Gallauer Spring 16

Spring 16, Math 33A/1, Quiz 5 (Week 6): 10 minutes

Nan	
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Day \ T.A.	David	Casey	Adam
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

10

Problem 1 (10 points). Compute the orthogonal projection of $18 \cdot \vec{e}_1$ onto the subspace of $\mathbb{R}^5$ spanned by the following vectors:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \\ -2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

$\vec{u}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{\sqrt{1+1+1+1}}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix}$ ✓

$\vec{w}_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \\ -2 \end{bmatrix} - \left(\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \\ -2 \end{bmatrix}\right)\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix}$

$\vec{w}_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \\ -2 \end{bmatrix} - 4\begin{bmatrix} 1/2 \\ -1/2 \\ 0/2 \\ 1/2 \\ -1/2 \end{bmatrix}$

$\vec{w}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ ✓

$\vec{u}_2 = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \\ 0 \\ 0 \end{bmatrix}$ ✓

$\vec{w}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}\right)\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} - \left(\begin{bmatrix} 2/3 \\ 2/3 \\ 1/2 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}\right)\begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - 0\vec{u}_1 - 0\vec{u}_2$

$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$\vec{u}_3 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ 0/2 \\ -1/2 \\ 1/2 \end{bmatrix}$ ✓

$\text{proj}_V\left(\begin{bmatrix} 18 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right) = \underbrace{\left(\begin{bmatrix} 18 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix}\right)}_{9}\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} + \underbrace{\left(\begin{bmatrix} 18 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \\ 0 \\ 0 \end{bmatrix}\right)}_{12}\begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \\ 0 \\ 0 \end{bmatrix} + \underbrace{\left(\begin{bmatrix} 18 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ -1/2 \\ 1/2 \end{bmatrix}\right)}_{9}\begin{bmatrix} 1/2 \\ -1/2 \\ -9/2 \\ 1/2 \end{bmatrix}$

$= \begin{bmatrix} 9/2 \\ -9/2 \\ 0 \\ 9/2 \\ -9/2 \end{bmatrix} + \begin{bmatrix} 8 \\ 8 \\ 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 9/2 \\ -9/2 \\ 0/2 \\ -9/2 \\ 9/2 \end{bmatrix} = \boxed{\begin{bmatrix} 17 \\ -1 \\ 4 \\ 0 \\ 0 \end{bmatrix}}$ ✓