MT1 Physics 1A(3), W17

Full Name (Printed) HE KAI LIM

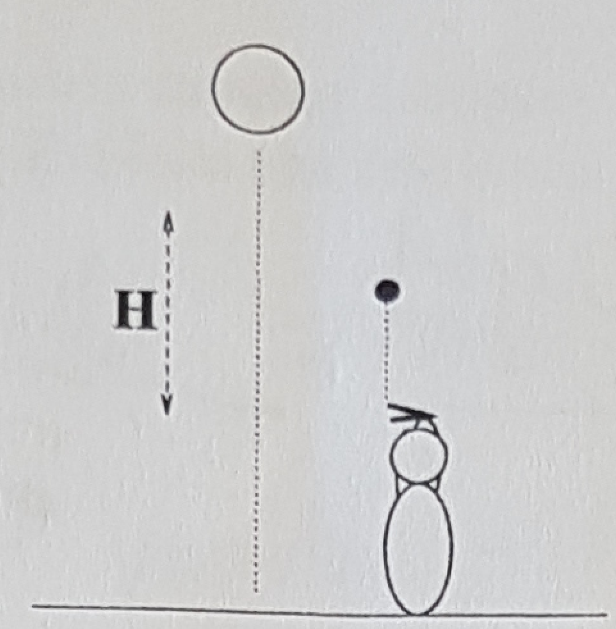
Full Name (Signature)

Student ID Number

Seat Number

Problem	Grade
1	30/30
2	17 /30
3	30 /30
Total	77 /90

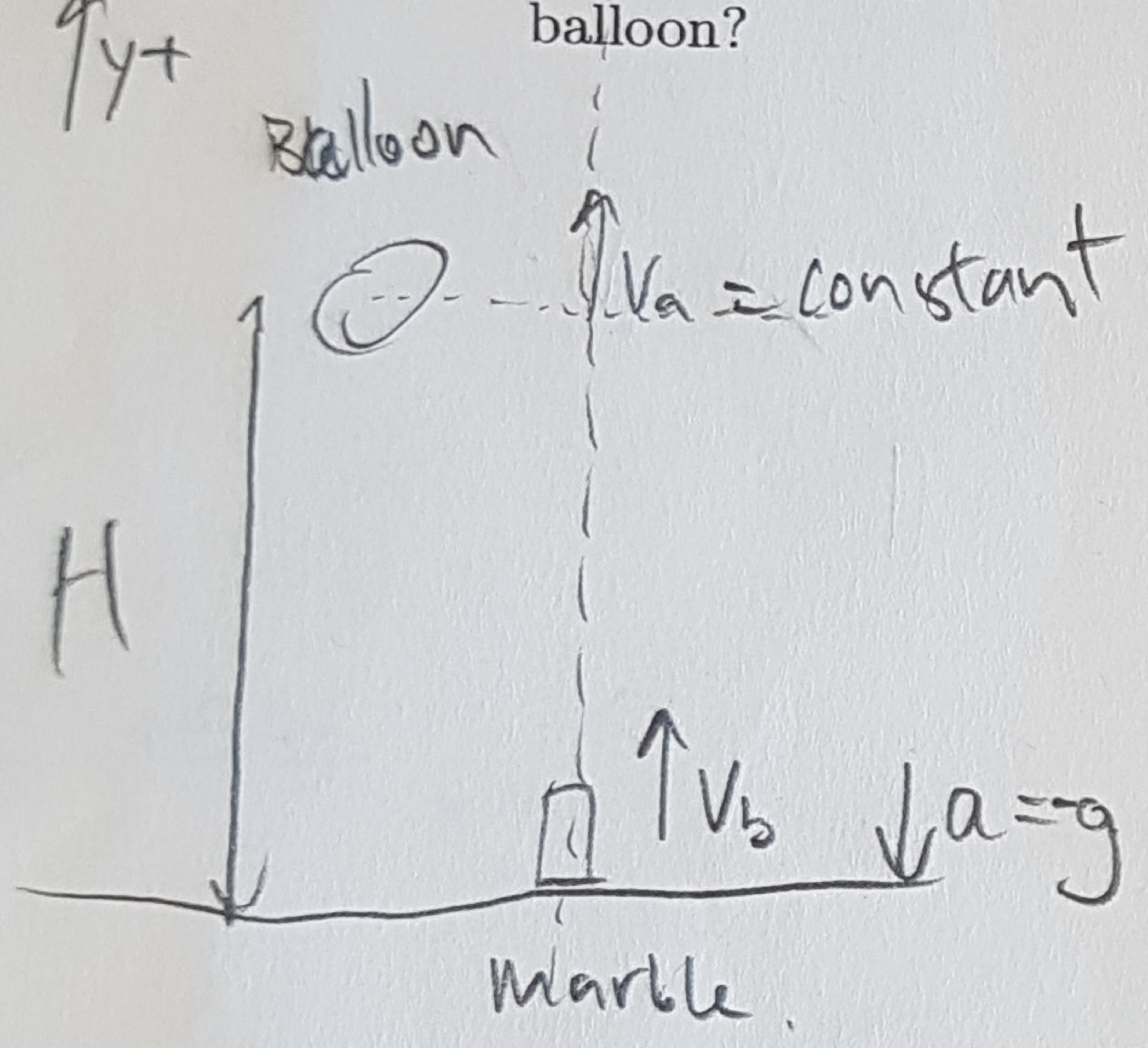
- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



+10/10

1) A small hot-air balloon slowly rises from the surface of the Earth at a constant speed v_a. Nearby, a young child holds a loaded slingshot above his head, pointed straight up. When the balloon reaches a height H above the slingshot, the child fires a marble with a large velocity v_b along a vertical path adjacent to that of the balloon.

- 1a) (10 points) How fast is the marble moving (relative to the child) when it first overtakes the balloon?



$V_r = V_{marble} - V_{balloon}$

$=$ relative V_{marble}, compared to balloon.

$\therefore V_r^2 = V_{r\,initial}^2 + 2\vec{a}\,s$

$V_r = \sqrt{(V_b - V_a)^2 - 2gH}$

$$\boxed{\begin{aligned}&\therefore V_{marble} \text{ relative to (stationary) child} = V_r + V_{balloon}\\ &= \sqrt{(V_b - V_a)^2 - 2gH} + V_a\end{aligned}}$$

+10/10

- 1b) (10 points) How far above the balloon will the marble appear to go?

Maximum ~~height~~ height coincides with $V_r = 0$

let time when balloon & marble are FIRST parallel (1st overtake) $= t_1$ and $y = 0$, Relative to Balloon ie. Balloon = frame of reference.

$r(t) = V_0(\Delta t) + \frac{1}{2}a(\Delta t)^2$

$= \sqrt{(V_b - V_a)^2 - 2gH}\,\Delta t - \frac{1}{2}g(\Delta t)^2$

$\frac{dr}{dt} = V(t) = \sqrt{(V_b - V_a)^2 - 2gH} - g\Delta t = 0$ ($\because$ max height $\equiv$ 0 relative velocity.)

$\therefore \Delta t = \frac{\sqrt{(V_b - V_a)^2 - 2gH}}{g}$

$\therefore r(t) = \sqrt{(V_b - V_a)^2 - 2gH}\left(\frac{\sqrt{(V_b - V_a)^2 - 2gH}}{g}\right) - \frac{1}{2}g(\Delta t)^2 =$ Flip over!

$$= \frac{(V_b - V_a)^2 - 2gH}{g} - \frac{1}{2}\frac{1}{g}\left[(V_b - V_a)^2 - 2gH\right]$$

$$\boxed{= \frac{(V_b - V_a)^2 - 2gH}{2g}}$$

greatest distance above the balloon. Explain the relevance of your answer.

$h = y_m - y_b$

+5/5

- 1c) (5 points) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon. Explain the relevance of your answer.

When marble = max height above Balloon,

$V_{relative} = 0 = V_{marble} - V_{balloon}$

$\therefore V_{marble} = V_{balloon}$ [inertial frame]

$\therefore V_{marble} = V_{balloon}$

$\boxed{= V_a}$

+5/5

- 1d) (5 points) How much time will elapse between the marble's first encounter with the balloon and it's last?

from (1b),

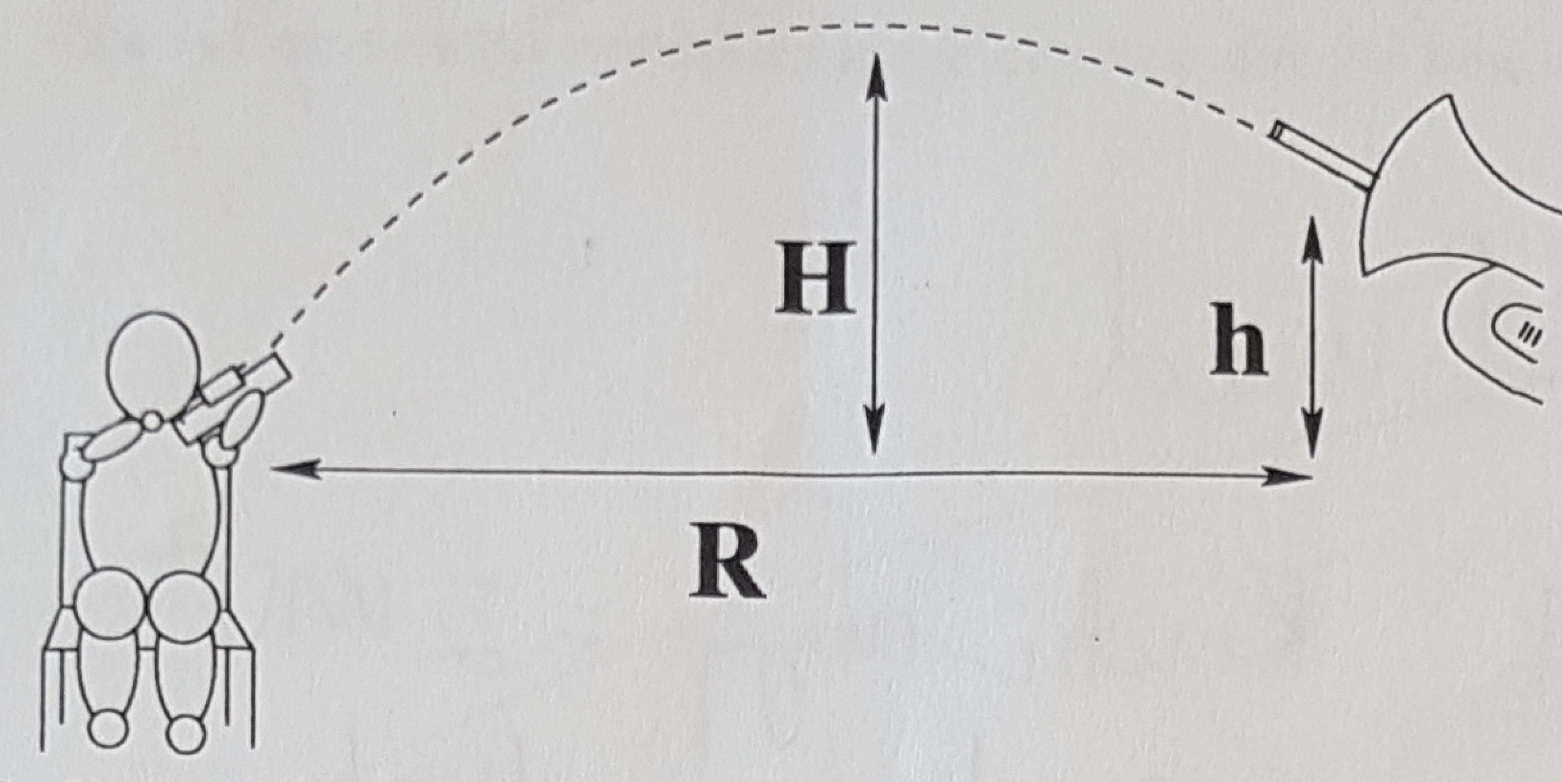
$r(t) = \sqrt{(V_b - V_a)^2 - 2gH}\,\Delta t - \frac{1}{2} g (\Delta t)^2$

For last encounter, $r(t) = 0$

$\therefore \sqrt{(V_b - V_a)^2 - 2gH}\,\cancel{\Delta t} = \frac{1}{2} g (\Delta t)^{\cancel{2}}$

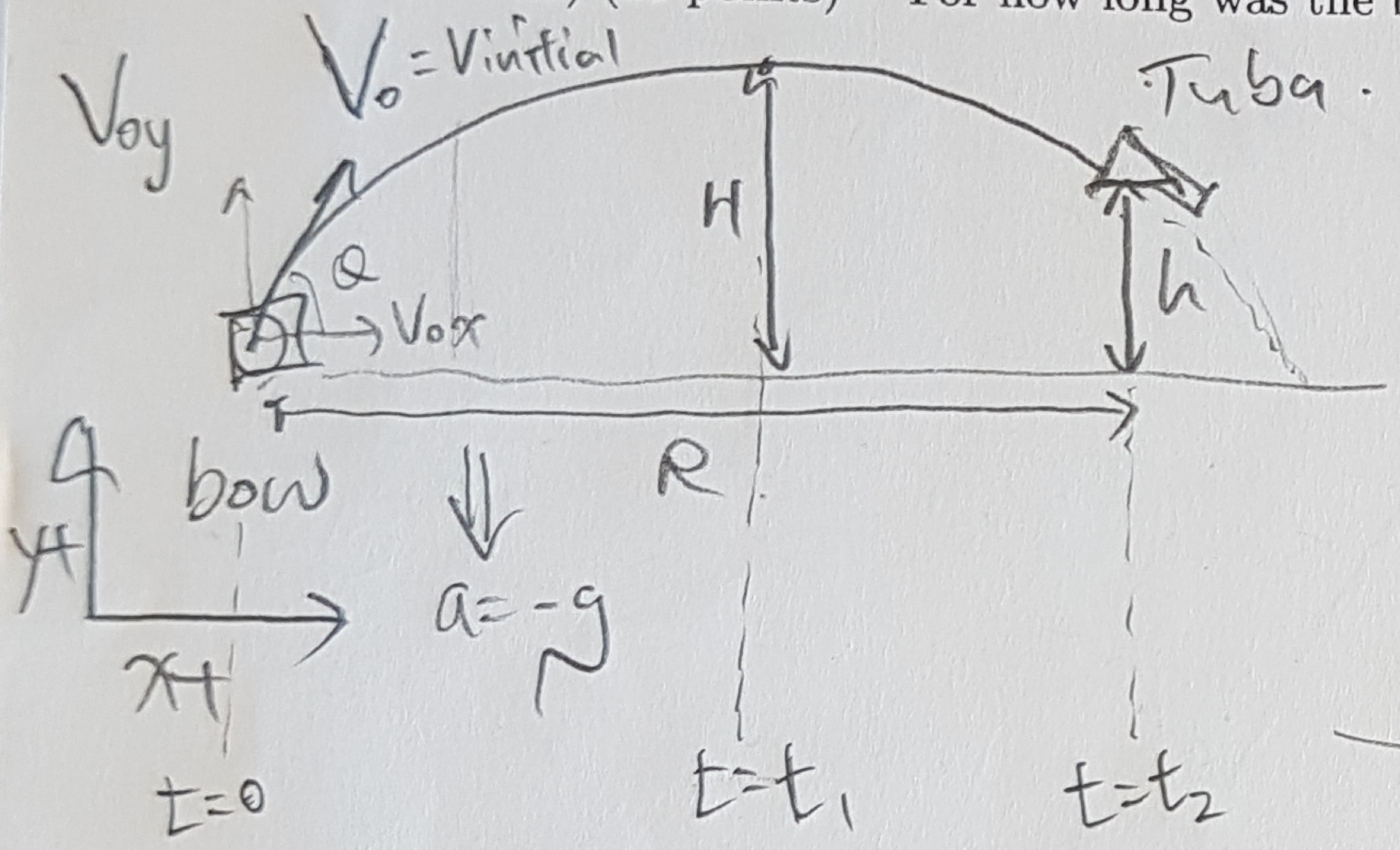
$$\boxed{\Delta t = \frac{2\sqrt{(V_b - V_a)^2 - 2gH}}{g}}$$

17



2) During a particularly lively solo the bow gets away from our virtuoso violinist. It rises to a maximum height H above the violin then descends into a nearby tuba. Assume the opening of the tuba lies a horizontal distance R from and a vertical height h above the violin and answer the following questions...

- 2a) (10 points) For how long was the bow in flight?



For max H, $V_y = 0$

$V_y^2 = V_{oy}^2 - 2gH = 0$ +2

$V_{oy} = \sqrt{2gH} = V_o \sin\theta$ +2

~~time = $\frac{R}{V_{ox}} = \frac{R}{V_o\cos\theta} = \frac{R\sin\theta}{\sqrt{2gH}\cos\theta} = \frac{R}{\sqrt{2gH}}\tan\theta$~~

- 2b) (10 points) With what speed did the bow leave the violin?

~~$V_o = \sqrt{2gH}/\sin\theta$~~

$\Delta t = \frac{R}{V_{ox}}$ +2

$V_{ox} = \frac{R}{g}\left(\sqrt{2gH} + \sqrt{2gH - 2gh}\right)$ +2

$\therefore V = \sqrt{V_{ox}^2 + V_{oy}^2}$ +2

$= \sqrt{\text{plug in}}$ +1

7

$r(t) = h = V_{oy}\Delta t - \frac{1}{2}g\Delta t^2$ +2

$= \sqrt{2gH}\,\Delta t - \frac{g}{2}\Delta t^2$

$\therefore \Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ +2

$$\boxed{= \frac{\sqrt{2gH} + \sqrt{2gH - 2gh}}{g}}$$

+2

10

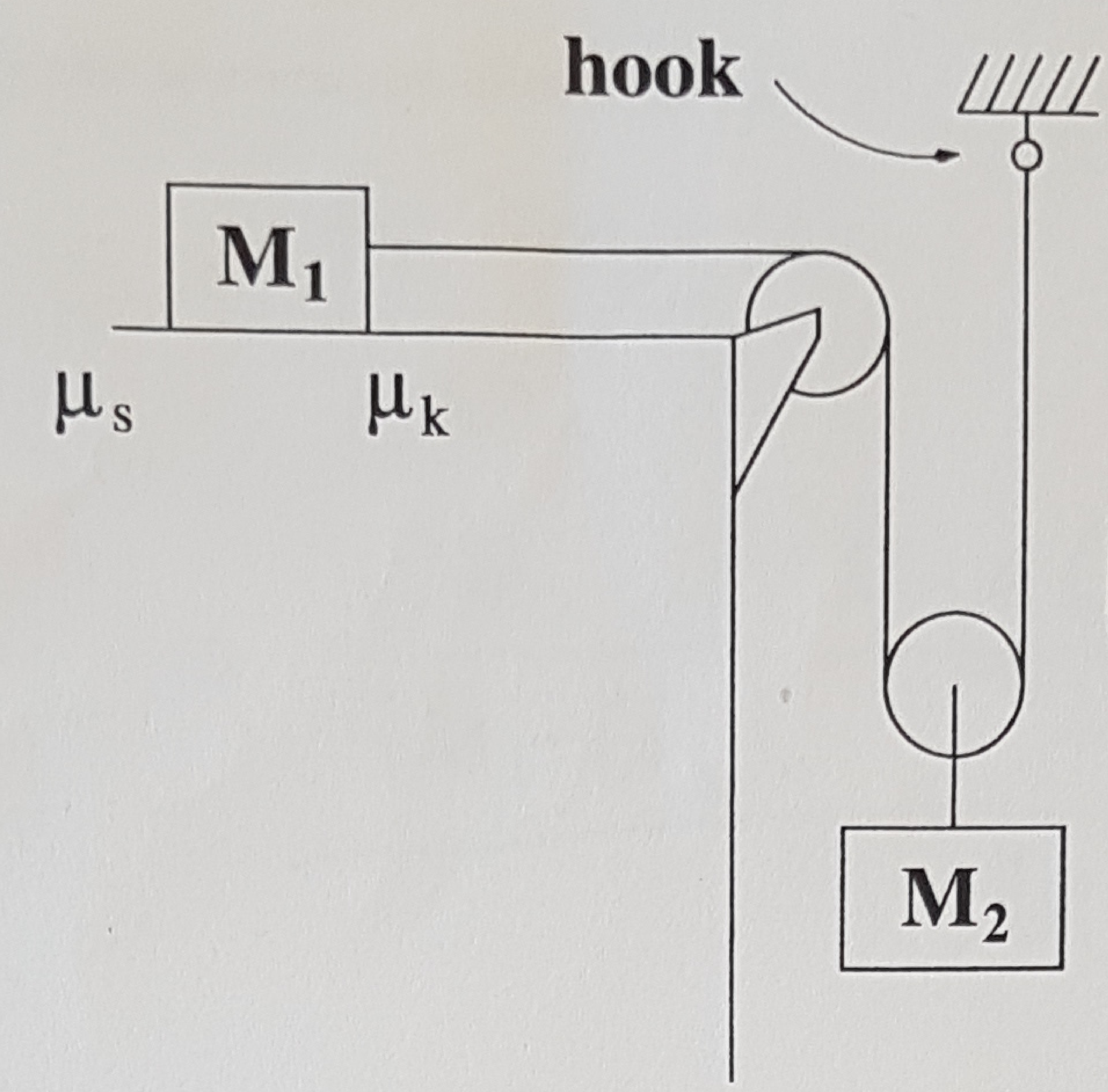
balloon

- 1c) (5 points) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon. Explain the relevance of your answer.

$h = y_m - y_h$

- 2c) (10 points) At what angle with respect to the horizontal did the bow enter the tuba?

• 1c) (5 points) How fast is the mach... greatest distan...



working

$$2-\frac{M_2}{2M_1}$$

$$\frac{4M_1-M_2}{2M_2}$$

Consider the apparatus shown above. The coefficients of friction between block one and the table are both known, as are the masses M_1 and M_2.

- 3a) (5 points) Identify the object (or objects) you're interested in, and draw free-body diagrams for each.

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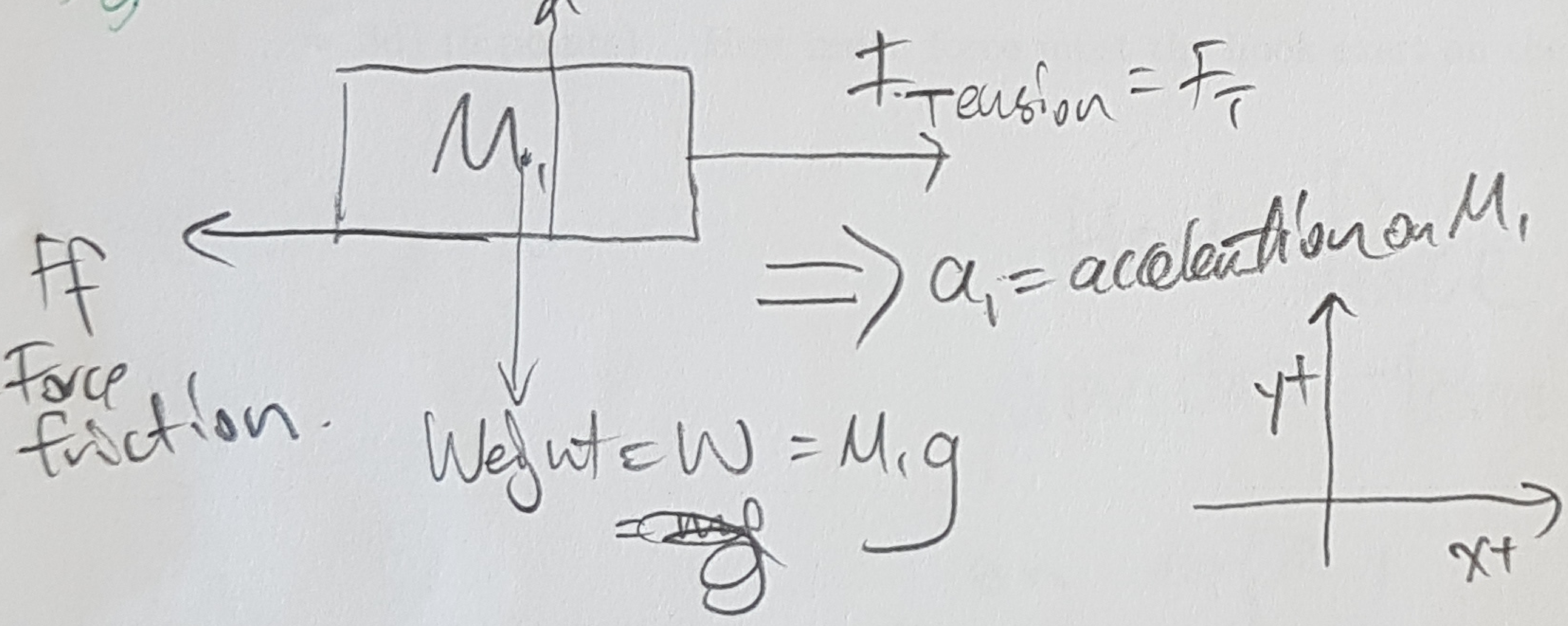
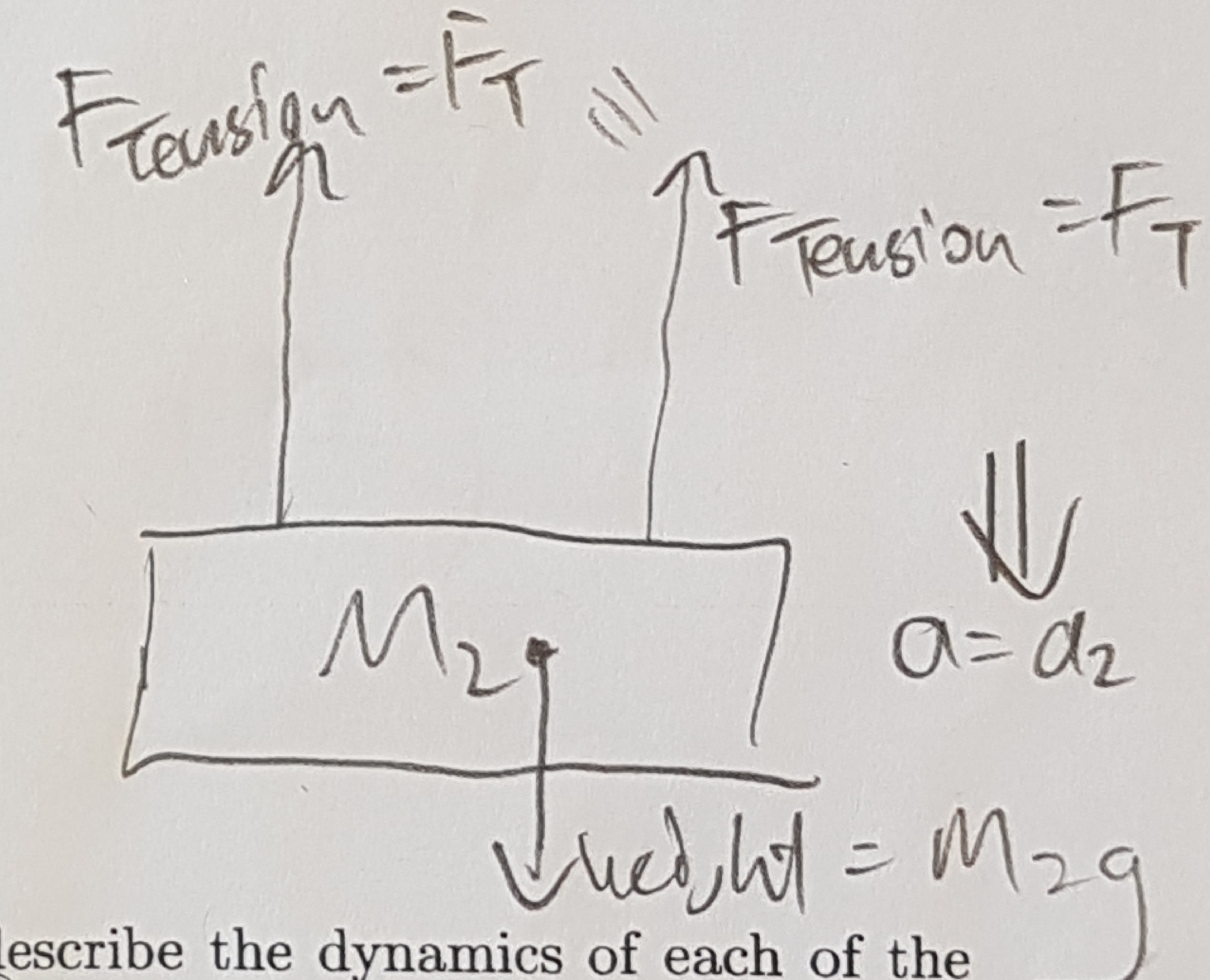
F_{N1} = Normal contact force.





- 3b) (10 points) Use Newton's laws to obtain equations ~~that~~ describe the dynamics of each of the objects of interest. Describe friction as F_f, and allow for acceleration of the body or bodies.

10/10

means is moving, i.e. dynamic.

$\sum \vec{F} = m\vec{a}$ (Newton's 2nd)

On M_2:

$\sum F = 2F_T - W$

$= 2F_T - M_2 g = M_2 a_2$

On M_1: $F_{N1} = W$ (By Newton's 3rd law)

$\sum F = F_T - F_f = M_1 a_1$

$= F_T - F_{N1}(\mu_k)$

$= F_T - M_1 g \mu_k$

$$a_1 = \frac{F_T - M_1 g \mu_k}{M_1}$$

On M_2: ∵ of pulley system.

$a_2 = \frac{1}{2} a \Rightarrow$

$\therefore 2F_T - M_2 g = M_2\left(\frac{F_T - M_1 g \mu_k}{2M_1}\right)$

$\left(2 - \frac{M_2}{2M_1}\right) F_T = \frac{M_2 g \mu_k}{2} + M_2 g$

$$F_T = \frac{2M_2}{4M_1 - M_2}\left(\frac{M_2 g \mu_k}{2} + M_2 g\right)$$

- 3c) (10 points) What is the acceleration of each block?

$$a_1 = \frac{F_T - M_1 g \mu_k}{M_1}$$

10/10

$$= \frac{\frac{2M_2}{4M_1 - M_2}\left(\frac{M_2 g \mu_k}{2} + M_2 g\right) - M_1 g \mu_k}{M_1}$$

Units are correct, will have to check + see if it simplifies to answer key.

$$a_2 = \frac{\frac{2M_2}{4M_1 - M_2}\left(\frac{M_2 g \mu_k}{2} + M_2 g\right) - M_1 g \mu_k}{2M_1}$$

- 3d) (5 points) How much force must the hook exert on the rope?

Conceptually correct!

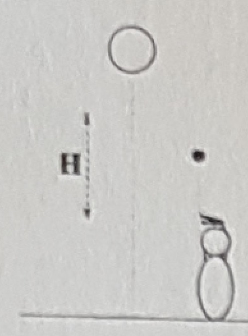
Hook force provides tension (T_+) on rope, and since rope top end is stationary,

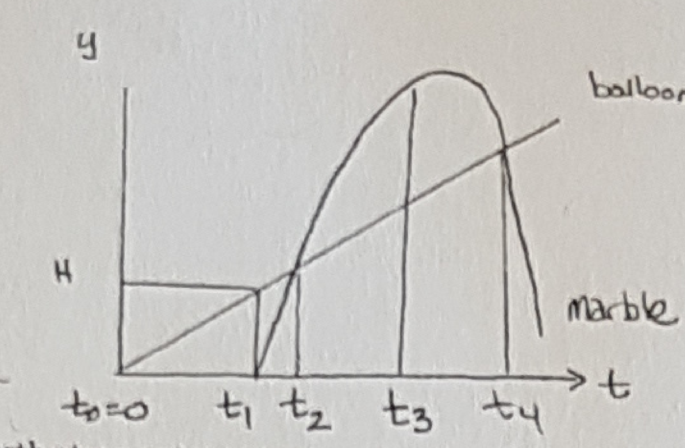
5/5

$\therefore$ Hook force $= F_T$

$$= \frac{2M_2}{4M_1 - M_2}\left(\frac{M_2 g \mu_k}{2} + M_2 g\right)$$

$y_b = V_a t$
$y_m = V_b(t-t_1) - \frac{1}{2}g(t-t_1)^2$
$V_{ym} = V_b - \frac{1}{2}g(t-t_1)$





1) A small hot-air balloon slowly rises from the surface of the Earth at a constant speed v_a. Nearby, a young child holds a loaded slingshot above his head, pointed straight up. When the balloon reaches a height H above the slingshot, the child fires a marble with a large velocity v_b along a vertical path adjacent to that of the balloon.

- 1a) (10 points) How fast is the marble moving (relative to the child) when it first overtakes the balloon?

$y_{b1} = V_a t_1$
$t_1 = H/V_a$

$y_b(t_2) = y_m(t_2)$
$V_a t_2 = V_b(t_2-t_1) - \frac{1}{2}g(t_2-t_1)^2$
$\frac{1}{2}g(t_2-t_1)^2 - (V_b-V_a)(t_2-t_1) + V_a t_1 = 0$
$t_2 - t_1 = \frac{V_b - V_a \pm \sqrt{(V_b-V_a)^2 - 2gV_a t_1}}{g}$

we want the earlier root: why? :)

$t_2 - t_1 = \frac{V_b - V_a - \sqrt{(V_b-V_a)^2 - 2gH}}{g}$

$V_{ym2} = V_b - \frac{1}{2}g(t_2-t_1)$

$\boxed{V_{ym2} = V_a + \sqrt{(V_b-V_a)^2 - 2gH}}$

- 1b) (10 points) How far above the balloon will the marble appear to go?

$h = y_m - y_b = V_b(t-t_1) - \frac{1}{2}g(t-t_1)^2 - V_a t$

Max: $\frac{dh}{dt} = V_b - V_a - g(t_3 - t_1) = 0$
$t_3 - t_1 = \frac{V_b - V_a}{g}$

$h_{max} = (V_b - V_a)(t_3 - t_1) - \frac{1}{2}g(t_3-t_1)^2 - V_a t_1$
$h_{max} = \frac{(V_b-V_a)^2}{g} - \frac{1}{2}\frac{(V_b-V_a)^2}{g} - H$

$\boxed{h_{max} = \frac{(V_b-V_a)^2}{2g} - H}$

- 1c) (5 points) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon. Explain the relevance of your answer.

$h = y_m - y_b$
$\frac{dh}{dt} = 0 = V_{ym} - V_{yb}$
$V_{ym} = V_{yb} = V_a$

$\boxed{V_{ym} = V_a}$

The instant at which the marble reaches its greatest distance from the balloon it is no longer gaining on the balloon (getting closer) or receeding away (getting farther)

Note this is not the same instant the marble reaches its greatest height relative to the child! (when $\frac{dy_m}{dt} = V_{ym} = 0$)

- 1d) (5 points) How much time will elapse between the marble's first encounter with the balloon and it's last?

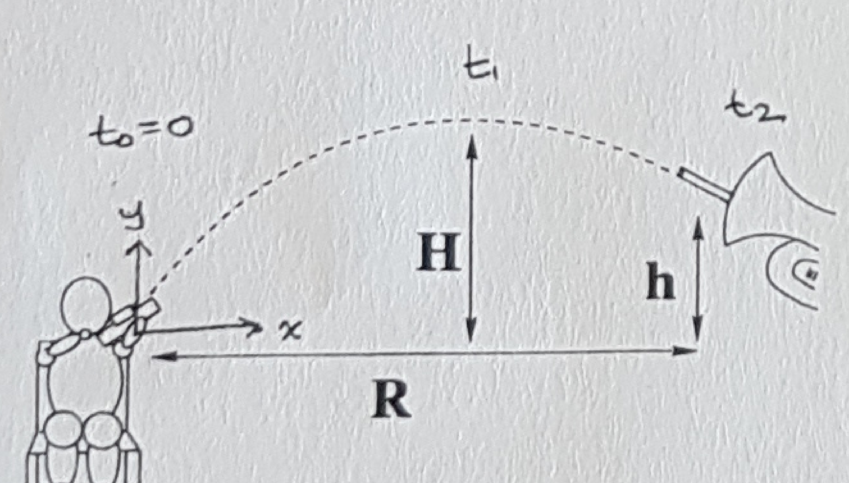
Take a look at the quadratic eqn we derived for $y_b = y_m$... the roots correspond to t_2 and t_4!

$t_4 - t_2 = (t_4 - t_1) - (t_2 - t_1)$

$t_4 - t_2 = \frac{V_b - V_a + \sqrt{(V_b-V_a)^2 - 2gH}}{g} - \frac{V_b - V_a - \sqrt{(V_b-V_a)^2 - 2gH}}{g}$

$\boxed{t_4 - t_2 = \frac{2\sqrt{(V_b-V_a)^2 - 2gH}}{g}}$

$x = V_{ox} t$
$y = V_{oy} t - \frac{1}{2}gt^2$
$V_y = V_{oy} - gt$
$V_y^2 = V_{oy}^2 - 2gy$



2) During a particularly lively solo the bow gets away from our virtuoso violinist. It rises to a maximum height H above the violin then descends into a nearby tuba. Assume the opening of the tuba lies a horizontal distance R from and a vertical height h above the violin and answer the following questions...

- 2a) (10 points) For how long was the bow in flight?

$\cancel{V_y^2} = V_{oy}^2 - 2gH$
$V_{oy} = \sqrt{2gH}$

$y_2 = V_{oy} t_2 - \frac{1}{2}gt_2^2$
$0 = -h + V_{oy} t_2 - \frac{1}{2}gt_2^2$
$t_2 = \frac{V_{oy} \pm \sqrt{V_{oy}^2 - 2gh}}{g}$

we're interested in the larger value of t_2: why?

$\boxed{t_2 = \sqrt{\frac{2H}{g}}\left(1 + \sqrt{1 - h/H}\right)}$

- 2b) (10 points) With what speed did the bow leave the violin?

$x_2 = V_{ox} t_2$
$V_{ox} = R/t_2$ $\Rightarrow$ $V_{ox} = R\sqrt{\frac{g}{2H}} \frac{1}{1 + \sqrt{1 - h/H}}$

$V_o^2 = V_{ox}^2 + V_{oy}^2 = \frac{gR^2}{2H} \frac{1}{(1 + \sqrt{1-h/H})^2} + 2gH$
$V_o^2 = 2gH\left[\frac{R^2}{4H^2} \frac{1}{(1+\sqrt{1-h/H})^2} + 1\right]$

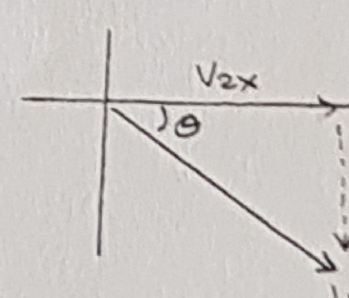
$\boxed{V_o = \sqrt{2gH} \cdot \sqrt{1 + \left\{\frac{R}{2H} \frac{1}{(1+\sqrt{1-h/H})}\right\}^2}}$

That's ugly! sorry :)

- 2c) (10 points) At what angle with respect to the horizontal did the bow enter the tuba?

$V_{2x} = V_{ox} = R\sqrt{\frac{g}{2H}} \frac{1}{1 + \sqrt{1-h/H}}$

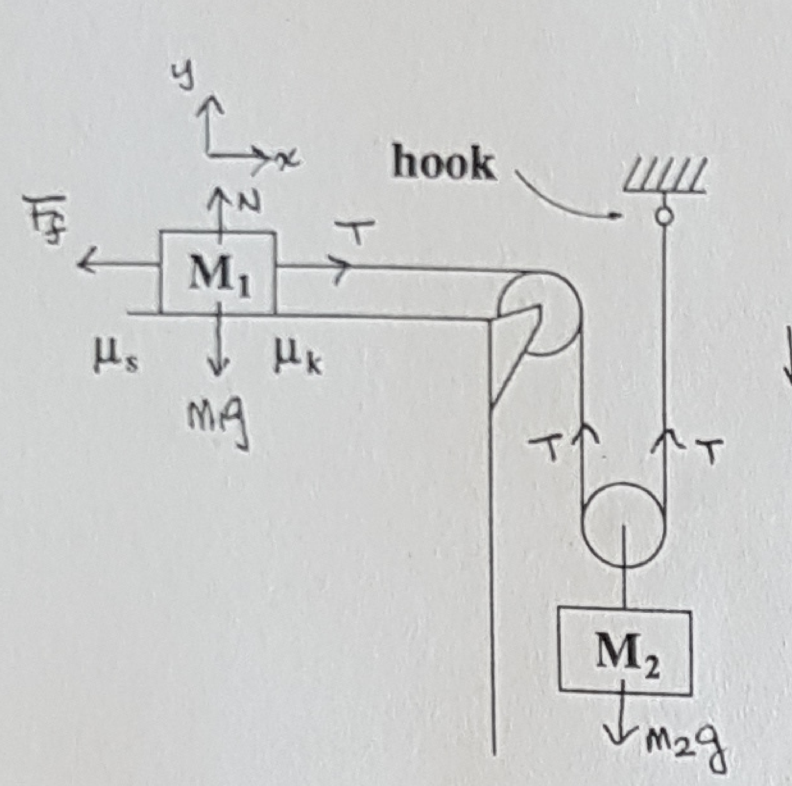
$V_{2y} = V_{oy} - gt_2$
$V_{2y} = \sqrt{2gH}\left(1 - 1 - \sqrt{1-h/H}\right)$
$V_{2y} = -\sqrt{2gH}\sqrt{1-h/H}$



$\tan\theta = \left|\frac{V_{2y}}{V_{2x}}\right|$

$\tan\theta = \left|\frac{V_{2y}}{V_{2x}}\right| = \sqrt{2gH}\sqrt{1-h/H} \frac{1}{R}\sqrt{\frac{2H}{g}}\left(1 + \sqrt{1-h/H}\right)$

$\boxed{\tan\theta = \frac{2H}{R}\left(\sqrt{1-h/H} + 1 - h/H\right)}$



if M_2 falls x
m_1 has to move $2x$

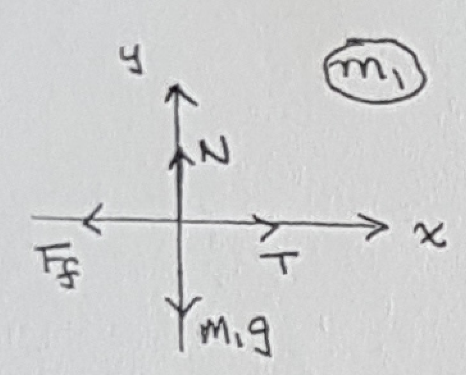
$x_1 = 2x_2$

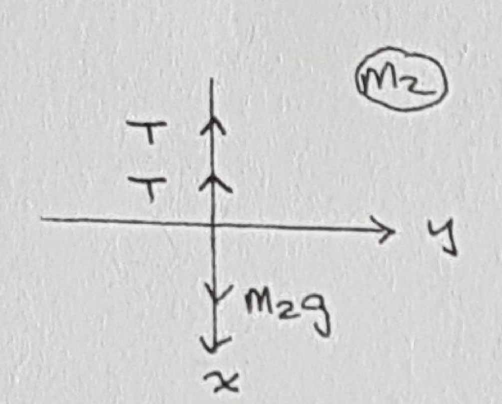
$\downarrow$

$a_{1x} = 2a_{2x}$

Consider the apparatus shown above. The coefficients of friction between block one and the table are both known, as are the masses M_1 and M_2.

- 3a) (5 points) Identify the object (or objects) you're interested in, and draw free-body diagrams for each.





- 3b) (10 points) Use Newton's laws to obtain equations that describe the dynamics of each of the objects of interest. Describe friction as F_f, and allow for acceleration of the body or bodies.

$$\boxed{\begin{aligned} &T - F_f = m_1 a_{1x} \\ &N - m_1 g = 0 \\ &m_2 g - 2T = m_2 a_{2x} \end{aligned}}$$

Also useful...

$a_{1x} = 2a_{2x}$

$F_K = \mu_K N$

$F_S \leq \mu_S N$

- 3c) (10 points) What is the acceleration of each block?

If the system is static, $a_{1x} = a_{2x} = 0$; otherwise...

$T - \mu_k m_1 g = 2m_1 a_{2x}$

$m_2 g - 2T = m_2 a_{2x}$

$(m_2 - 2\mu_k m_1) g = (4m_1 + m_2) a_{2x}$

$$\boxed{\begin{aligned} a_{1x} &= \frac{2(m_2 - 2\mu_k m_1)}{4m_1 + m_2} g \\ a_{2x} &= \frac{m_2 - 2\mu_k m_1}{4m_1 + m_2} g \end{aligned}}$$

- 3d) (5 points) How much force must the hook exert on the rope?

$N3 \Rightarrow F_{hook} = T = \frac{1}{2} m_2 g \left(1 - \frac{a_{2x}}{g}\right)$

$= \frac{1}{2} m_2 g \left(1 - \frac{m_2 - 2\mu_k m_1}{4m_1 + m_2}\right)$

$$\boxed{F_{hook} = \frac{m_1 m_2 g (2 + \mu_K)}{4m_1 + m_2}}$$