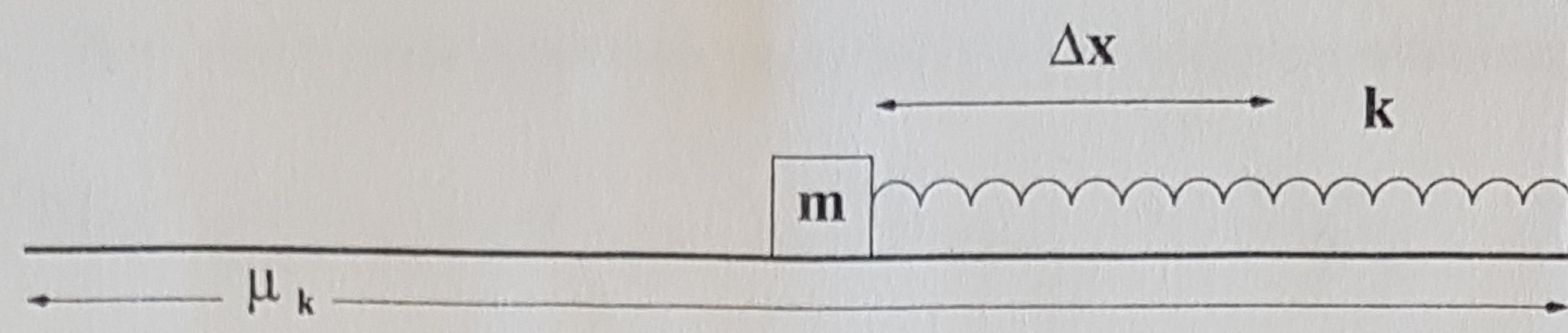
Full Name (Signature) [illegible]

Studen

Seat Number 110

Problem	Grade
1	29 /30
2	25 /30
3	08 /30
Total	62 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



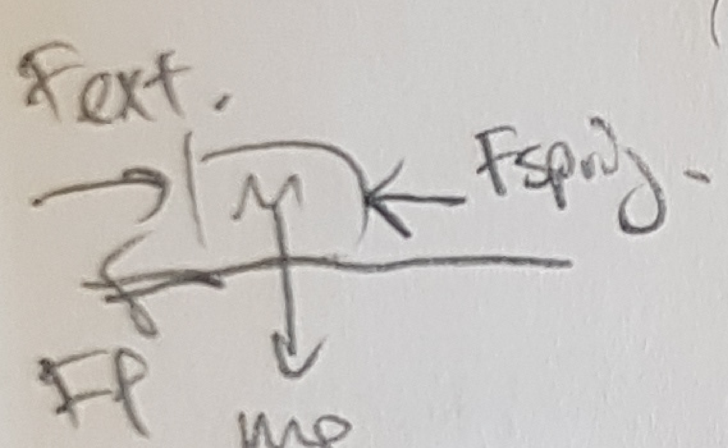
1) A block of mass m is set at rest on a rough surface (μ_k) up against a spring (k) that is initially uncompressed. Someone then comes along and pushes the mass into the spring until the spring is compressed an amount Δx, and holds it there...

- 1a) (5 points) What is the total amount of work done on the block during the compression?

+5/5

Block initial ~~mechanical~~ mechanical energy = KE + PE$^{\text{gravity}}$ = 0

Block Final mechanical energy = KE + PE$^{\text{gravity}}$ = 0.



From diagram, since Δspeed = 0,

$-F_{\text{external}} = F_{\text{spring}} + F_{\text{friction}}$.

$\therefore \Sigma F_{TOT} = F_{ext} + F_{spring} + F_f = 0$

$\therefore$ No work was done on the block

BUT work was done by spring, by friction, and by external force.

+5/5

- 1b) (5 points) How much of that work (done on the block during compression) was done by the spring?

Work Done by spring

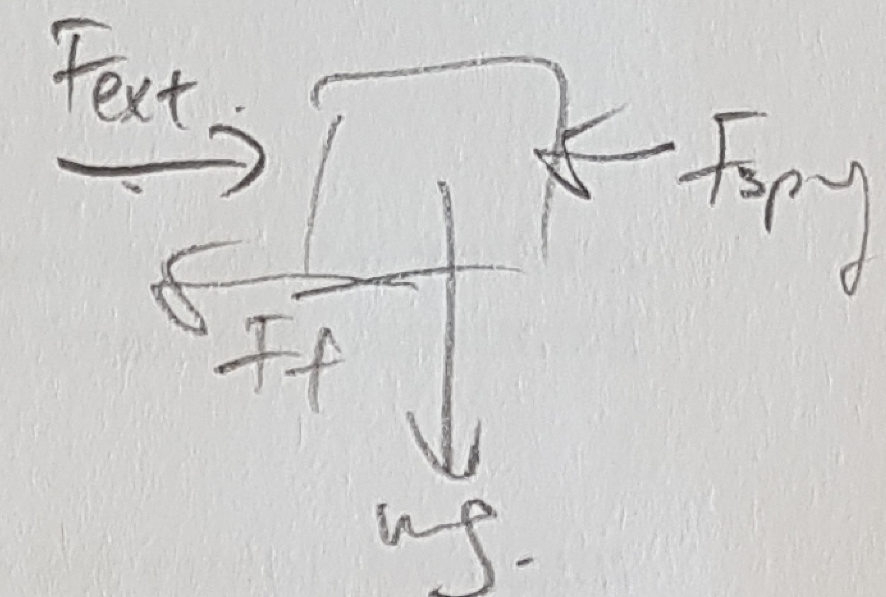
= $-\Delta$potential spring energy.

$= \boxed{-\frac{1}{2}kx^2}$

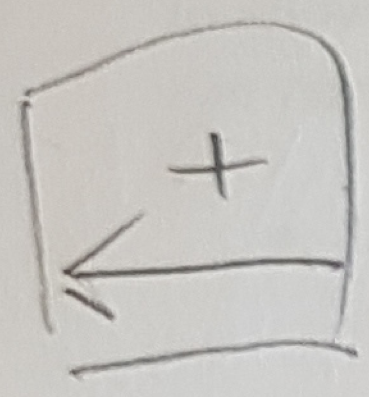
- 1c) (10 points) How much work was done (in total) by the someone who compressed the spring?

+9/10

From (1a), $-F_{ext} = F_{spg} + F_{fric}.$



$\therefore W_{ext} = \int F_{ext}\, dx$



$= \int F_{spring} + F_{fric}\, dx$

$= \int_{x_0}^{x_f} -kx - mg\mu_k\, dx$

$= -\frac{1}{2}kx^2 - mg\mu_k x \Big|_{x_0}^{x_{final}}$

$\boxed{= +\frac{1}{2}k(\Delta x)^2 + mg\mu_k(\Delta x)}$

+10/10

- 1d) (10 points) If the block is then released, how far from its initial position (when the spring was compressed) will it move before coming to rest?

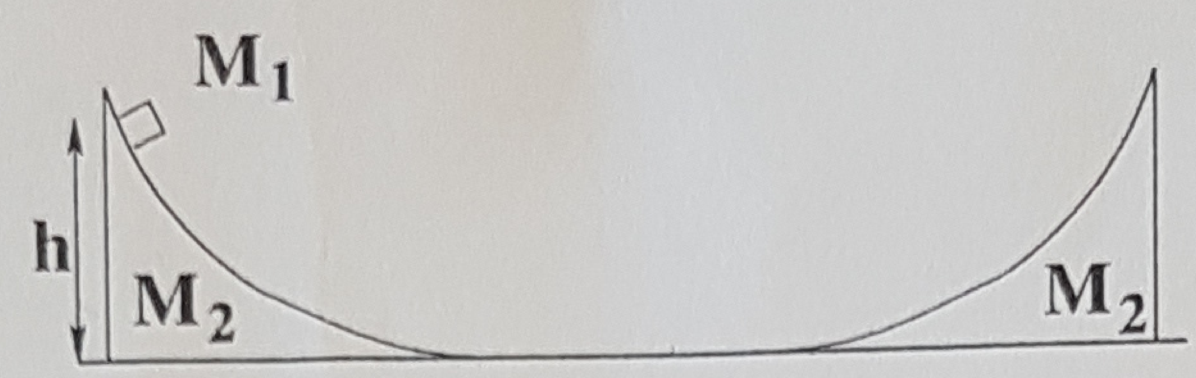
Upon release, (assume massless spring)

Stored energy in spring → Kinetic energy of block → work done by Friction ~~at~~ at final rest

let x_1 = final position,

$\therefore \frac{1}{2}k(\Delta x)^2 = mg\mu_k x_1$

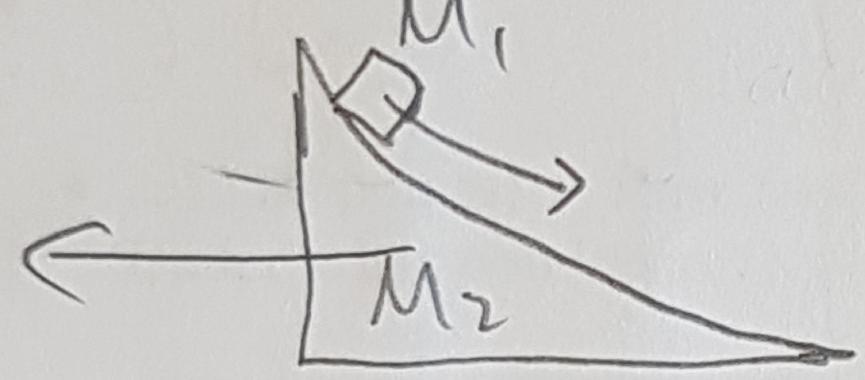
$$\boxed{x_1 = \frac{k(\Delta x)^2}{2 mg \mu_k}}$$



All surfaces are frictionless

2) A pair of identical ramps of mass M_2 sit at rest, facing one another, on a frictionless horizontal surface, as shown. A small block of mass M_1 is placed at a height h on the left ramp and released from rest. Assuming the contact between the block and each ramp is also frictionless...

- 2a) (10 points) How fast is the block moving when it reaches the bottom of the left ramp? (It might be a good idea to check your answer in the limits $M_1 << M_2$ and $M_1 >> M_2$).



$\uparrow y+$, Only gravity acts.

$\vec{x+}$, No net force,

consv. of momentum

$\therefore P_x$ final $= P_x$ initial

(1) $\rightarrow M_1V_1 + M_2V_2 = 0$

Energy is conserved ∵ elastic explosion/collision

$E_{final} = E_{initial}$

(2) $\rightarrow \frac{1}{2}(M_1V_1^2 + M_2V_2^2) = M_1gh$

From (1):

$V_2 = \frac{-M_1V_1}{M_2}$ — (1a)

Sub into (2).

$\frac{1}{2}\cancel{M_1}V_1^2 + \frac{1}{2}M_2\frac{M_1^2V_1^2}{M_2^2} = \cancel{M_1}gh$

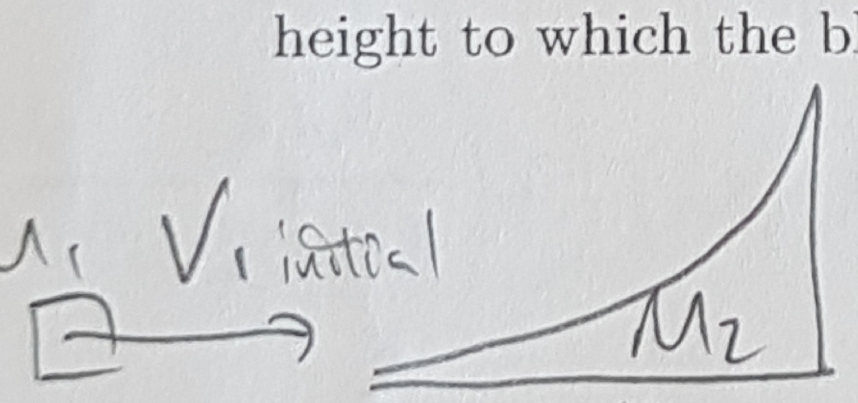
$\frac{1}{2}V_1^2 + \frac{1}{2}\frac{M_1V_1^2}{M_2} = gh$

$V_1^2(1 + \frac{M_1}{M_2}) = 2gh$

$\boxed{V_1 = \sqrt{\frac{2ghM_1}{M_1+M_2}}}$

+10

- 2b) (10 points) How are the horizontal components of the block's and the right ramp's velocities related when the block has traveled as far up the right ramp as it's going to go? Find the greatest height to which the block climbs on the right ramp.



stationary, $V_{3\,initial} = 0$

At top position,

V_1 final $= V_3$ final

because both M_1 & M_2 move together

∴

Apply conservation of momentum:

$P_{initial} = P_{final}$

$M_1V_{1\,initial} = M_1V_{1\,final} + M_2V_{3\,final}$

$= (M_1 + M_2)V_{1\,final}$

$\boxed{V_{1\,final}} = \frac{M_1V_{1\,initial}}{M_1+M_2}$

$= \boxed{\frac{M_1}{M_1+M_2}\sqrt{\frac{2gHM_1}{M_1+M_2}}}$

Energy TOTAL is conserved,

$\therefore \Sigma E_{initial} = \Sigma E_{final}$

$\frac{1}{2}M_1V_1^2 = \frac{1}{2}(M_1+M_2)V_{1\,final}^2 + M_1g\Delta h$

Flip

Cont.

$$\frac{1}{2}M_1V_1^2 = \frac{1}{2}(M_1+M_2)\left(\frac{M_1}{(M_1+M_2)}\sqrt{\frac{2gHM_1}{M_1+M_2}}\right)^2 + M_1 g \Delta h$$

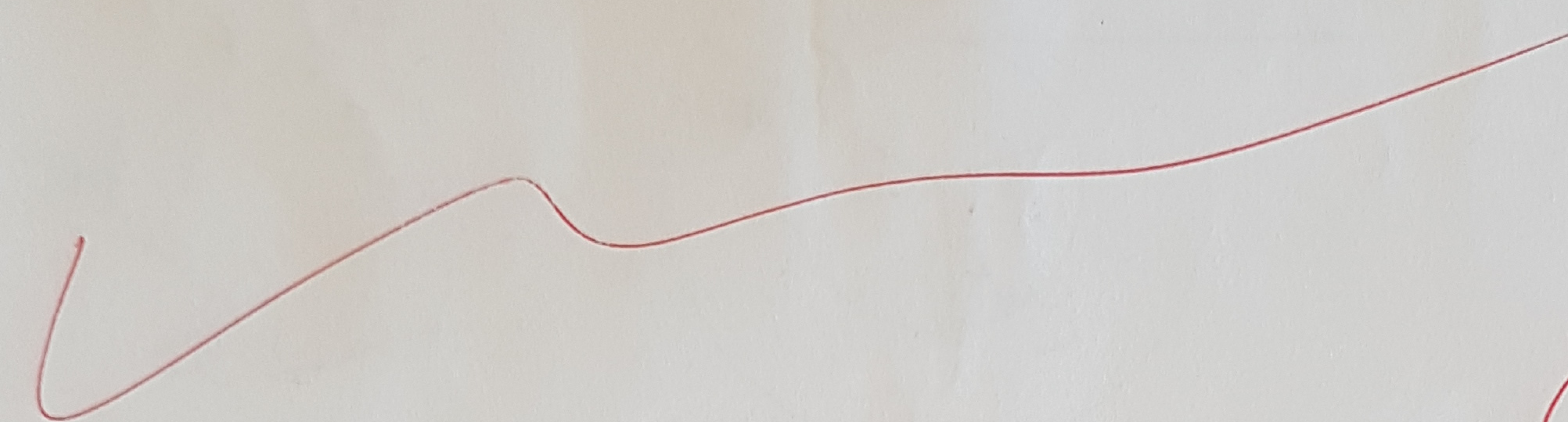
$$= \left(\frac{M_1^2}{2(M_1+M_2)}\right)\left(\frac{2ghM_1}{M_1+M_2}\right) + M_1 g \Delta h$$

$$\Delta h = \left[\frac{1}{2}M_1V_1^2 - \frac{2ghM_1^3}{2(M_1+M_2)^2}\right]\frac{1}{M_1 g}$$

$$= \frac{1}{g}\left[\frac{1}{2}\left(\frac{2ghM_1}{M_1+M_2}\right) - \frac{2ghM_1^2}{2(M_1+M_2)^2}\right]$$

$$= \boxed{\left[\frac{hM_1}{M_1+M_2} - \frac{hM_1^2}{2(M_1+M_2)^2}\right]}$$

- 2b) (*continued*)



+9

- 2c) (10 points) How fast will the block be traveling when it leaves the bottom of the right ramp? Under what condition(s) will it return to the left ramp?

Apply logic from 2a again.

cons. momentum

P_x ~~initial~~ $= P_x$ final ✓

① → $(M_1+M_2)\left(\frac{M_1}{M_1+M_2}\right)\sqrt{\frac{2ghM_1}{M_1+M_2}} = M_1 V_{1\,new} + M_2 V_{3\,new}$

② Elastic explosion, energy is conserv.

$E_{initial} = E_{final} \Rightarrow mgh + KE = KE_1 + KE_2$

$\therefore$

$$M_1 g\left(\frac{hM_1}{M_1+M_2} - \frac{hM_1^2}{2(M_1+M_2)^2}\right) + \frac{1}{2}(M_1+M_2)\left(\frac{M_1}{M_1+M_2}\right)^2 \frac{2ghM_1}{M_1+M_2} = \frac{1}{2}M_1 V_{1new}^2 + \frac{1}{2}M_2 V_{3new}^2$$

from ①, (1c) → $V_{3new} = \dfrac{M_1\sqrt{\frac{2ghM_1}{M_1+M_2}} - M_1 V_{1new}}{M_2}$

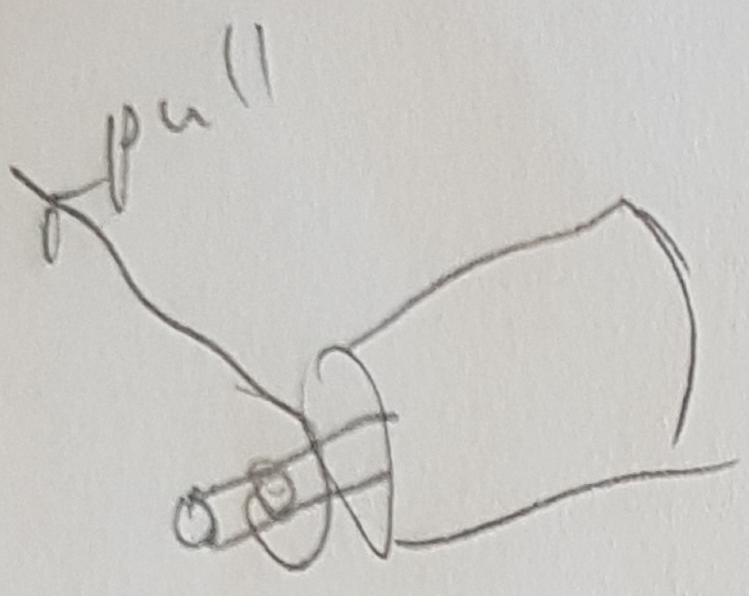
sub (1c) into ②,

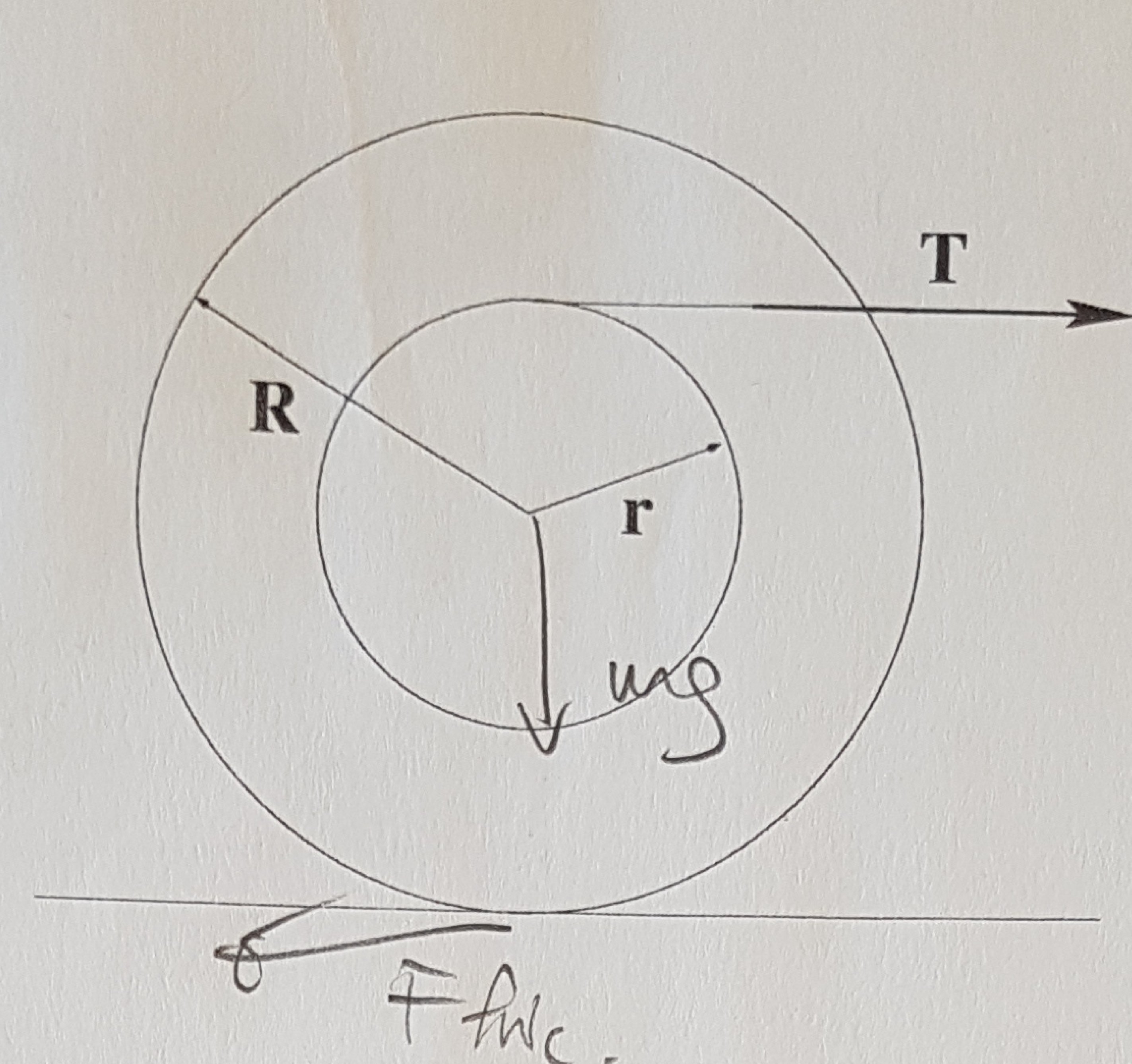
$$\frac{M_1^2 gh}{M_1+M_2} - \frac{hM_1^3 g}{2(M_1+M_2)^2} + \frac{M_1^3 gh}{(M_1+M_2)^2} = \frac{1}{2}M_1 \boxed{V_{1new}}^2 + \frac{1}{2}M_2\left(M_1\sqrt{\frac{2gHM_1}{M_1+M_2}} - M_1\boxed{V_{1new}}\right)$$

$$\boxed{\text{answer} = \text{solve the quadratic equation}}$$

+6

8





3) A heavy, uniform cylinder has a mass m and a radius R. It is accelerated by a force of magnitude T that is applied via a rope wound around a light (read negligible) drum of radius r that is attached to the cylinder. The coefficient of static friction is sufficient to allow the cylinder to roll without slipping.

clockwise

(+↻)

- 3a) (15 pts) Find the frictional force acting on the cylinder.

$\Sigma \tau = Tr + F_{fric} R$ +3 (center of cylinder = axis of rot.)

$\tau = I\alpha = Tr + F_{fric} R$ +3

$Tr + F_f R = (N M R^2)(\alpha)$

$\alpha = \dfrac{Tr + F_f R}{N M R^2}$

- 3b) (5 pts) Find the acceleration of the center-of-mass of the cylinder.

$F = mg$

$Tr + F_f R = I\alpha$ +2

$\alpha = \dfrac{Tr + F_f R}{N M R^2}$

$a = r\alpha$ plug in,

- 3c) (5 pts) If you were to pull on a block that has the same mass as the cylinder with a horizontal force T, what would be the largest acceleration you could give it? Show that, with a proper choice of r, you can give the cylinder a larger acceleration than the block!

$$F = ma$$

$$T - F_{fric} = ma$$

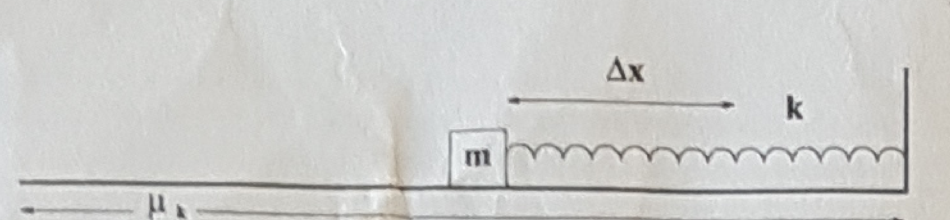
$$\boxed{a = \cancel{\frac{T - F_{fric}}{m}}}$$

- 3d) (5 pts) In what direction will the force of friction point when the cylinder is accelerating under the circumstances laid out in part c?

Not rolling

$\therefore$ only slip

ie. friction is dynamic.



1) A block of mass m is set at rest on a rough surface (μ_k) up against a spring (k) that is initially uncompressed. Someone then comes along and pushes the mass into the spring until the spring is compressed an amount Δx, and holds it there...

- 1a) (5 points) What is the total amount of work done on the block during the compression?

$\Delta K = W_{TOT}$

The block starts and ends at rest, so...

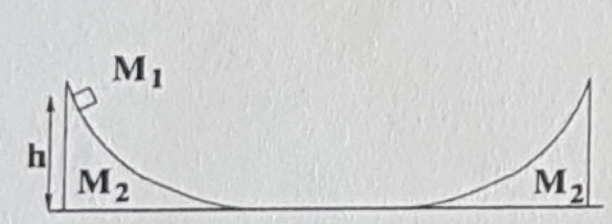
$$\boxed{W_{TOT} = 0}$$

- 1b) (5 points) How much of that work (done on the block during compression) was done by the spring?

$W_{sp} = -\Delta U_{sp}$

$W_{sp} = -\frac{1}{2}k(x_f^2 - x_i^2)$ $\quad x_i = 0$, $x_f = \Delta x$

$$\boxed{W_{sp} = -\frac{1}{2}k\,\Delta x^2}$$



All surfaces are frictionless

2) A pair of identical ramps of mass M_2 sit at rest, facing one another, on a frictionless horizontal surface, as shown. A small block of mass M_1 is placed at a height h on the left ramp and released from rest. Assuming the contact between the block and each ramp is also frictionless...

- 2a) (10 points) How fast is the block moving when it reaches the bottom of the left ramp? (It might be a good idea to check your answer in the limits $M_1 << M_2$ and $M_1 >> M_2$).

$\Sigma P_i = \Sigma P_f$

$0 = m_1 V_{1Ax} + m_2 V_{2Ax}$

$V_{2Ax} = -\frac{m_1}{m_2} V_{1Ax}$

$\Delta E = \cancel{W_{nc}}^{\,0}$ (1-D) (Components OK)

$m_1 g h = \frac{1}{2} m_1 V_{1Ax}^2 + \frac{1}{2} m_2 V_{2Ax}^2$

$m_1 g h = \frac{1}{2} m_1 V_{1Ax}^2 \left(1 + \frac{m_1}{m_2}\right)$

$$\boxed{V_{1Ax} = \sqrt{2gh\,\frac{m_2}{m_1+m_2}}}$$

Check:
$m_1 >> m_2$ $\quad V_{1x} \sim 0$
$m_1 << m_2$ $\quad V_{1x} \sim \sqrt{2gh}$ ✓

- 2b) (10 points) How are the horizontal components of the block's and the right ramp's velocities related when the block has traveled as far up the right ramp as it's going to go? Find the greatest height to which the block climbs on the right ramp.

At the instant the block reaches its highest point, it is no longer moving (up or down) with respect to the ramp, $V_{1Bx} = V_{2Bx}$

$\Sigma P_{xi} = \Sigma P_{xf}$

$m_1 V_{1Ax} = (m_1 + m_2) V_{12Bx}$

$V_{12Bx} = \frac{m_1}{m_1+m_2} V_{1Ax}$

$\Delta E = W_{nc}$

$\frac{1}{2} m_1 V_{1Ax}^2 = \frac{1}{2}(m_1+m_2) V_{12Bx}^2 + m_1 g h_B$

$\frac{1}{2} m_1 V_{1Ax}^2 = \frac{1}{2} m_1 V_{1Ax}^2 \left(\frac{m_1}{m_1+m_2}\right) + m_1 g h_B$

$h_B = \frac{1}{2g} V_{1Ax}^2 \left(1 - \frac{m_1}{m_1+m_2}\right)$

$$\boxed{h_B = h\left(\frac{m_2}{m_1+m_2}\right)^2}$$

This is actually kind of cool!

- 1c) (10 points) How much work was done (in total) by the someone who compressed the spring?

$W_{TOT} = W_N + W_g + W_{sp} + W_{fric} + W_{ext}$

$0 = 0 + 0 + (-\frac{1}{2}k\Delta x^2) + (-\mu_k m g \Delta x) + W_{ext}$

$W_{fric} = \int |\vec{F}_f||d\vec{r}|\cos(180°) = \mu_k mg\,\Delta x\,(-1)$

$$\boxed{W_{ext} = \frac{1}{2}k\Delta x^2 + \mu_k m g \Delta x}$$

That is - Work was done to compress the spring and move the block over friction.

- 1d) (10 points) If the block is then released, how far from its initial position (when the spring was compressed) will it move before coming to rest?

$\Delta E = W_{NC}$

$\Delta K + \Delta U_g + \Delta U_s = W_{fric}$

$0 + 0 + \frac{1}{2}k(0 - \Delta x^2) = -\mu_k m g d$

$$\boxed{d = \frac{k\,\Delta x^2}{2\mu_k m g}}$$

- 2b) (continued)

- 2c) (10 points) How fast will the block be traveling when it leaves the bottom of the right ramp? Under what condition(s) will it return to the left ramp?

We can find the velocity of the block as it leaves the right ramp by analyzing the elastic collision it made with the ramp (that is, consider the incident block, the departing block and ramp, ignore the moments when the block is climbing the ramp)

$\Sigma P_{xi} = \Sigma P_{xf}$

$m_1 V_{1Ax} = m_1 V_{1Cx} + m_2 V_{2Cx}$

$m_1 V_{1Ax} = m_1 V_{1Cx} + m_2 (V_{1Ax} + V_{1Cx})$

$V_{1Cx} = \frac{M_1 - M_2}{M_1 + M_2} V_{1Ax}$

$\Sigma K_i = \Sigma K_f$? 1D Elastic shortcut

$V_{1Ax} + V_{1Cx} = \cancel{V_{2Ax}} + V_{2Cx}$

→ (from part a) $V_{2Ax} = -\frac{m_1}{m_2} V_{1Ax}$

Both masses have negative components. To catch up - V_{1Cx} needs to be more negative than V_{2Ax}

$\frac{m_1 - m_2}{m_1 + m_2} < -\frac{m_1}{m_2}$

The 'easiest' answer is that $m_1 < m_2$

But you can do better (for full credit ☺)

$m_1^2 + 2m_2 m_1 - m_2^2 < 0$

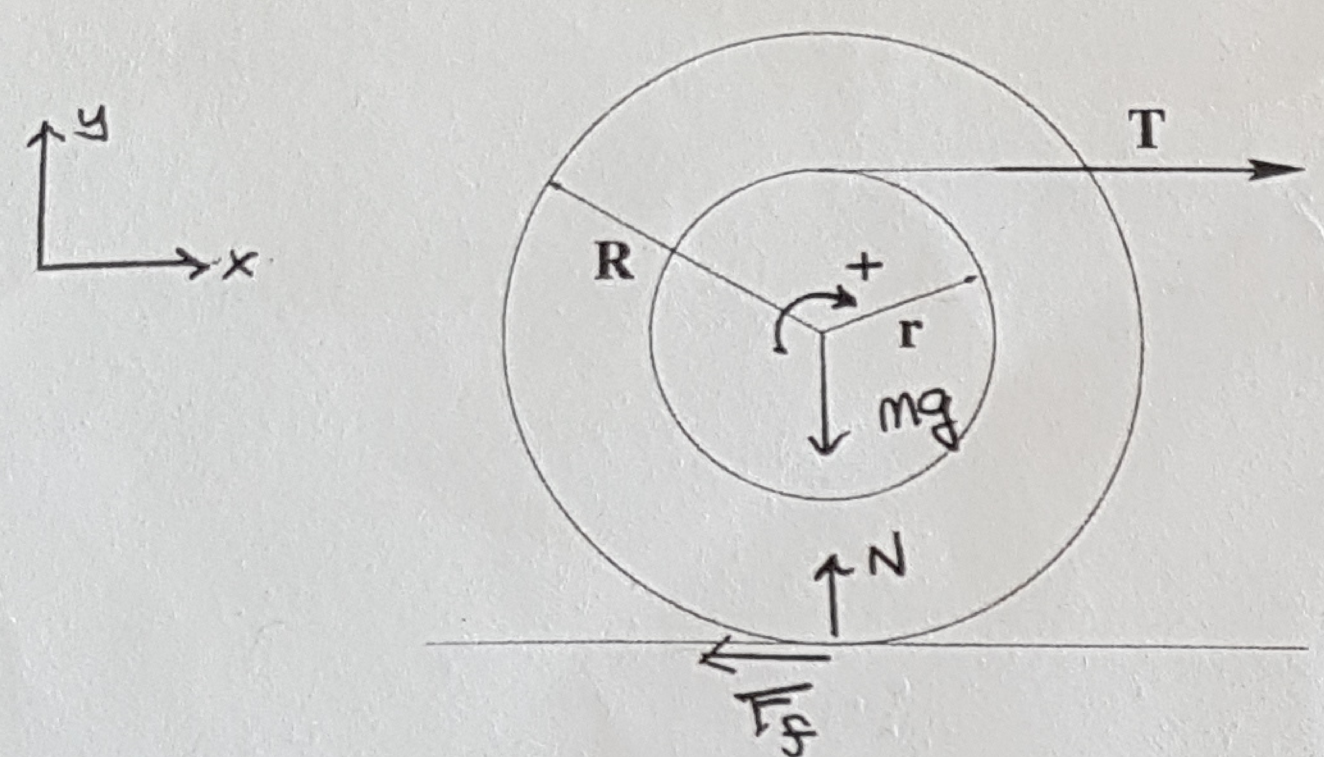
$m_1^2 + 2m_1 m_2 - m_2^2 = 0$

$m_1 = m_2(-1 \pm \sqrt{2})$

$(m_1 - m_2(-1-\sqrt{2}))(m_1 - m_2(-1+\sqrt{2})) < 0$

$$\boxed{m_1 < m_2(\sqrt{2} - 1)}$$

FOR FULL CREDIT



No-Slip

$a_x = R\alpha$

3) A heavy, uniform cylinder has a mass m and a radius R. It is accelerated by a force of magnitude T that is applied via a rope wound around a light (read negligible) drum of radius r that is attached to the cylinder. The coefficient of static friction is sufficient to allow the cylinder to roll without slipping.

- 3a) (15 pts) Find the frictional force acting on the cylinder.

$\Sigma F_x = ma_x$

$T - F_f = ma_x$

$\Sigma F_y = m\cancel{a_y}$

$N - mg = 0$

$a_x = R\alpha$

$\Sigma \vec{\tau} = I\vec{\alpha}$

$rT + RF_f = \frac{1}{2}mR^2\left(\frac{a_x}{R}\right)$

$rT + RF_f = \frac{1}{2}mR\left(\frac{T}{m} - \frac{F_f}{m}\right)$

$rT + RF_f = \frac{1}{2}RT - \frac{1}{2}RF_f$

$\frac{3}{2}RF_f = \left(\frac{1}{2}R - r\right)T$

$$\boxed{F_f = T\,\tfrac{1}{3}\left(1 - 2\,r/R\right)}$$

- 3b) (5 pts) Find the acceleration of the center-of-mass of the cylinder.

$a_x = \frac{1}{m}T\left(1 - F_f/T\right)$

$a_x = \frac{T}{m}\left[1 - \frac{1}{3} + \frac{2}{3}\,r/R\right]$

$$\boxed{a_x = T/m\;\tfrac{2}{3}\left(1 + r/R\right)}$$

- 3c) (5 pts) If you were to pull on a block that has the same mass as the cylinder with a horizontal force T, what would be the largest acceleration you could give it? Show that, with a proper choice of r, you can give the cylinder a larger acceleration than the block!

The maximum acceleration you could give a block: T/m

For the cylinder, we can exceed this if:

$\frac{2}{3}(1 + r/R) > 1$

$\Rightarrow \boxed{r > R/2}$

- 3d) (5 pts) In what direction will the force of friction point when the cylinder is accelerating under the circumstances laid out in part c?

If $r > R/2$; $F_f \rightarrow$ negative... that is, it points opposite the direction we assumed – it is pointing forward (why?? :') → This, of course is what gives the center-of-mass its extra boost!