Gallauer Spring 16

Spring 16, Math 33A, Lecture 1, Quiz 3 (Week 3): 10 minutes

Day \ T.A.	David	Casey	Adam
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

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Problem 1 (5 points). Consider the linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$, $U : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T(\vec{x}) = \frac{1}{5}\begin{bmatrix} 5 & 2 \\ 10 & -11 \end{bmatrix} \cdot \vec{x}, \qquad U(\vec{x}) = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \vec{x}.$$

Find the matrix representing the composition $TU : \mathbb{R}^2 \to \mathbb{R}^2$ and interpret it geometrically. (I.e. is it a scaling, orthogonal projection, reflection, rotation, ...?)

Problem 2 (5 points). Is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 4 \\ 3 & -1 & -1 \end{bmatrix}$$

invertible? If so, find its inverse.

① $TU = \begin{bmatrix} 1 & 2/5 \\ 2 & -11/5 \end{bmatrix}\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 5 & 2 \\ 10 & -11 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 10+2 & 5+4 \\ 20-11 & 10-22 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 12 & 9 \\ 9 & -12 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 4/5 & 3/5 \\ 3/5 & -4/5 \end{bmatrix}}$

This is a $\boxed{\text{reflection}}$ ✓

② $\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 1 \end{array}\right] \div 2 \to \left[\begin{array}{ccc|ccc} 1 & 0 & .5 & .5 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -2(\mathrm{I}) \\ -3(\mathrm{I}) \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & .5 & .5 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & -1 & -2.5 & -1.5 & 0 & 1 \end{array}\right] +(\mathrm{II}) \to$

$\left[\begin{array}{ccc|ccc} 1 & 0 & .5 & .5 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & .5 & -2.5 & 1 & 1 \end{array}\right] \cdot 2 \to \left[\begin{array}{ccc|ccc} 1 & 0 & .5 & .5 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 2 \end{array}\right] \begin{array}{l} -\frac{1}{2}(\mathrm{III}) \\ -3(\mathrm{III}) \\ \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & 14 & -5 & -6 \\ 0 & 0 & 1 & -5 & 2 & 2 \end{array}\right]$

$\boxed{\text{Yes;}}$ $\boxed{A^{-1} = \begin{bmatrix} 3 & -1 & -1 \\ 14 & -5 & -6 \\ -5 & 2 & 2 \end{bmatrix}}$ ✓